The interference channel

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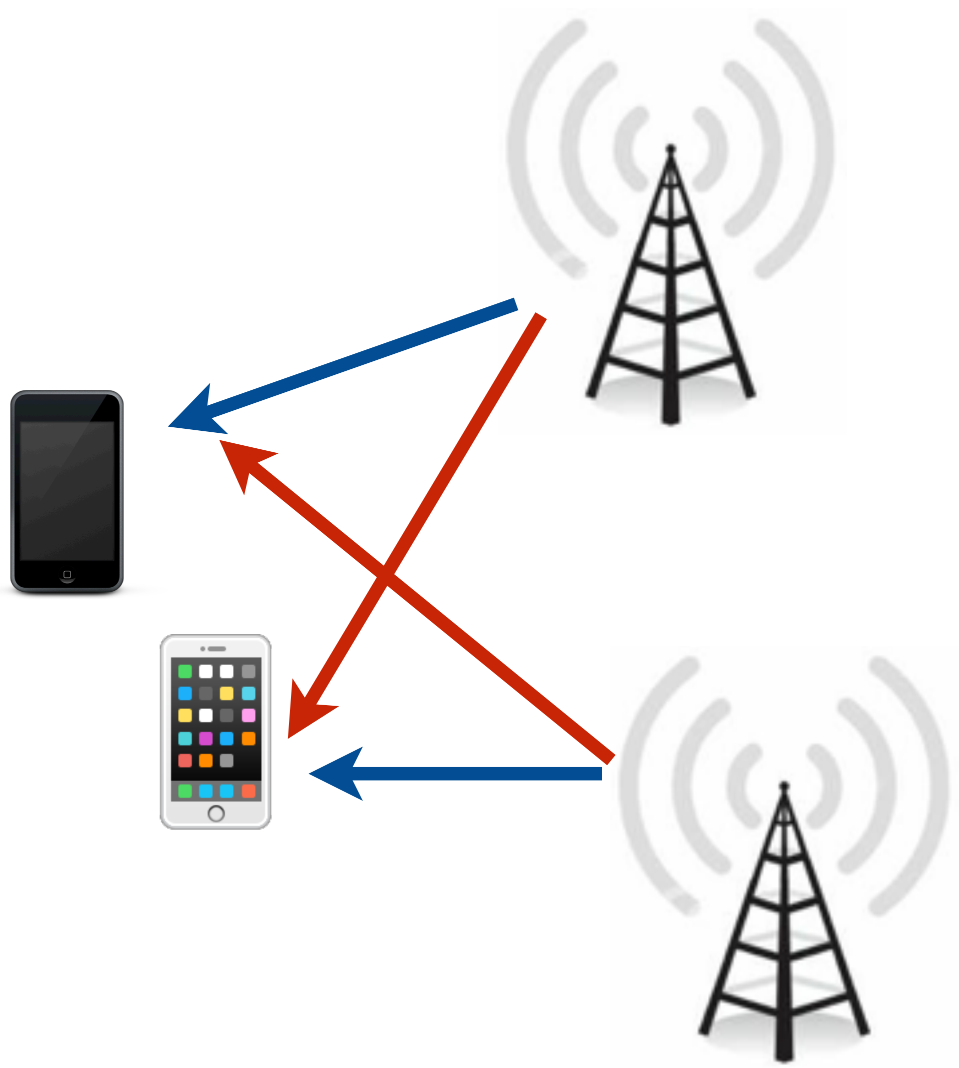
148 slides

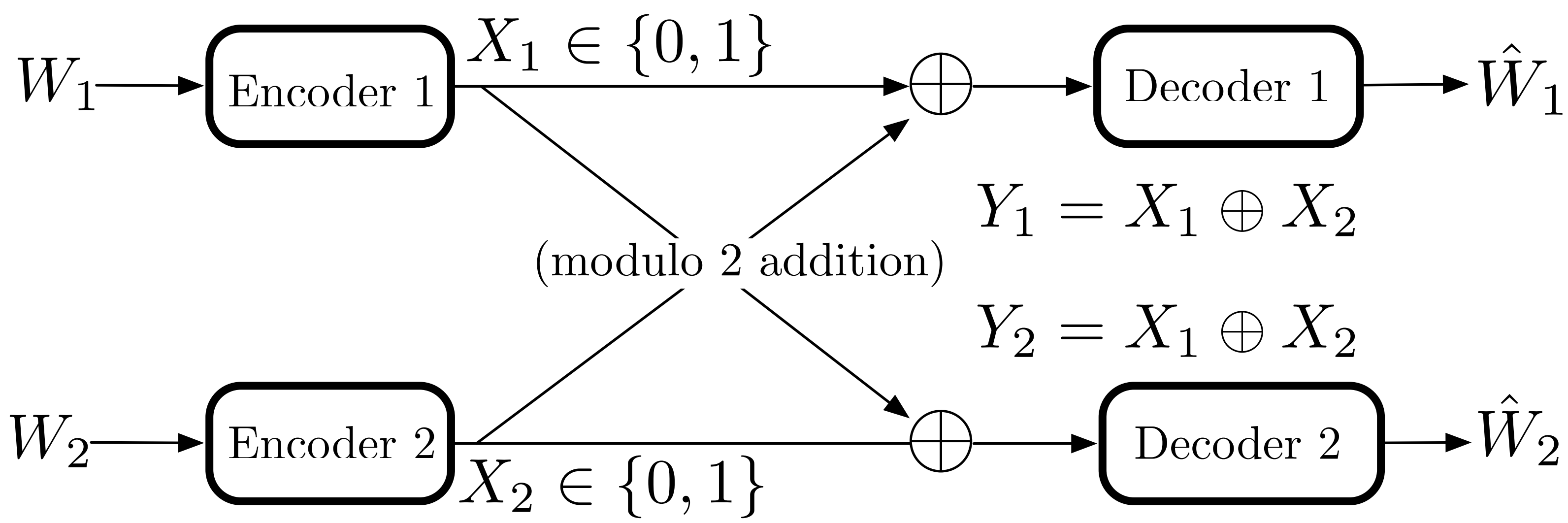


What to do?

What is information theoretically optimal?

OPEN PROBLEM: THE CAPACITY REGION OF THE INTERFERENCE CHANNEL

Example 1: binary adder channel



What rate pairs $(\log(\#W_1), \log(\#W_2))$ can be transmitted reliably?

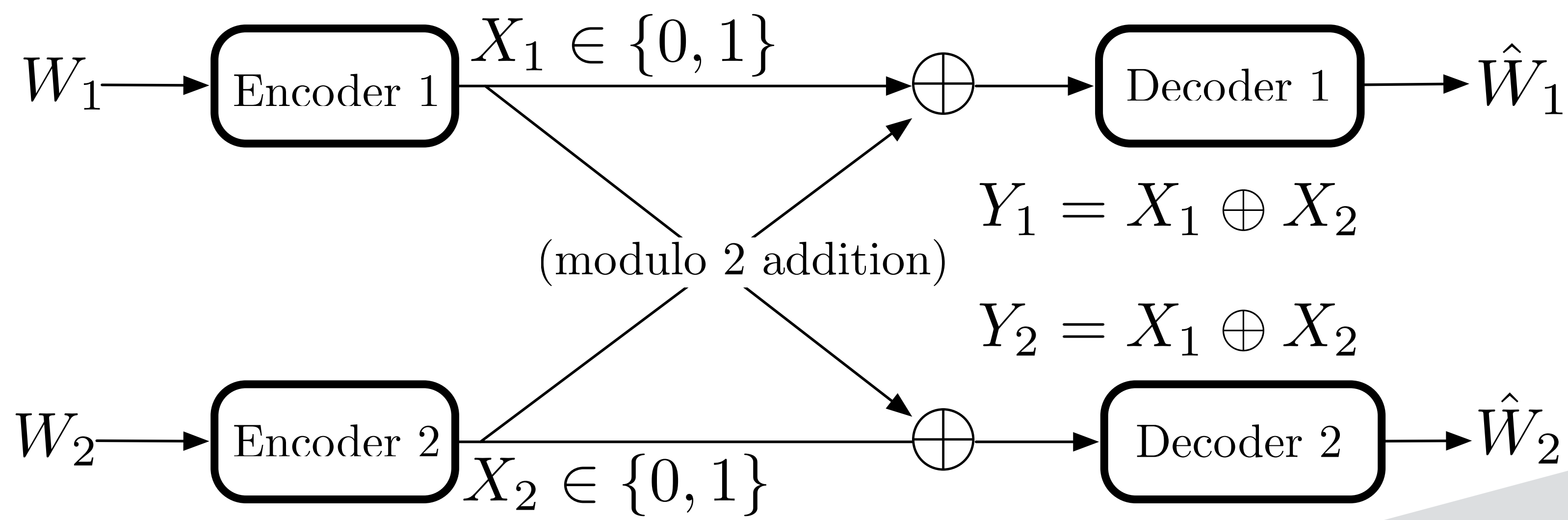
Y	$X_1 = 0$	$X_1 = 1$
$X_2 = 0$	0	1
$X_2 = 1$	1	0

