In Class Problem Set 3

Problems:

1. Determine the number of vectors $(x_1, \ldots, x_n)$ such that each x_i is a positive integer and

$$\sum_{i=1}^{n} x_i \leq k$$

   where $k \geq n$.

2. **St. Petersburg paradox.** You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

3. The game of craps is played as follows: A player rolls two dice. If the sum of the dice is either a 2, 3, or 12, the player loses; if the sum is either a 7 or an 11, he or she wins. If the outcome is anything else, the player continues to roll the dice until he or she rolls either the initial outcome or a 7. If the 7 comes first, the player loses; whereas if the initial outcome reoccurs before the 7, the player wins. Compute the probability of a player winning at craps.

   Hint: Let E_i denote the event that the initial outcome is i and the player wins. The desired probability is $\sum_{i=2}^{12} \Pr(E_i)$, define the events $E_{i,n}$ to be the event that the initial sum is i and the player wins on the nth roll. Argue that $\Pr(E_i) = \sum_{n=1}^{\infty} \Pr(E_{i,n})$.

4. Let X be a random variable with probability density function

$$f(x) = \begin{cases} C\left(1 - x^2\right) & -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) What is the value of C?

   (b) What is the cumulative distribution of X?

5. Let X have the PDF

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda |x|},$$

   where λ is a positive scalar. Verify that f_X satisfies the normalization condition, and evaluate the mean and variance of X.