ECEN 455 Lab 3: Simulating a Communication System

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Overview:

Computing the relationship between the reliability of a communication system and the available physical resources, such as energy and time, is one of the main challenges in digital communications. For relatively simple communication systems, such a relationship can be obtained analytically and precisely. For example, suppose we want to transmit one bit information using two voltages across an additive Gaussian noise channel. The average error probability for the maximum-likelihood (ML) receiver can be derived analytically and expressed using the Gaussian Q-Function.

If, however, either the communication scheme or the channel model is too complicated to allow for the precise performance analysis, computer simulations become a valuable alternative. In this lab, we start with BPSK, the simplest possible example, and learn how to simulate a digital communication system. We also learn that it is important to verify that our simulation is correct by comparing with known analytical bounds. The same ideas are also extended to more complicated constellation diagrams, such as QPSK, 16-QAM and 8-PSK. Once again the simulation results are then compared to the theoretical results. While QPSK and 16-QAM both have closed form expressions for the exact probability of error, the 8-PSK expression is only an upper bound.

Exercises:

1. Prepare a generic MATLAB function: $\hat{x} = \text{mlDecisions}(\text{constellation}, y)$, which returns the ML estimate of x for a given constellation and the received voltage y. The decoder will make a decision in favor of any signal point if its distance from the received point is minimum.

2. Write a MATLAB program to simulate a communication system: which produces complex signal at the transmitter, passes it through an additive complex Gaussian noise channel and makes a decision based on the ML decision rule at the receiver. Randomly transmit one out of M points on the signal constellation. Estimate the probability of error P_e by counting the frequency of the error events out a total of $N = 10^4$ simulations. For communication systems, the SNR is typically stated as E_s/N_0 where E_s is the energy per symbol and the N_0 is the noise spectral density. In this case, E_s is the second moment (average absolute value squared) of the constellation and $N_0 = 2\sigma^2$ (where σ^2 is variance of the AWGN in each dimension). Plot the estimated error probabilities as a function of $E_s/N_0 = 10^{(-1:0.1:1)}$. Compare your simulation results with the analytical expressions given on the next page.

The signal constellations and expressions for error probability are given on the next page.

Exercises (cont'd):

3. The results obtained in part 2 give the symbol error rate (SER) versus E_s/N_0. Now, we will compute similar results for the bit error rate (BER). First, use Gray mapping to label the constellations. Gray mapping is a technique for assigning binary labels to integer sequences so that the binary labels of adjacent integers differ only in 1 bit. For example, the Gray mappings, for 4 and 8 elements, are given by

$$\{0, 1, 2, 3\} \to \{00, 01, 11, 10\}$$

$$\{0, 1, 2, 3, 4, 5, 6, 7\} \to \{000, 001, 011, 010, 110, 111, 101, 100\}.$$

Modify your program to use Gray mapping and count both bit errors and symbol errors. Now, repeat your simulations (for $N = 10^4$ symbols) to estimate the BER. Finally, plot the results to show the relationship between the BER and E_b/N_0, where E_b is energy transmitted per bit (i.e., E_s normalized by the number of bits).

Constellation Data

1. Signal Constellation:
   (a) QPSK: $\{1 + j, 1 - j, -1 - j, -1 + j\}$
   (b) 16-QAM: $\{a + b\,j \,|\, a, b \in \{-3, -1, 1, 3\}\,\}$
   (c) 8-PSK: $\left\{1, -1, j, -j, \frac{1}{\sqrt{2}}(-1 - j), \frac{1}{\sqrt{2}}(-1 + j), \frac{1}{\sqrt{2}}(1 - j), \frac{1}{\sqrt{2}}(1 + j)\right\}$
2. Probability of error:
   (a) QPSK: $P\,(e) = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)$
   (b) 16-QAM: $P\,(e) = 3Q\left(\sqrt{\frac{1}{5}\frac{E_s}{N_0}}\right) - \frac{9}{4}Q^2\left(\sqrt{\frac{1}{5}\frac{E_s}{N_0}}\right)$
   (c) 8-PSK: $P\,(e) \leq 2Q\left(\sqrt{\frac{2E_s}{N_0}\sin^2\frac{\pi}{8}}\right)$

Extra Credit: Plot the Voronoi regions of the 3 constellations. The performance of a 16-point constellation (at high SNR) can be improved moving the points so that the minimum distance between any two points is the same, but the average energy is lower. Find such a constellation and compute its average energy and the minimum distance between any two points.

MATLAB functions: Here's a short list of MATLAB commands that may be useful for this lab: `min, abs, randn, erfq, voronoi`