$$Y_1 = Y_2 = 0 \to (X_1, X_2) \in \{(0,0), (1,1)\}$$

$$Y_1 = Y_2 = 1 \to (X_1, X_2) \in \{(0,1), (1,0)\}$$

Can only transmit 1 bit in total

Example 1: binary adder channel



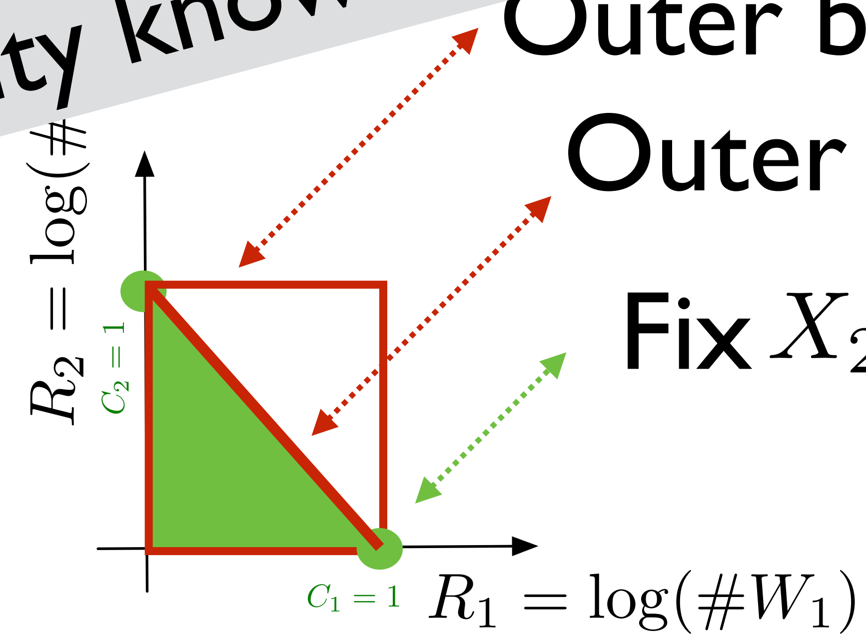
What rate pairs $(\log(\#W_1), \log(\#W_2))$ ca… …smitted reliably?

Sometimes capacity known, and time sharing is optimal!

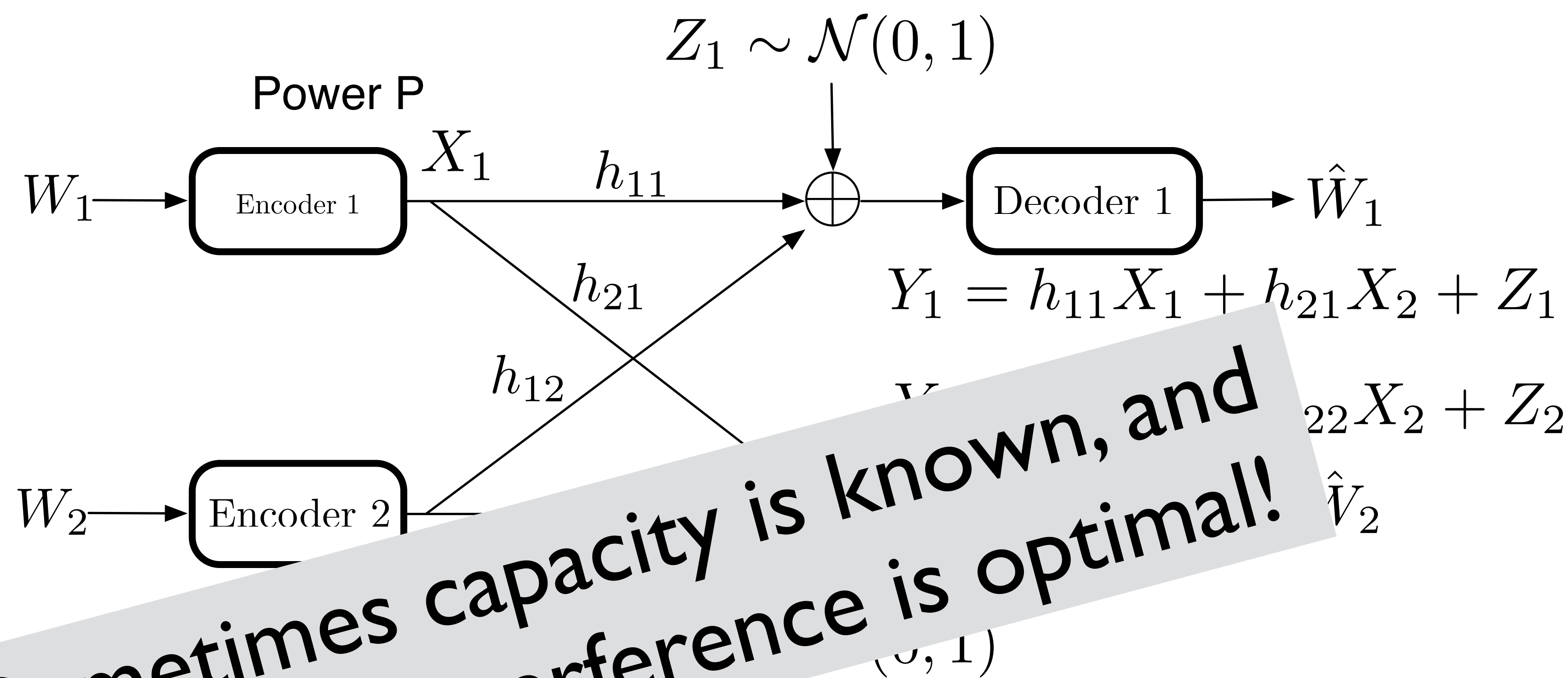
Outer bound (individual)

Outer bound (sum)

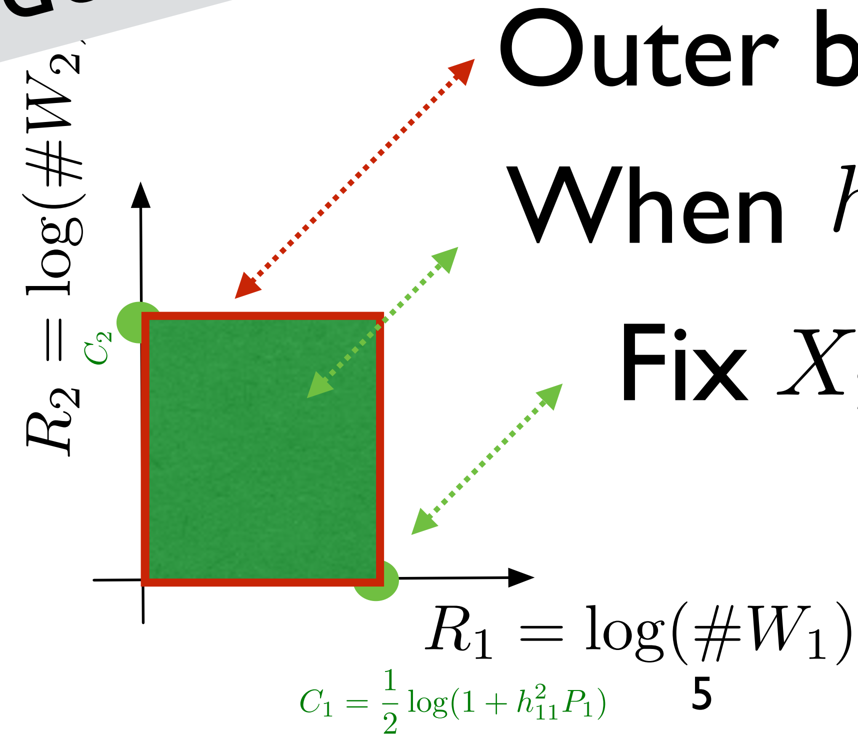
Fix $X_2 = 0$ then $Y_1 = X_1$



Example 2: AWGN channel



Sometimes capacity is known, and decoding interference is optimal!



