Electrical and Computer Engineering Course Syllabus

Instructor:	Prof. Henry Pfister	**E-mail:**	`henry.pfister@duke.edu`
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Class Room:	FCIEMAS Schiciano B	**Class Time:**	MW 10:15 - 11:30 PM

Course Name: **ECE 586**

Course Title: **Vector Space Methods with Applications**

Prerequisite(s): Undergraduate Linear Algebra

Required Text(s): *Engineering Fundamentals* (EF) by Chamberland and Pfister
Linear Algebra Done Right (LADR) by Axler
Proofs and Fundamentals (PAF) by Bloch

Other Text(s): *Mathematical Methods and Algorithms for Signal Processing* (MMA) by Moon and Stirling
Topology (TOP) by Munkres
Linear Algebra (LA) by Hoffman and Kunze
Optimization by Vector Space Methods (OVSM) by Luenberger

Course Objectives:

1. Explore fundamental concepts of logic including sets, axioms, quantifiers, implications, necessary and sufficient conditions. Illustrate valid proof methods such as proofs by contradiction, proofs by contrapositive, the principle of mathematical induction and counter examples.
2. Establish basic notions of topology in the context of metric spaces. Study formal definitions for open sets, closed sets, convergence, limit points, completeness and continuous functions.
3. Review linear algebra, combinations of vectors, independence, bases and dimensions. Distinguish between vector spaces, normed spaces and inner-product spaces. Discuss the fundamental subspaces associated with a matrix. Introduce the projection theorem and illustrate its applications.
4. Apply vector space methods to signal processing, machine learning, optimization, least-squares filtering, and minimum mean-square error estimation. Acquire the ability to recognize, formulate and solve pertinent engineering problems using vector space methods and Hilbert spaces.
5. Review basic optimization theory from a vector space perspective. Discuss constrained optimization, the Lagrangian approach, and duality.
6. Introduce the notions of linear operators, fundamental subspaces, matrix representations, inverses and pseudoinverses. Examine the properties of characteristic polynomials, eigenvalues, eigenvectors and eigenfunctions. Develop the theory of the singular value decomposition.
7. Gain proficiency at using high-level scientific programming such as Python and Matlab.
8. Engage the student in an active learning experience. Prepare the student to become an active contributor to the common body of knowledge.

Rules and Guidelines:

The class shall follow all established policies of Duke University.

Course Topics and Hours:

Unit	Topics	Hours
1	Logic and Set Theory	4.5
2	Metric Spaces / Topology	6
3	Vector Spaces	4.5
4	Norms and Inner Products	6
5	Linear Operators	3
6	Projections and Applications	6
7	Singular Value Decomposition	3
8	Convex Optimization	3
	Total Hours	36

Student Evaluation:

Flip Video Questions	10%	Watch flip video and submit questions before lecture
Homework / Quizzes	26%	8-10 assignments throughout the semester
Midterm Exams	26%	Two equally weighted midterm exams
Final Exam	16%	Final exam on **Wednesday, December 8th 9 AM - Noon**
Mini-Projects	22%	Use the tools acquired in this class to solve engineering problems

Course Outline

1. Mathematical Review
   (a) Logic
   (b) Set Theory
   (c) Functions
2. Metric Spaces and Topology
   (a) Metric Spaces
   (b) Introduction to Topology
   (c) Continuity and Completeness
   (d) Contraction Mapping Theorem
3. Linear Algebra
   (a) Fields, Matrices, and Vector Spaces
   (b) Norms and Inner Products
   (c) Orthogonal Projections
   (d) Banach and Hilbert Spaces
   (e) Linear Operators / Matrix Norms
4. Representations and Approximations
   (a) Projections in Hilbert Spaces
   (b) Matrix Representations
   (c) Applications and Examples
   (d) Projections onto Convex Sets
5. Linear Transformations and Operators
   (a) Linear Transformations and Operator Norms
   (b) Fundamental Subspaces and Pseudoinverses
   (c) Singular Value Decomposition
   (d) Eigenvalues and Eigenvectors
6. Optimization
   (a) Convex Functions
   (b) Constrained Optimization
   (c) Karush-Kuhn-Tucker Conditions
   (d) *Lagrangian Duality

Schedule:

Date	Topic	Reading	Assignment	Video
8/23/21	Propositional Logic	EF: Section 1.1-1.2.1	HW1	0 and 1
8/25/21	Predicate Logic	EF: Section 1.2.2-1.3		2
8/30/21	Set Theory	EF: Section 1.4	HW2	3
9/1/21	Relations and Functions	EF: Section 1.4-1.5	Drop/Add Ends Fri	4
9/6/21	Metric Spaces and Topology	EF: Section 2.1-2.1.2	HW3 - long deadline	5
9/8/21	Topology / Completeness	EF: Section 2.1.3		6
9/13/21	Contraction / Compactness	EF: Section 2.1.3		7
9/15/21	Compactness / Functions	EF: Section 2.1.4-2.1.5	Practice Midterm	8
9/20/21	Catchup and Review		Midterm	
9/22/21	Fields and Matrices	EF: Section 3.1-3.2		9
9/27/21	Vector Spaces	EF: Section 3.3-3.3.3	HW4	10
9/29/21	Linear Transforms	EF: Section 3.4		11
10/6/21	Markov Chains	Project Handout	HW5	12
10/11/21	Normed Vector Spaces	EF: Section 3.5	Markov Chain Project	13
10/13/21	Operator Norms	EF Section 6.3	HW6	14
10/18/21	Inner-Product Spaces	EF: Section 3.6-3.7		15
10/20/21	Derivatives / Optimization	EF Section 5.1-5.2	Practice Midterm	16
10/25/21	Catchup and Review		Midterm	
10/27/21	Best Approximation Theorem	EF: Section 4.1		17
11/1/21	Best Approximation Formulas	EF: Section 4.1	HW7 / LSQ Project	18
11/3/21	Orthogonal Projection	EF: Section 4.2-4.3	W-Drop Ends Fri	19
11/8/21	Four Fundamental Subspaces	Paper on Website	HW8	20
11/10/21	SVD and Pseudo-Inverses	EF Section 9.1-9.3	EigSVD Project	21
11/15/21	Projection onto Convex Sets	EF Section 4.6	HW9	22
11/17/21	Alternating Projections	Project Handout	AltProj Project	23
11/22/21	Convex Optimization	EF Section 5.3-5.4		24