

# ELECTRICAL AND COMPUTER ENGINEERING COURSE SYLLABUS

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<b>Room Class/Disc:</b>	Teer 115 / Hudson 125	<b>Time Class/Disc:</b>	TTH 11:45-1:00 / M 11:45-1:00

<b>Course Name:</b>	<b>ECE 586</b>
<b>Course Title:</b>	<b>Vector Space Methods with Applications</b>
<b>Prerequisite(s):</b>	Undergraduate Linear Algebra
<b>Required Text(s):</b>	<i>Engineering Fundamentals</i> (EF) by Chamberland and Pfister <i>Linear Algebra Done Right</i> (LADR) by Axler <i>Proofs and Fundamentals</i> (PAF) by Bloch
<b>Other Text(s):</b>	<i>Mathematical Methods and Algorithms for Sig. Proc.</i> (MMA) by Moon and Stirling <i>Topology</i> (TOP) by Munkres <i>Linear Algebra</i> (LA) by Hoffman and Kunze <i>Optimization by Vector Space Methods</i> (OVSM) by Luenberger

## Course Objectives:

1. Explore fundamental concepts of logic including sets, axioms, quantifiers, implications, necessary and sufficient conditions. Illustrate valid proof methods such as proofs by contradiction, proofs by contrapositive, the principle of mathematical induction and counter examples.
2. Establish basic notions of topology in the context of metric spaces. Study formal definitions for open sets, closed sets, convergence, limit points, completeness and continuous functions.
3. Review linear algebra, combinations of vectors, independence, bases and dimensions. Distinguish between vector spaces, normed spaces and inner-product spaces. Discuss the fundamental subspaces associated with a matrix. Introduce the projection theorem and illustrate its applications.
4. Apply vector space methods to signal processing, machine learning, optimization, least-squares filtering, and minimum mean-square error estimation. Acquire the ability to recognize, formulate and solve pertinent engineering problems using vector space methods and Hilbert spaces.
5. Review basic optimization theory from a vector space perspective. Discuss constrained optimization, the Lagrangian approach, and duality.
6. Introduce the notions of linear operators, fundamental subspaces, matrix representations, inverses and pseudoinverses. Examine the properties of characteristic polynomials, eigenvalues, eigenvectors and eigenfunctions. Develop the theory of the singular value decomposition.
7. Gain proficiency at using high-level scientific programming in Python (or equivalent).
8. Engage the student in an active learning experience. Prepare the student to become an active contributor to the common body of knowledge.

## Rules and Guidelines:

The class shall follow all established policies of Duke University.

## Course Topics and Hours:

Unit	Topics	Hours
1	Logic and Set Theory	4.5
2	Metric Spaces / Topology	6
3	Vector Spaces	4.5
4	Norms and Inner Products	6
5	Linear Operators	3
6	Projections and Applications	6
7	Singular Value Decomposition	3
8	Convex Optimization	3
	<b>Total Hours</b>	<b>36</b>

## Student Evaluation:

Flipped Questions	3%	Review video or slides and answer questions before lecture
Homework / Quizzes	25%	8-10 assignments throughout the semester
Midterm Exams	30%	Two equally weighted midterm exams
Final Exam	20%	Final exam on <b>Friday, December 15th from 2pm - 5pm</b>
Mini-Projects	22%	Use the tools acquired in this class to solve engineering problems

## Course Outline

1. Mathematical Review
  - (a) Logic
  - (b) Set Theory
  - (c) Functions
2. Metric Spaces and Topology
  - (a) Metric Spaces
  - (b) Introduction to Topology
  - (c) Continuity and Completeness
  - (d) Contraction Mapping Theorem
3. Linear Algebra
  - (a) Fields, Matrices, and Vector Spaces
  - (b) Norms and Inner Products
  - (c) Orthogonal Projections
  - (d) Banach and Hilbert Spaces
  - (e) Linear Operators / Matrix Norms
4. Representations and Approximations
  - (a) Projections in Hilbert Spaces
  - (b) Matrix Representations
  - (c) Applications and Examples
  - (d) Projections onto Convex Sets
5. Linear Transformations and Operators
  - (a) Linear Transformations and Operator Norms
  - (b) Fundamental Subspaces and Pseudoinverses
  - (c) Singular Value Decomposition
  - (d) Eigenvalues and Eigenvectors
6. Optimization
  - (a) Convex Functions
  - (b) Constrained Optimization
  - (c) Karush-Kuhn-Tucker Conditions
  - (d) \*Lagrangian Duality

Date	Topic	Reading	Assignment	Slides
08/28/23	Propositional Logic	EF: Section 1.1-1.2.1	HW1	0 & 1
08/29/23	Predicate Logic	EF: Section 1.2.2-1.3		2
08/31/23	Set Theory	EF: Section 1.4	HW2	3
09/04/23	Labor Day			
09/05/23	discussion makeup		Tue @ Teer 115	
09/07/23	canceled		Add/Drop Ends Fri	
09/12/23	Relations and Functions	EF: Section 1.4-1.5		4
09/14/23	Metric Spaces and Topology	EF: Section 2.1-2.1.2	HW3 - long deadline	5
09/18/23	Continuity, $\mathbb{R}$ , and functions	EF: Section 2.1.3	Mon @ HH 125	6
09/19/23	Completeness / Contraction	EF: Section 2.1.3		7
09/21/23	Compactness	EF: Section 2.1.4-2.1.5		8
09/26/23	Fields and Matrices	EF: Section 3.1-3.2	Practice Midterm	9
09/28/23	discussion makeup	Review Session	Thur @ Teer 115	
10/03/23	Vector Spaces	EF: Section 3.3-3.3.3	HW4	10
10/05/23	Midterm 1			
10/10/23	Linear Transforms	EF: Section 3.4		11
10/12/23	Markov Chains	Project Handout	HW5 / Markov Project	12
10/13/23	Fall Break Begins		Ends 10/18/23	
10/19/23	Normed Vector Spaces	EF: Section 3.5		13
10/24/23	Operator Norms	EF Section 6.3		14
10/26/23	Inner-Product Spaces 1	EF: Section 3.6-3.7	HW6 – Due 11/02	15
10/31/23	Inner-Product Spaces 2	EF: Section 3.6-3.7		15
11/02/23	Derivatives / Optimization	EF Section 5.1-5.2	Practice Midterm	16
11/07/23	Best Approximation Theorem	EF: Section 4.1	Review Session	17
11/09/23	Best Approximation Formulas	EF: Section 4.2-4.3	HW7 / LSQ Project	18
11/14/23	Midterm 2			
11/16/23	Four Fundamental Subspaces	Paper on Website		19
11/21/23	SVD and Pseudo-Inverses	EF Section 9.1-9.3	HW8	20
11/23/23	Thanksgiving Break			
11/28/23	Projection onto Convex Sets	EF Section 4.6	PCA / Tensor Project	21
11/30/23	Alternating Projections	Project Handout	HW9	22
12/05/23	Convex Optimization	EF Section 5.3-5.4	AltProj Project	23