Outer bound (individual)

When h_{21}, h_{12} are "big"

Fix $X_2 = 0$ then $Y_1 = h_{11}X_1 + Z_1$

Goals of this lecture

forced jokes

1) understand what is understood

forced travel pics

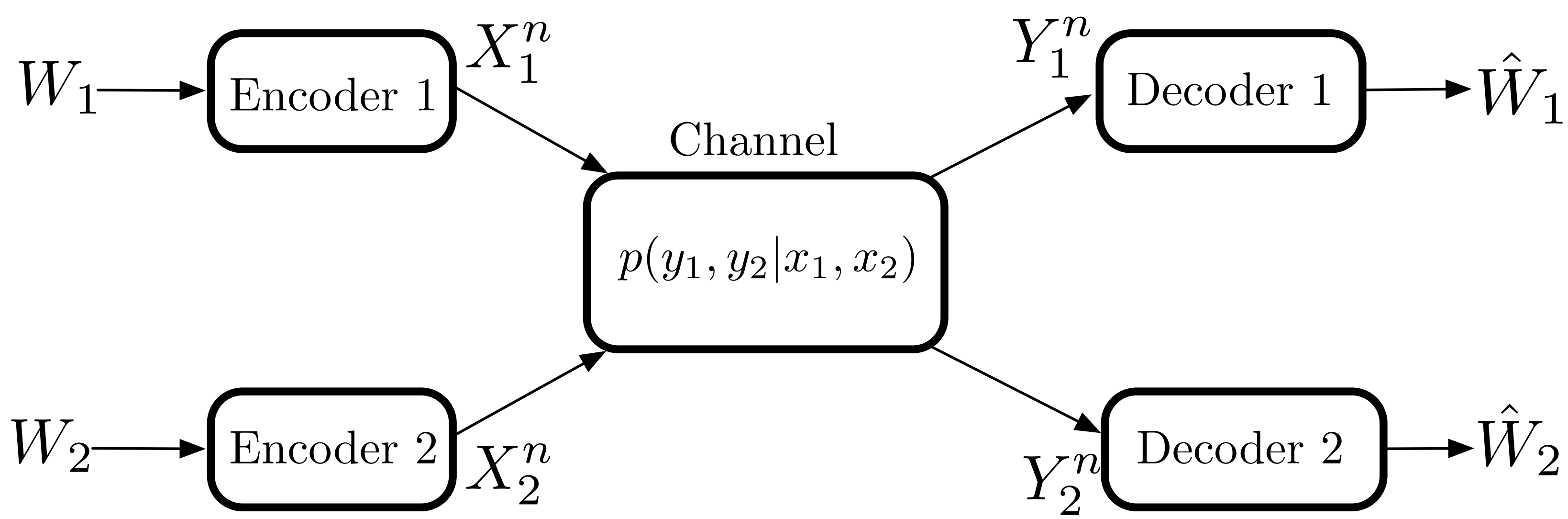
2) understand 3 outer bound and 1 inner bound proof techniques

forced UIC advertisement

3) understand different ways of handling interference

4) ask questions and relate to your own research

Formal definition

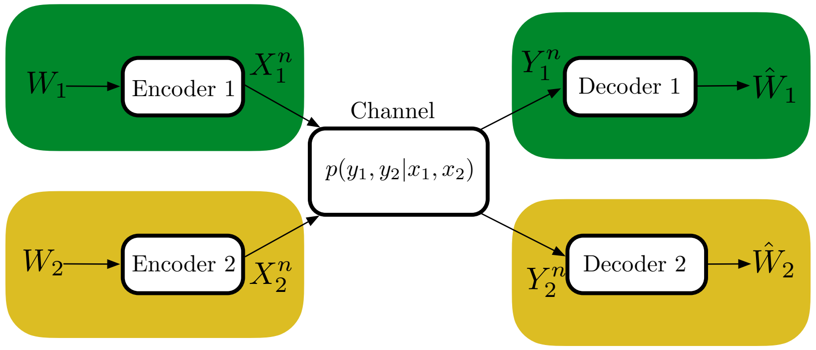


- a discrete memoryless interference channel (DM-IC) $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ consists of 4 finite sets/alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a collection of conditional pmfs $p(y_1, y_2|x_1, x_2)$

- sender $j = 1, 2$ sends an independent message W_i to receiver j

- lower case x is an instance of random variable X in calligraphic alphabet $\mathcal{X}$

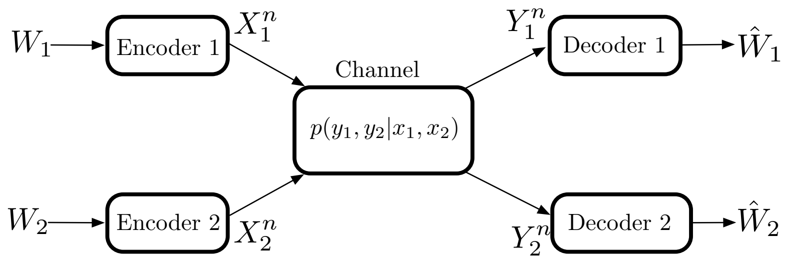
Formal definition



- A $(2^{nR_1}, 2^{nR_2}, n)$ code for the IC consists of:
  1. Two message sets $[1 : 2^{nR_1}]$, and $[1 : 2^{nR_2}]$
  2. Two encoders:
  $$w_1 \in [1 : 2^{nR_1}] \to x_1^n(w_1)$$
  $$w_2 \in [1 : 2^{nR_2}] \to x_2^n(w_2)$$
  3. Two decoders:
  $$y_1^n \to [1 : 2^{nR_1}] \cup \text{error}$$
  $$y_2^n \to [1 : 2^{nR_2}] \cup \text{error}$$

- we assume W_1 and W_2 are uniformly distributed on $[1 : 2^{nR_1}]$ and $[1 : 2^{nR_2}]$ respectively

Formal definition



- average probability of error:

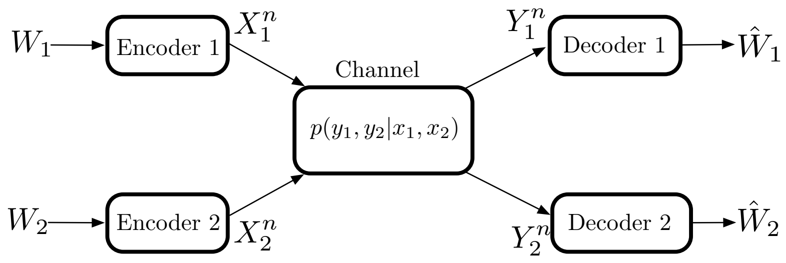
$$P_e^{(n)} := \mathrm{P}\{(\widehat{W_1}, \widehat{W_2}) \neq (W_1, W_2)\}$$

