ECE 586: Vector Space Methods
Lecture 1 Flip Video: Propositional Logic

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1: Logic

- Statement (or proposition)
  - A declarative sentence that is true or false, but not both
  - Ex. "This video was recorded for a course at Duke University" ✓
  - Ex. "The real number $\sqrt{2}$ is rational" ✗
  - Ex. "Wash your hands before dinner" not a statement!
- Combining statements
  - One can also form new statements from old ones using English expressions: and; or; not; if, then; if and only if
  - Ex. "Duke is located in Durham, NC or all real numbers are rational"
  - Note: symbols $P, Q, R, \ldots$ used to denote abstract statements

1.1: Basic Definitions

- Conjunction of P, Q (i.e., P AND Q)
  - Binary operation on logical propositions (denoted $P \wedge Q$) that: is true only if both statements are true, and is false otherwise
- Disjunction of P, Q (i.e., P OR Q)
  - Binary operation on logical propositions (denoted $P \vee Q$) that: is true if either statement is true, and false otherwise
- Negation of P (i.e., NOT P)
  - Unary operation on a logical proposition (denoted $\neg P$) that: is true if the statement is false, and is false otherwise
- Truth Tables

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	$\neg P$
T	F
F	T

1.2: Conditional Statements (1)

- Conditional Connective $P \to Q$ (i.e., if P, then Q)
  - Binary operation on logical propositions that:
    is false if P is true and Q is false, and is true otherwise

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

  - P is called the antecedent and Q is called the consequent
- Meaning
  - When P is false, some people guess the truth value should be undefined. But, the values shown above are universally accepted in logic
  - Intuitively, one can think of $P \to Q$ as a promise that Q is true whenever P is true. When P is false, the promise is kept by default
  - Ex. Suppose a friend promises "if it is sunny tomorrow, I will ride my bike". We say their statement is true if they keep their promise. So, if it rains and they don't ride their bike, then most people would agree they've kept their promise.

1.2: Conditional Statements (2)

- Biconditional $P \leftrightarrow Q$ (i.e., P if and only if Q)
  - Binary operation on logical propositions that is:
    true if P and Q have the same truth value, and false otherwise

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

  - Identical truth values as: $(P \rightarrow Q) \wedge (Q \rightarrow P)$
  - Ex. "John graduates this term if and only if he passes this class"
- Variations of the conditional connective $P \rightarrow Q$
  - The converse of $P \rightarrow Q$ is the statement $Q \rightarrow P$
  - The contrapositive of $P \rightarrow Q$ is the statement $\neg Q \rightarrow \neg P$

1.2: Compound Statements

- It is also useful to consider compound logical statements like

$$(P \to R) \land (Q \lor \neg R)$$

- In general, there is a mechanical way to compute the truth table:

P	Q	R	$(P$	$\to$	$R)$	$\land$	$(Q$	$\lor$	$\neg R)$
T	T	T	T	T	T	T	T	T	F
T	T	F	T	F	F	F	T	T	T
T	F	T	T	T	T	F	F	F	F
T	F	F	T	F	F	F	F	T	T
F	T	T	F	T	T	T	T	T	F
F	T	F	F	T	F	T	T	T	T
F	F	T	F	T	T	F	F	F	F
F	F	F	F	T	F	T	F	T	T
			1	5	2	7	3	6	4

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1.2: Meta Statements

- A meta statement is a logical statement about logical statements
- Ex. A tautology is a compound statement (e.g., $R(P, Q)$) that
  - is true for every valuation of its propositional variables
  - Ex. $R(P, Q) = P \vee \neg P \vee Q$ is a tautology
- Ex. A contradiction is a compound statement (e.g., $R(P, Q)$) that
  - is false for every valuation of its propositional variables
  - Ex. $R(P, Q) = P \wedge \neg P \wedge Q$ is a contradiction
- An implication $R \Rightarrow S$ is a meta statement (e.g., for $R(P, Q), S(P, Q)$)
  - that states $R(P, Q) \to S(P, Q)$ is always true (i.e., $R \to S$ is a tautology)
  - Ex. $(P \to Q) \wedge P \Rightarrow Q$ because $(P \to Q) \wedge P \to Q$ is a tautology
- An equivalence $R \Leftrightarrow S$ is a meta statement (e.g., for $R(P, Q), S(P, Q)$)
  - that states $R(P, Q) \leftrightarrow S(P, Q)$ is always true (i.e., $R \leftrightarrow S$ is a tautology)
  - Ex. $P \to Q \Leftrightarrow \neg P \vee Q$ because $(P \to Q) \leftrightarrow (\neg P \vee Q)$ is a tautology

Next Steps

- To continue studying after this video –
  - Try the suggested reading: Course Notes EF 1-1.2.2
  - Or the optional reading: PAF 1.1-2.6
  - Also, look at the problems in Assignment 1