

ECE 586: Vector Space Methods
Lecture 1 Flip Video: Propositional Logic

Henry D. Pfister
Duke University

1: Logic

- **Statement** (or proposition)
 - An assertion that is true or false, but not both
 - Ex. “This video was recorded for a course at Duke University” ✓
 - Ex. “The real number $\sqrt{2}$ is rational” ✗
 - Ex. “Wash your hands before dinner” not a statement!
- Combining statements
 - One can also form new statements from old ones using English expressions: and; or; not; if, then; if and only if
 - Ex. “Duke is located in Durham, NC or all real numbers are rational”
 - Note: symbols P, Q, R, \dots used to denote abstract statements

1.1: Basic Definitions

- Conjunction of P, Q (i.e., P AND Q)
 - Binary operation on logical propositions (denoted $P \wedge Q$) that:
is true only if both statements are true, and is false otherwise
- Disjunction of P, Q (i.e., P OR Q)
 - Binary operation on logical propositions (denoted $P \vee Q$) that:
is true if either statement is true, and false otherwise
- Negation of P (i.e., NOT P)
 - Unary operation on a logical proposition (denoted $\neg P$) that:
is true if the statement is false, and is false otherwise
- Truth Tables

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

1.2: Conditional Statements (1)

- Conditional Connective $P \rightarrow Q$ (i.e., if P , then Q)
 - Binary operation on logical propositions that:
 - is false if P is true and Q is false, and is true otherwise

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- P is called the **antecedent** and Q is called the **consequent**
- Meaning
 - When P is false, some people guess the truth value should be undefined. But, the values shown above are **universally accepted** in logic
 - Intuitively, one can think of $P \rightarrow Q$ as a **promise that Q is true whenever P is true**. When P is false, the promise is kept by default
 - Ex. Suppose a friend promises “**if it is sunny tomorrow, I will ride my bike**”. We say their statement is **true if they keep their promise**. So, if it rains and they don't ride their bike, then most people would agree they've kept their promise.

1.2: Conditional Statements (2)

- Biconditional $P \leftrightarrow Q$ (i.e., P if and only if Q)
 - Binary operation on logical propositions that is:
true if P and Q have the same truth value, and false otherwise

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- Identical truth values as: $(P \rightarrow Q) \wedge (Q \rightarrow P)$
 - Ex. "John graduates this term if and only if he passes this class"
- Variations of the conditional connective $P \rightarrow Q$
 - The **converse** of $P \rightarrow Q$ is the statement $Q \rightarrow P$
 - The **contrapositive** of $P \rightarrow Q$ is the statement $\neg Q \rightarrow \neg P$

1.2: Compound Statements

- It is also useful to consider **compound logical statements** like

$$(P \rightarrow R) \wedge (Q \vee \neg R)$$

- In general, there is a mechanical way to compute the truth table:

P	Q	R	$(P \rightarrow R)$	\wedge	$(Q \vee \neg R)$
T	T	T	T	T	F
T	T	F	F	F	T
T	F	T	T	F	F
T	F	F	F	F	T
F	T	T	T	T	F
F	T	F	F	T	T
F	F	T	T	F	F
F	F	F	F	F	T
			1	5	2
				7	3
					6
					4

1.2: Meta Statements

- A meta statement is a logical statement about logical statements
- Ex. A **tautology** is a compound statement (e.g., $R(P, Q)$) that
 - is true for every valuation of its propositional variables
 - Ex. $R(P, Q) = P \vee \neg P \vee Q$ is a tautology
- Ex. A **contradiction** is a compound statement (e.g., $R(P, Q)$) that
 - is false for every valuation of its propositional variables
 - Ex. $R(P, Q) = P \wedge \neg P \wedge Q$ is a contradiction
- An **implication** $R \Rightarrow S$ is a meta statement (e.g., for $R(P, Q), S(P, Q)$)
 - that states $R(P, Q) \rightarrow S(P, Q)$ is always true (i.e., $R \rightarrow S$ is a tautology)
 - Ex. $(P \rightarrow Q) \wedge P \Rightarrow Q$ because $(P \rightarrow Q) \wedge P \rightarrow Q$ is a tautology
- An **equivalence** $R \Leftrightarrow S$ is a meta statement (e.g., for $R(P, Q), S(P, Q)$)
 - that states $R(P, Q) \leftrightarrow S(P, Q)$ is always true (i.e., $R \leftrightarrow S$ is a tautology)
 - Ex. $P \rightarrow Q \Leftrightarrow \neg P \vee Q$ because $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$ is a tautology

- To continue studying after this video –
 - Try the suggested reading: Course Notes EF 1-1.2.2
 - Or the optional reading: PAF 1.1-2.6
 - Also, look at the problems in Assignment 1