ECE 586: Vector Space Methods
Lecture 4: Frequently Asked Questions

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Here we give a list of questions, and their answers, that were submitted by students after watching the flip video.

1 Relations and Functions

Do we always mean that the relation between x, y is reflexive, symmetric and transitive when we use $\sim$? In general, the symbol $\sim$ is often used for equivalence relations but not always. Thus, one should say specifically that $\sim$ is an *equivalence* relation in order for it to have those properties.

Could you talk more about the inverse image and its use? For a function $f : X \to Y$, the inverse image of a set $B \subseteq Y$ is the set of all x such that $f(x) \in B$ – more compactly, $f^{-1}(B) \triangleq \{x \in X | f(x) \in B\}$. In advanced math classes, it is used to define a number of things. For example, a function $f : X \to Y$ is continuous if and only if the inverse image of any open set in Y is open in X. In advanced probability, random variables are functions $X : \Omega \to \mathbb{R}$ and it is used to compute the probability $\Pr(X \in A) = \mathbb{P}(X^{-1}(A))$ where $\mathbb{P}$ gives the probability for subsets of the sample space Ω.

What is one example in which a function is not surjective? Consider the function $f(x) = x^2+1$, and define the domain to be $X \triangleq \mathbb{R}$ and the codomain to be $Y \triangleq [0, \infty)$. The function is not surjective since its range is $[1, \infty)$, which is not the same as Y.

How can A and B be disjoint, but $f(B)$ is a subset of $f(A)$? Perhaps a concrete example would help to clarify this: Consider $B = [-2, -1]$ and $A = [0, 4]$ and the function $f(x) = x^2$. Note that $f(B) = [1, 4]$ and $f(A) = [0, 16]$, so $f(B)$ is a subset of $f(A)$ even though A and B are disjoint.

The name "equivalence relation" implies that such relations are similar in function to the "=" relation we commonly use. Are equivalence relations always used in such a way, or is the category more broad than that? Equivalence relations can be understood as an abstraction of equality. Let's say we define 2 integers as being equivalent if they have the same remainder after division by 3. If that's the case, then 4 and 19 are equivalent by my definition of equivalence (they both have remainder 1 by division by 3). Note that 4 and 19 are not equal, but they are equivalent per our specific definition of equivalence.

Can we have some more examples in our tutorial? If you're looking for examples of relations, then I suggest Chapter 5 of the textbook Proofs and Fundamentals (PAF) whose link is available on the course website. If your question is more general, then you can inquire about a particular subject and we can suggest resources with more examples.

When can we access the slides shown in this video? The slides used to make the video (modulo small corrections) are always posted to the website when the video is posted. If you're having trouble finding or accessing these, please e-mail me.