

## In Class Problem Set 4

### Problems:

1. Choose a number  $U$  from the unit interval  $[0, 1]$  with uniform distribution. Find the cumulative distribution and density for the random variables
  - (a)  $Y = U + 2$ ,
  - (b)  $Y = U^3$ .
2. Alvin throws darts at a circular target of radius  $r$  and is equally likely to hit any point in the target. Let  $X$  be the distance of Alvin's hit from the center.
  - (a) Find the PDF, the mean, and the variance of  $X$ .
  - (b) The target has an inner circle of radius  $t$ . If  $X \leq t$ , Alvin gets a score of  $S = 1/X$ . Otherwise his score is  $S = 0$ . Find the CDF of  $S$ . Is  $S$  a continuous random variable?
3. (a) A fire station is to be located along a road of length  $A$ ,  $A < \infty$ . If fires will occur at points uniformly chosen on  $(0, A)$ , where should the station be located so as to minimize the expected distance from the fire? That is, choose  $a$  so as to
$$\min E[|X - a|]$$
when  $X$  is uniformly distributed over  $(0, A)$ .  
(b) Now suppose that the road is infinite length – stretching from point 0 outward to  $\infty$ . If the distance of a fire from point 0 is exponentially distributed with rate  $\lambda$ , where should the fire station now be located? That is, we want to minimize  $E[|X - a|]$ , where  $x$  is now exponential with rate  $\lambda$ .

4. Consider a triangle and a point chosen within the triangle according to the uniform probability law. Let  $X$  be the distance from the point to the base of the triangle. Given the height of the triangle, find the CDF and the PDF of  $X$ .