Electrical and Computer Engineering Course Syllabus

Instructor:	Prof. Henry Pfister	**E-mail:**	henry.pfister@duke.edu
Office / Hour:	305 Gross Hall / TH 2:00-3:00 PM	**Phone:**	(919) 660-5288
Class Room:	Hudson Hall 218	**Class Time:**	MW 1:25 - 2:40 PM

Course Name: **ECE 590.06**

Course Title: **Vector Space Methods with Applications**

Prerequisite(s): Undergraduate Linear Algebra

Required Text(s): *Linear Algebra Done Right* (LADR) by Axler
Proofs and Fundamentals (PAF) by Bloch

Other Text(s): *Mathematical Methods and Algorithms for Signal Processing* (MMA) by Moon and Stirling
Topology (TOP) by Munkres
Linear Algebra (LA) by Hoffman and Kunze
Optimization by Vector Space Methods (OVSM) by Luenberger

Course Objectives:

1. Explore fundamental concepts of logic including sets, axioms, quantifiers, implications, necessary and sufficient conditions. Illustrate valid proof methods such as proofs by contradiction, proofs by contrapositive, the principle of mathematical induction and counter examples.
2. Establish basic notions of topology in the context of metric spaces. Study formal definitions for open sets, closed sets, convergence, limit points, completeness and continuous functions.
3. Review linear algebra, combinations of vectors, independence, bases and dimensions. Distinguish between vector spaces, normed spaces and inner-product spaces. Discuss the fundamental subspaces associated with a matrix. Introduce the projection theorem and illustrate its applications.
4. Apply vector space methods to signal processing, optimization, least-squares filtering, and minimum mean-square error estimation. Acquire the ability to recognize, formulate and solve pertinent engineering problems using vector space methods and Hilbert spaces.
5. Review basic optimization theory from a vector space perspective. Discuss constrained optimization, the Lagrangian approach, and duality.
6. Introduce the notions of linear operators, fundamental subspaces, matrix representations, inverses and pseudoinverses. Examine the properties of characteristic polynomials, eigenvalues, eigenvectors and eigenfunctions. Develop the theory of the singular value decomposition.
7. Gain proficiency at using high-level programming languages such as Matlab and Mathematica.
8. Engage the student in an active learning experience. Expose the student to search engines, scholastic resources, research tools, indexes and databases. Prepare the student to become an active contributor to the common body of knowledge.

Course Topics and Hours:

Unit	Topics	Hours
1	Logic and Set Theory	4.5
2	Topology	4.5
3	Linear Algebra	6
4	Projections	4.5
5	Convex Optimization	3
6	Linear Operators	3
7	Matrix Factorizations	3
8	Canonical Forms	4.5
9	Singular Value Decomposition	3
	Total Hours	36

Student Evaluation:

Homework / Quizzes	24%	Roughly 10 assignments throughout the semester
Midterm Exams	36%	Two equally weighted midterm exams
Final Exam	24%	Final exam on **Sunday, December 18th, 2-5 PM**
Mini-Projects	16%	Use the tools acquired in this class to solve an engineering problem

Rules and Guidelines:

The class shall follow all established policies of Duke University.

Course Outline

1. Mathematical Review
   (a) Logic
   (b) Set Theory
   (c) Functions
2. Metric Spaces and Topology
   (a) Metric Spaces
   (b) Introduction to Topology
   (c) Continuity and Completeness
   (d) Contraction Mapping Theorem
3. Linear Algebra
   (a) Fields, Matrices and Vector Spaces
   (b) Norms and Inner Products
   (c) Orthogonal Projections
   (d) Banach and Hilbert Spaces
4. Representations and Approximations
   (a) Projections in Hilbert Spaces
   (b) Matrix Representations
   (c) Applications and Examples
   (d) Projections onto Convex Sets
5. Optimization
   (a) Convex Functions
   (b) Constrained Optimization
   (c) Karush-Kuhn-Tucker Conditions
   (d) Lagrangian Duality
6. Linear Transformations and Operators
   (a) Linear Transformations and Operator Norms
   (b) Linear Functionals and Adjoints
   (c) Fundamental Subspaces and Pseudoinverses
7. Matrix Factorizations
   (a) Perturbation Bounds
   (b) LU, LDLT, and Cholesky Decomposition
8. Canonical Forms
   (a) Eigenvalues and Eigenvectors
   (b) The Jordan Form
9. Singular Value Decomposition