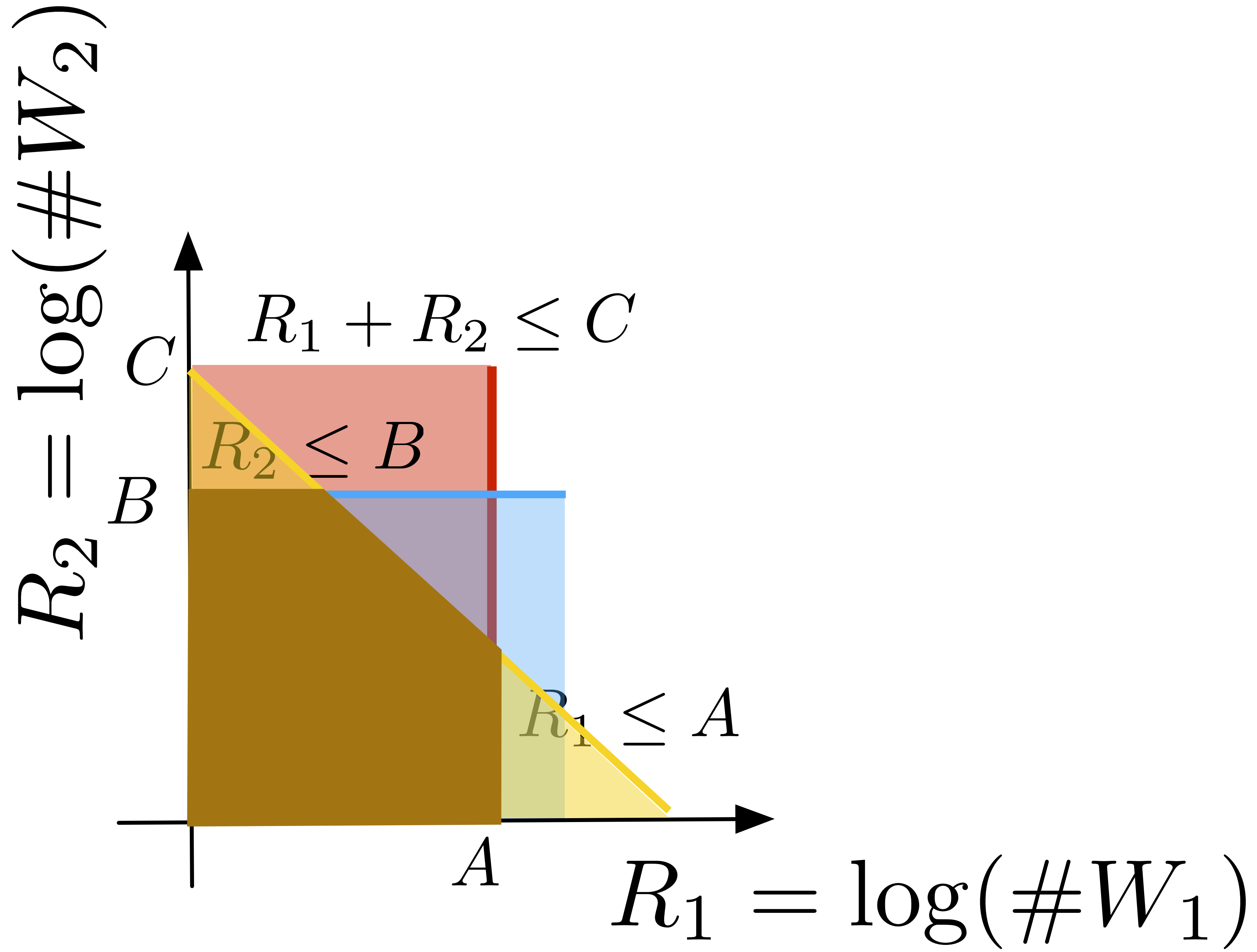
- Rate pair (R_1, R_2) is *achievable* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e^{(n)} \to 0$ as $n \to \infty$

- The *capacity region* of the DM-IC is the closure of the set of achievable rate pairs (R_1, R_2)

- Note: capacity region depends on $p(y_1, y_2|x_1, x_2)$ only through the marginals $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$

Rate regions



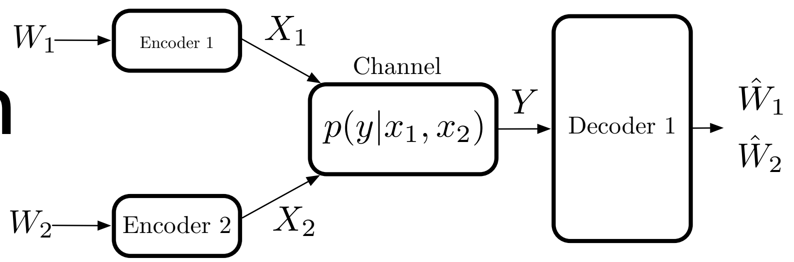


Reminder

Point-to-point capacity

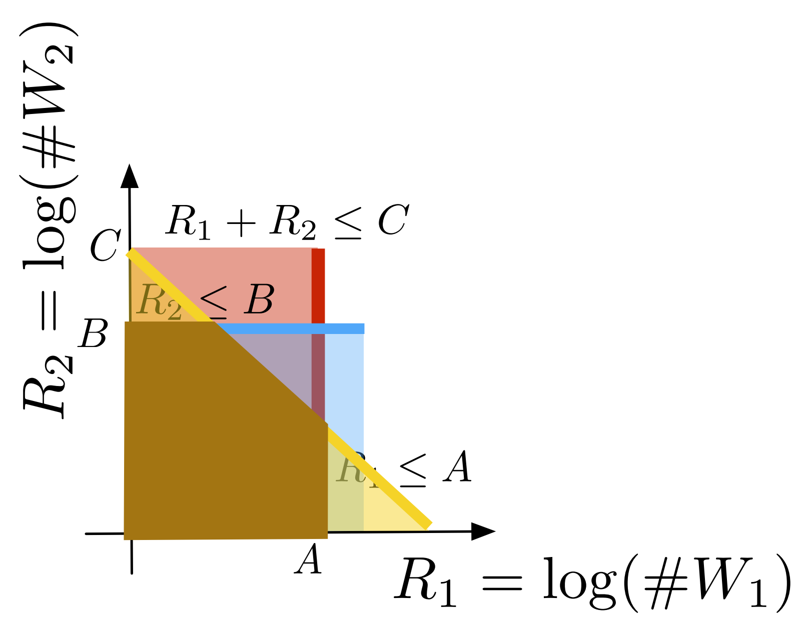
$$R \leq C := \max_{p(x)} I(X;Y)$$

Multiple-access channel (MAC) capacity region

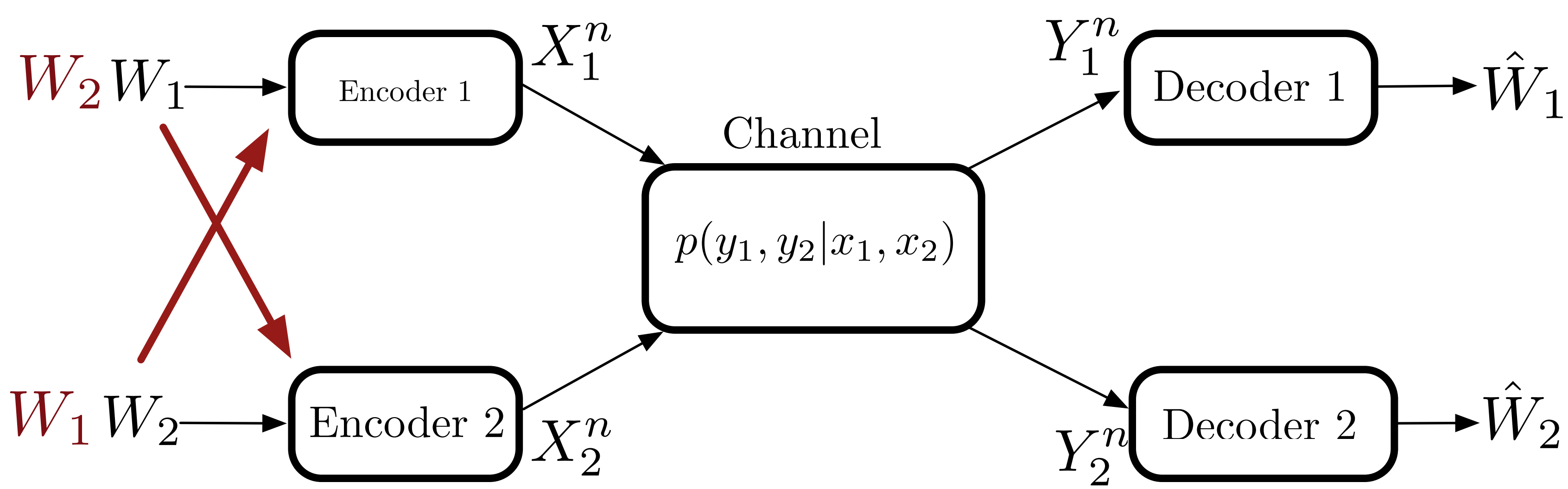


$$R_1 \leq I(X_1;Y|X_2,Q)$$
$$R_2 \leq I(X_2;Y|X_1,Q)$$
$$R_1 + R_2 \leq I(X_1,X_2;Y|Q)$$

