ECEN 455 Lab 1: Simulation of Random Events

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Overview:

Many systems designed by electrical engineers involve elements that either have inherent randomness or cannot be characterized precisely. One example is that amplifiers in communication systems are subject to thermal noise caused by molecular vibrations. Another is that the fabrication of CMOS circuits is affected by very small shifts in the mask and imperfections in the silicon. While the effect of these impairments might be exactly computable, it is more effective to treat them as random quantities and evaluate the average number of parts (on a silicon wafer) that meet the performance specifications.

Before building such a system, one generally likes estimate how that sytem will perform. To do this, engineers typically run simulations of the system using randomly chosen values for the unknown quantities. By repeating the simulations multiple times, one obtains a "Monte Carlo" estimate of the system performance. In this lab, you will use MATLAB to perform a number experiments involving probability. This will allow you to develop a hands-on feel for the simulation of random events.

Exercises:

1. **Birthday Problem:** This problem uses combinatorics (study of counting) for the purpose of calculating probability. A probability class has N students enrolled. What is the probability that at least two of the students have the same birthday? Assume that each student in the class is equally likely to be born on any day of the year. The MATLAB code for solving this probability is

```
event = zeros(10000,1);
% initialize to no successful event
N = 10; % No. of students
for ntrial = 1:10000
for i = 1:N
x(i,1) = ceil(365*rand(1,1));
% chooses birthdays at random
% (ceil rounds up to the nearest integer)
end
y = sort(x);
% arranges birthdays in ascending order
z = y(2:N) - y(1:(N-1)); % compares succesive birthdays
w = find(z==0);
% flags same birthdays
```

```
if length(w) > 0
event(ntrial) = 1; % event occurs if one or more birthdays is same
end
end
prob = sum(event)/10000;
```

This code calculates the probability for 10 students. You are supposed to do the following:

(a) Plot this probablity for total number of students ranging from 1-50. If you feel the iterations take a long time to run, reduce the total no. of iterations.

(b) For how many students is the probability close to 0.5? Does the answer seem intuitive to you? Explain.

(c) Also give the mathematical expression for this probability for N students.

2. **Bernoulli Sequence:** Any sequence of M independent subexperiments with each subexperiment producing two possible outcomes is called a Bernoulli sequence.

   Binomial Probability Law: The binomial probability law is used for determining the probability of k successes among M independent Bernoulli trials. The probability can be calculated using the formula

$$\Pr[k] = \binom{M}{k} p^k (1-p)^{M-k} \qquad k = 0, 1, \ldots, M,$$

   where p is the probability of success. Write a MATLAB code and plot the binomial probability law for

   (a) $M = 10$ and $p = 0.5$,

   (b) $M = 20$ and $p = 0.7$.

   Geometric Probability Law: This law helps in determining the probability that the first sucess will occur at trial k.

$$\Pr[k] = (1-p)^{k-1} p \qquad k = 1, 2, \ldots,$$

   For k up to 10, plot the geometric probability law for

   (a) $p = 0.25$,

   (b) $p = 0.5$.

   Telephone Calling Example: A fax machine dials a phone number that is typically busy 80% of the time. The machine dials it every 5 minutes until the line is clear and the fax is transmitted. What is the probability that the fax machine will have to dial the number 9 times?

3. **Approximation of the Binomial PMF by Poisson PMF:** This exercise will help you understand that the binomial and Poisson PMFs are related to each other under certain conditions. This relationship helps to explain why the Poisson PMF is used in various applications, primarily in traffic modelling. The relationship is as follows. For a binomial PMF,

suppose we let $M \to \infty$ and $p \to 0$ such that the product $Mp \to \lambda$ is a constant. Then, the sequence of binomial distributions converges a Poisson distribution with parameter λ gives by

$$\Pr[k] = \frac{\lambda^k}{k!} e^{-\lambda}.$$

Using Matlab plot the Binomial PMF for $M = 20, 40, 60$ and $Mp = 3$ and its approximation using Poisson PMF.

4. **Mean and Variance of Linear Functions of Random Variables:** Generate a linear function of two random variables of the form $Z = aX + bY$, where a and b are constants. For simplicity, we assume that the random variables X and Y are binomial with $p = 0.5$ and $M = 10$. Therefore, they can be obtained by using `x=sum(rand(1,M)<p)` in MATLAB. Then, use a simulation to estimate the mean and variance of Z. Do this exercise for

   (a) $a = 3$ and $b = 2$

   (b) $a = -5$ and $b = 7$.

   What do you expect the answers to be and what answers do you obtain through simulations? How can you make both the answers tally to a precision of 2nd decimal place? Use `help` command to understand more about the `sum` and `rand` functions.