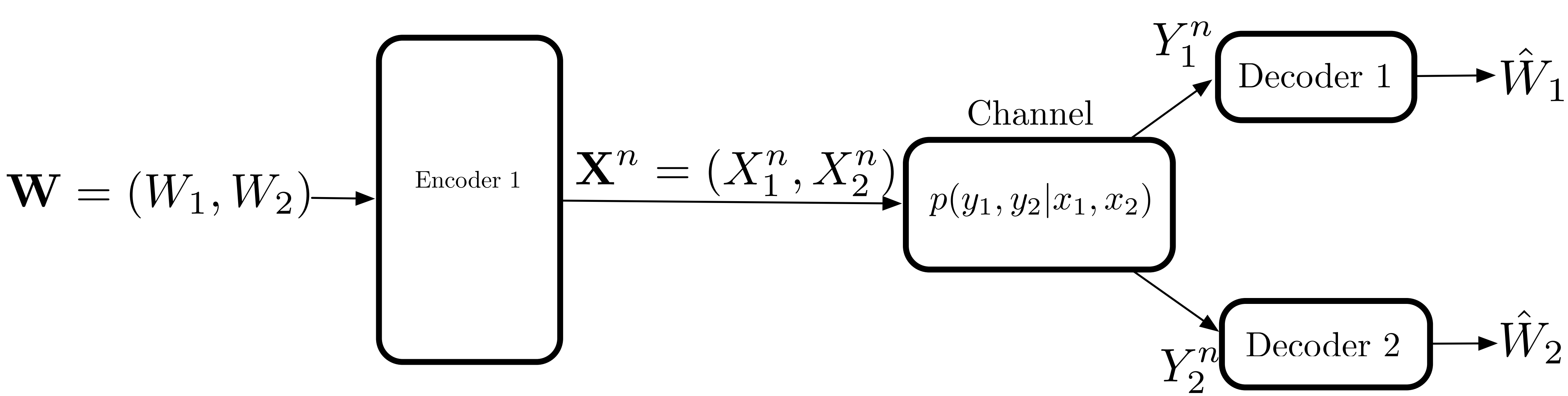
union taken over all $p(q)p(x_1|q)p(x_2|q)$



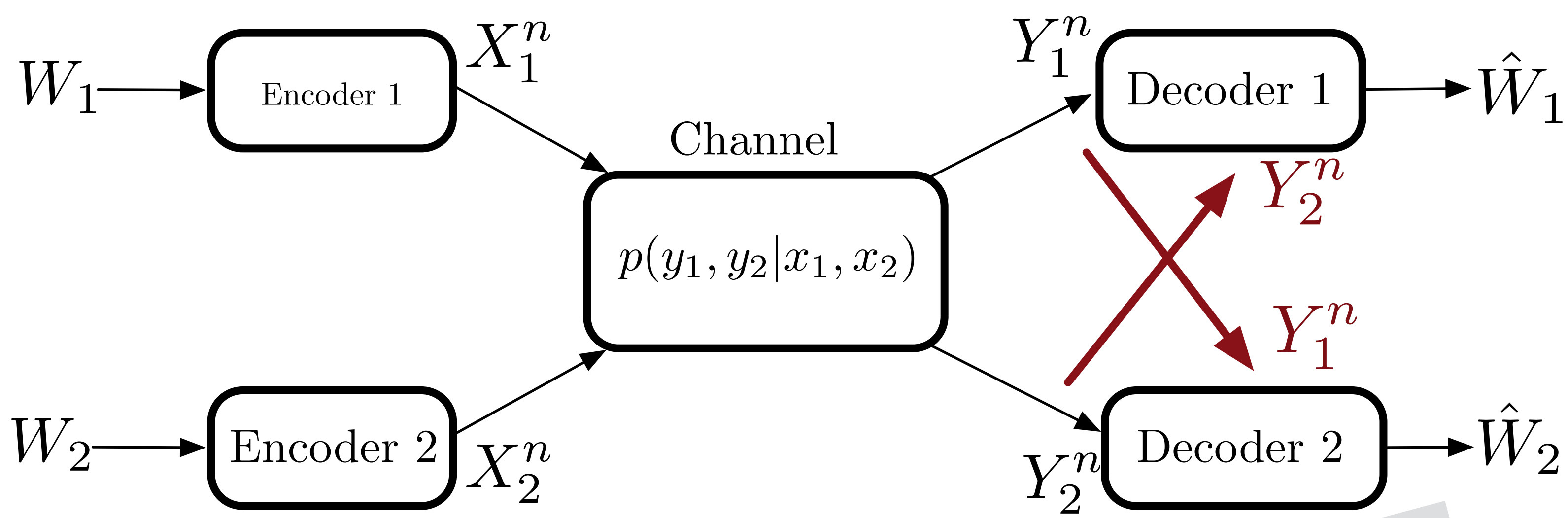
What if the transmitters cooperate?



becomes a 2Tx antenna broadcast channel (solved for AWGN)

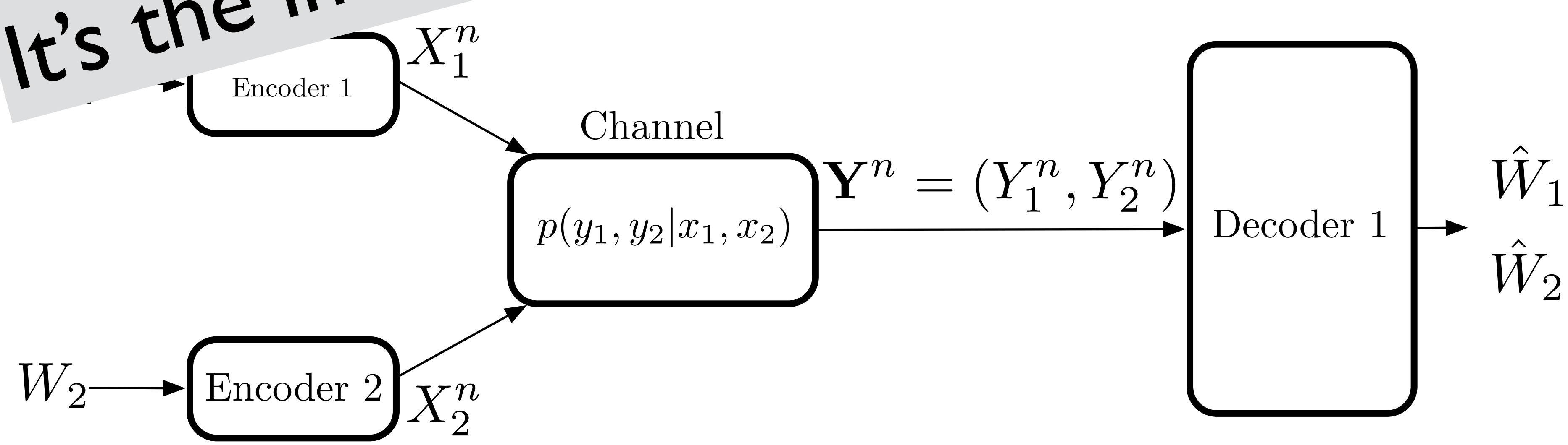


What if the receivers cooperate?



becomes a 2Rx anten ... ccess channel (solved)

It's the interference that is difficult!



Early work

[H. Sato, “Two-user communication channels,” IEEE Trans. Inf. Theory, vol. 23, no. 3, pp. 295–304, 1977.]

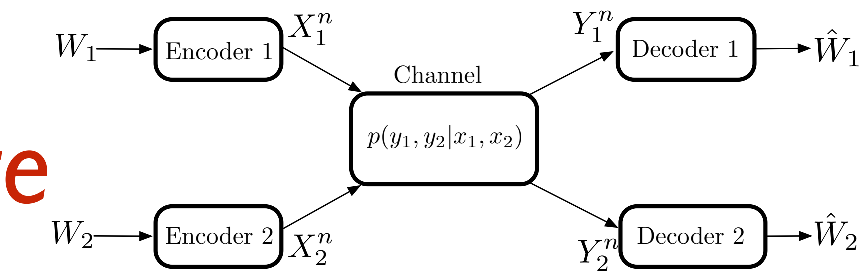
Devroye fail

[R. Ahlswede, “The capacity region of a channel with two senders and two receivers,” Ann. Probability, vol. 2, pp. 805–814, 1974.]

[A. B. Carleial, “A case where interference does not reduce capacity,” IEEE Trans. Inf. Theory, vol. 21, no. 5, pp. 569–570, 1975.]

[A. B. Carleial, “Interference channels,” IEEE Trans. Inf. Theory, vol. 24, no. 1, pp. 60-70, 1978.]

DM-IC: Basic inner and outer bounds



"Single-user" outer bound: *FIX interference*

- Maximal (achievable) individual rates:

$$R_1 \leq C_1 := \max_{p(x_1), x_2} I(X_1; Y_1 | X_2 = x_2), \quad R_2 \leq C_2 := \max_{p(x_2), x_1} I(X_2; Y_2 | X_1 = x_1)$$

"Basic genie" outer bound: *GIVE interference*

- Union over all $p(q)p(x_1|q)p(x_2|q)$ of

$$R_1 \leq I(X_1; Y_1 | X_2, Q), \quad R_2 \leq I(X_2; Y_2 | X_1, Q)$$

"Single-user" time-sharing inner bound: *AVOID interference*

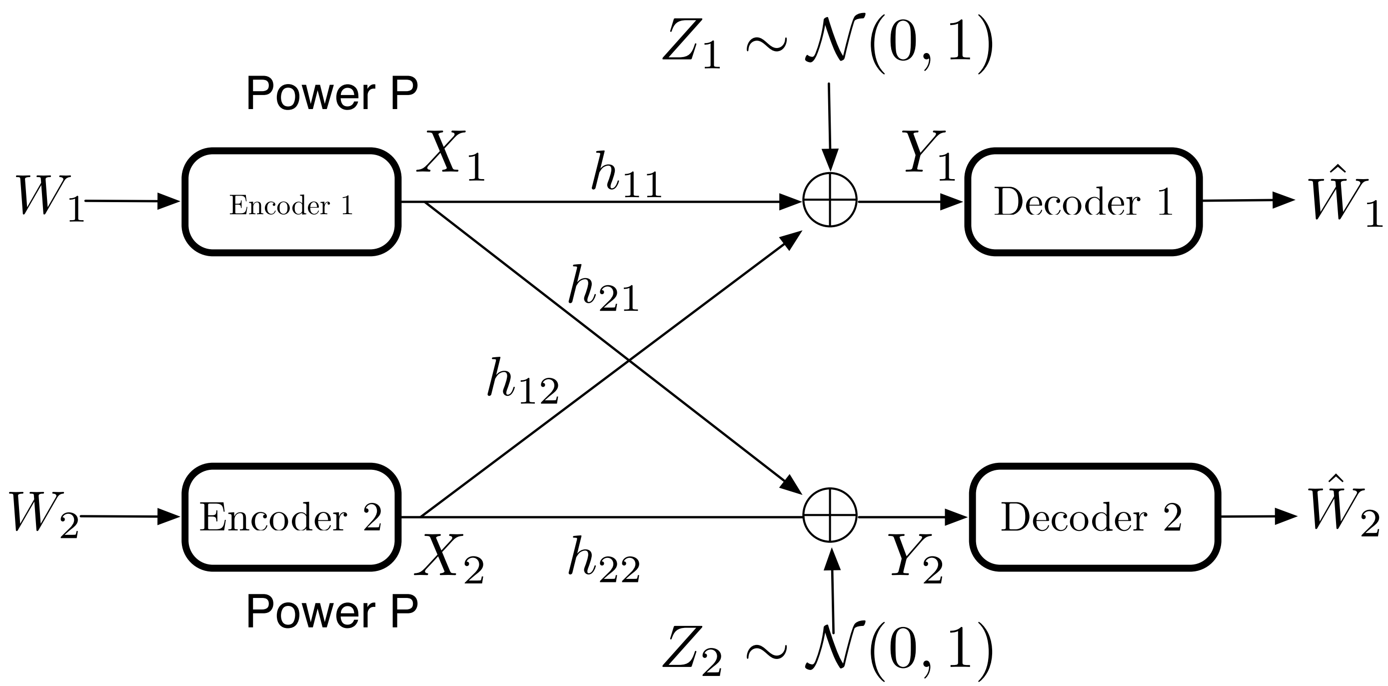
- Union over all $t \in [0,1]$ of: $R_1 \leq t\, C_1, \quad R_2 \leq (1-t)\, C_2$

Treating interference as noise inner bound: *"SUFFER" interference*

- Union over all $p(q)p(x_1|q)p(x_2|q)$ of

$$R_1 \leq I(X_1; Y_1 | Q), \quad R_2 \leq I(X_2; Y_2 | Q)$$

AWGN Gaussian IC



$$S_1 = h_{11}^2 P$$
$$I_1 = h_{21}^2 P$$

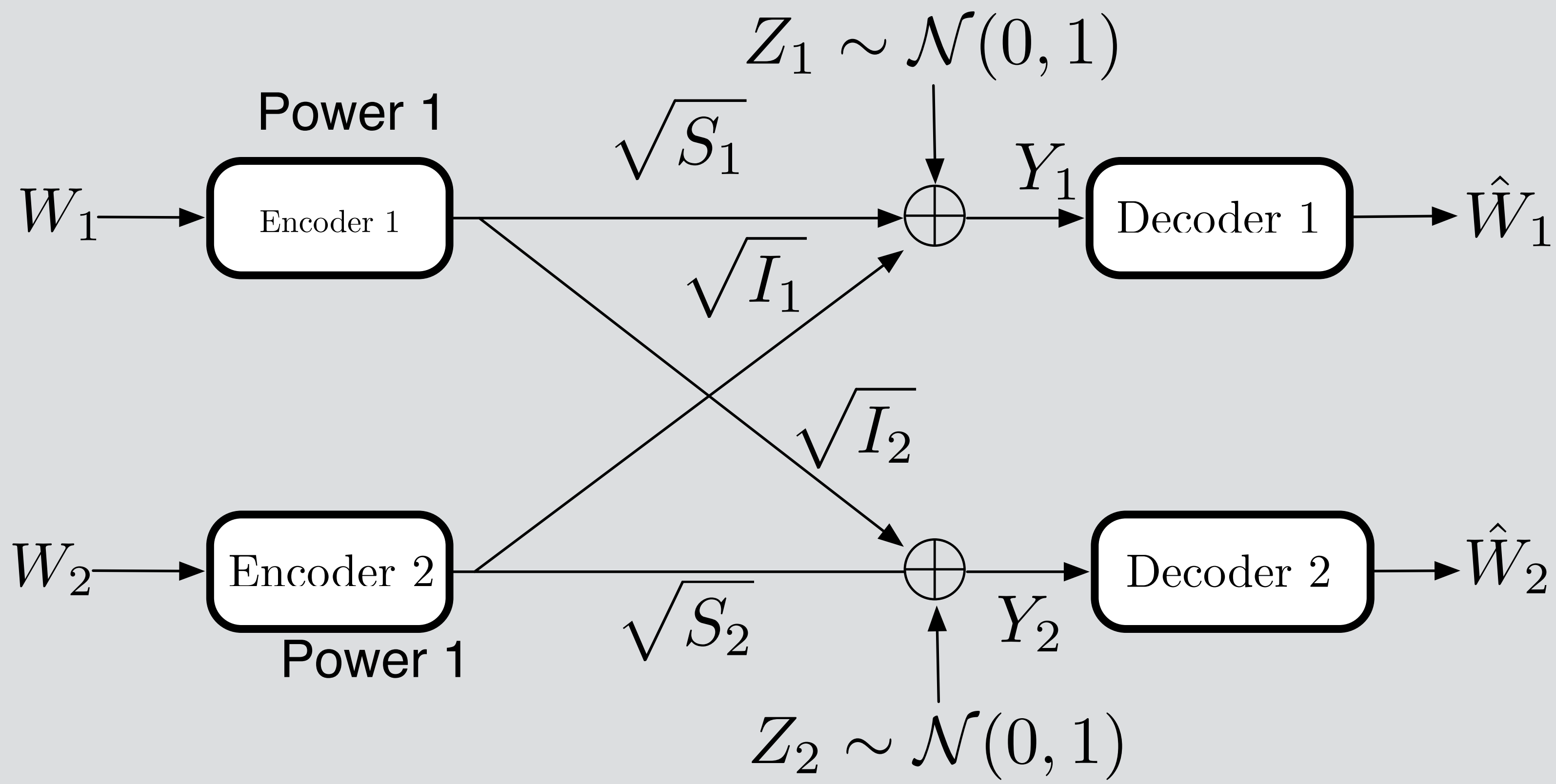
$$S_2 = h_{22}^2 P$$
$$I_2 = h_{12}^2 P$$

Average transmit power constraint: for all codewords $x_1^n(w_1)$ and $x_2^n(w_2)$,

$$\sum_{j=1}^{n} x_{1j}^2(w_1) \le nP, \quad \sum_{j=1}^{n} x_{2j}^2(w_2) \le nP,$$

Practically relevant

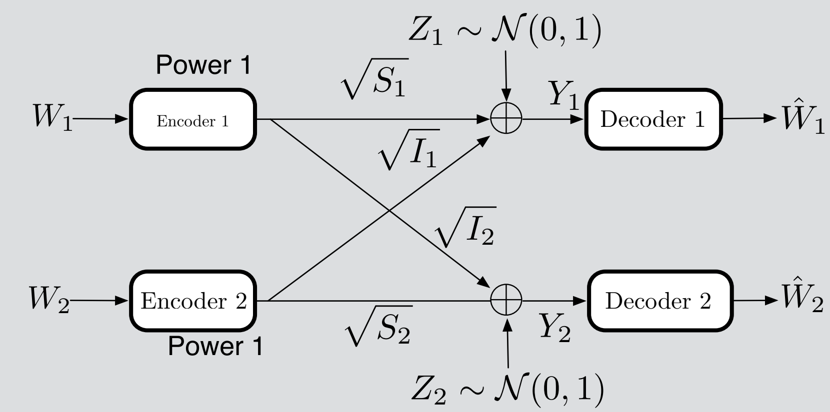
AWGN Gaussian IC



Average transmit power constraint: for all codewords $x_1^n(w_1)$ and $x_2^n(w_2)$,

$$\frac{1}{n}\sum_{j=1}^{n} x_{1j}^2(w_1) \le 1, \quad \frac{1}{n}\sum_{j=1}^{n} x_{2j}^2(w_2) \le 1,$$

Gaussian-IC: Basic inner and outer bounds



Average transmit power constraint: for all codewords $x_1^n(w_1)$ and $x_2^n(w_2)$,

$$\frac{1}{n}\sum_{j=1}^{n} x_{1j}^2(w_1) \le 1, \quad \frac{1}{n}\sum_{j=1}^{n} x_{2j}^2(w_2) \le 1,$$

“Single-user” outer bound:

- Maximal (achievable) individual rates:

$$R_1 \le C_1 := \frac{1}{2}\log(1+S_1), \quad R_2 \le C_2 := \frac{1}{2}\log(1+S_2)$$

“Single-user” time-sharing (with power control) inner bound:

- Union over all $\alpha \in [0,1]$:

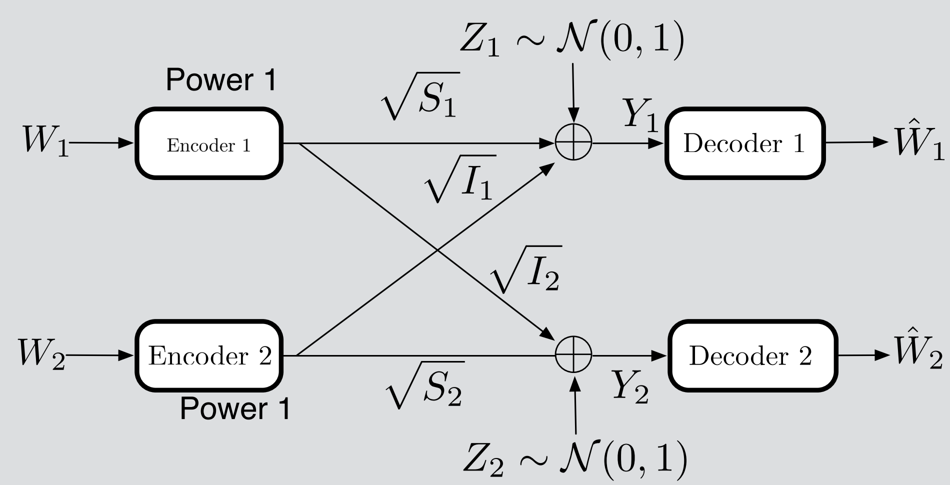
$$R_1 \le \frac{\alpha}{2}\log\left(1+\frac{S_1}{\alpha}\right), \quad R_2 \le \frac{(1-\alpha)}{2}\log\left(1+\frac{S_2}{(1-\alpha)}\right)$$

Treating interference as noise inner bound (with Gaussian inputs):

$$R_1 \le \frac{1}{2}\log\left(1+\frac{S_1}{1+I_1}\right), \quad R_2 \le \frac{1}{2}\log\left(1+\frac{S_2}{1+I_2}\right)$$

The main problem with the Gaussian-IC

What inputs are optimal?



Average transmit power constraint: for all codewords $x_1^n(w_1)$ and $x_2^n(w_2)$,

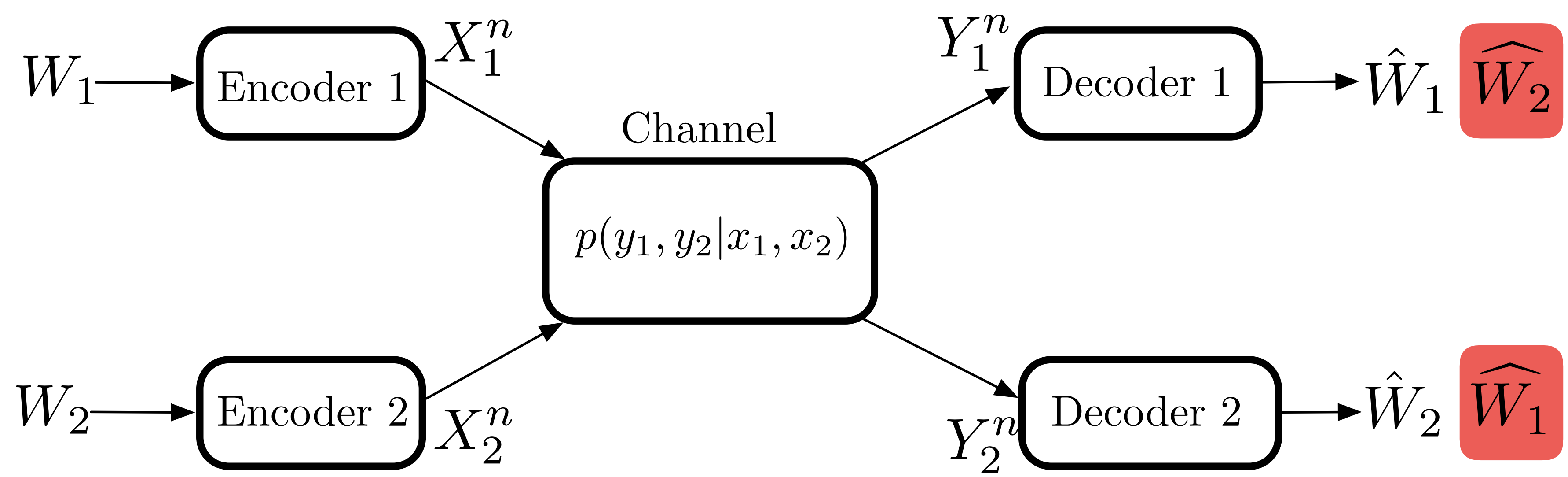
$$\frac{1}{n}\sum_{j=1}^{n} x_{1j}^2(w_1) \le 1, \quad \frac{1}{n}\sum_{j=1}^{n} x_{2j}^2(w_2) \le 1,$$

Max entropy and EPI:
Gaussian inputs are *best inputs*
but also *worst noise*

Tension between the 2 users!

DM-IC: Simultaneous decoding inner bound

"Compound MAC" inner bound: (force both to decode both)



- Union over all $p(q)p(x_1|q)p(x_2|q)$ of

$$
\begin{aligned}
R_1 &\leq \min\{I(X_1;Y_1|X_2,Q), I(X_1;Y_2|X_2,Q)\} \\
R_2 &\leq \min\{I(X_2;Y_1|X_1,Q), I(X_2;Y_2|X_1,Q)\} \\
R_1 + R_2 &\leq \min\{I(X_1,X_2;Y_1|Q), I(X_1,X_2;Y_2|Q)\}
\end{aligned}
$$

Can be tightened….. how?

DM-IC: Simultaneous decoding inner bound

"Compound MAC" inner bound: (force both to decode both)

- Union over all $p(q)p(x_1|q)p(x_2|q)$ of

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\end{aligned}
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Can be tightened….. how?

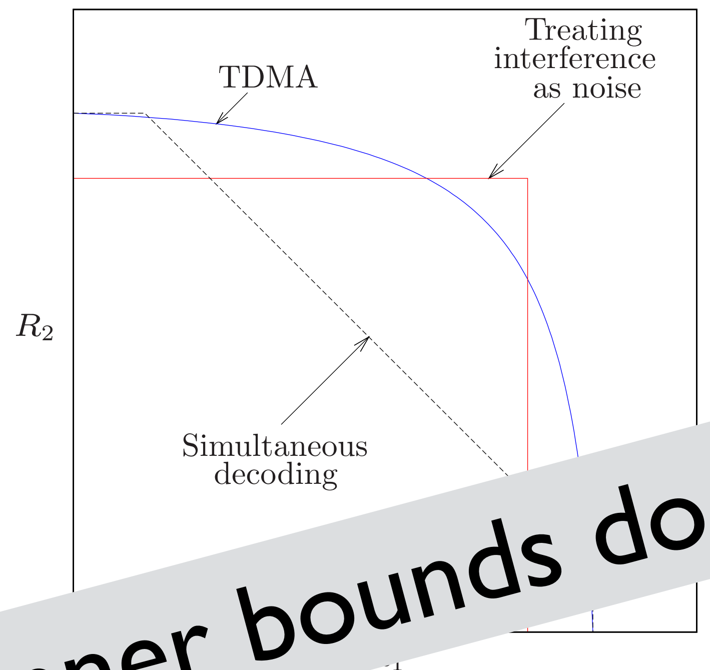
- Union over all $p(q)p(x_1|q)p(x_2|q)$ of

$$
\begin{aligned}
R_1 &\leq I(X_1;Y_1|X_2,Q) \\
R_2 &\leq I(X_2;Y_2|X_1,Q) \\
R_1 + R_2 &\leq \min\{I(X_1,X_2;Y_1|Q), I(X_1,X_2;Y_2|Q)\}
\end{aligned}
$$

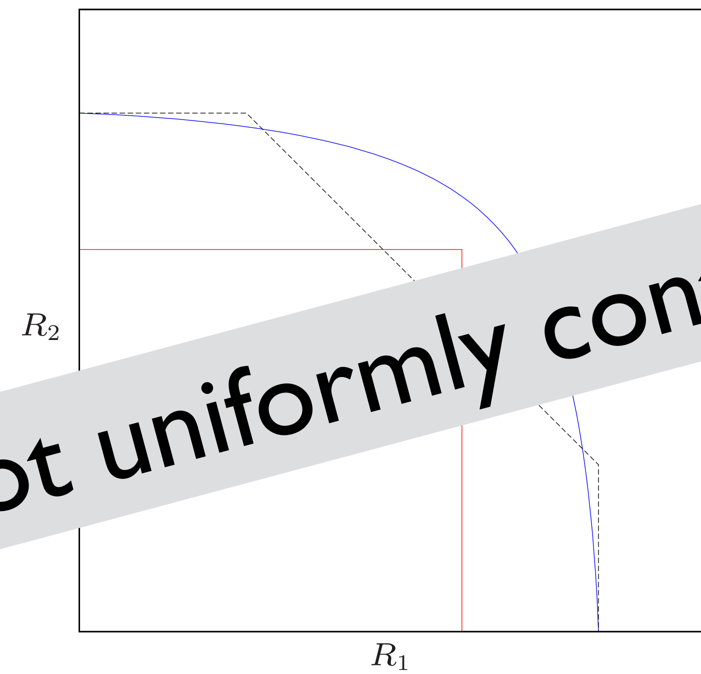
Comparison of the Bounds

- We consider the symmetric case ($S_1 = S_2 = S = 1$ and $I_1 = I_2 = I$)

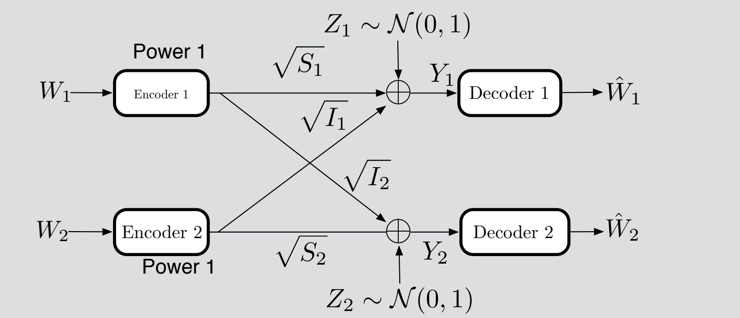
low interference $I = 0.2$



lowish interference $I = 0.5$



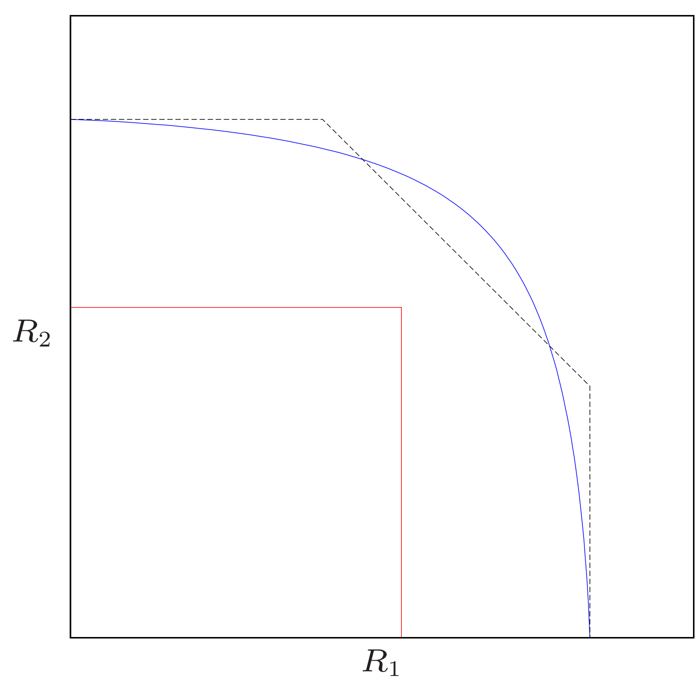
inner bounds do not uniformly contain each other!



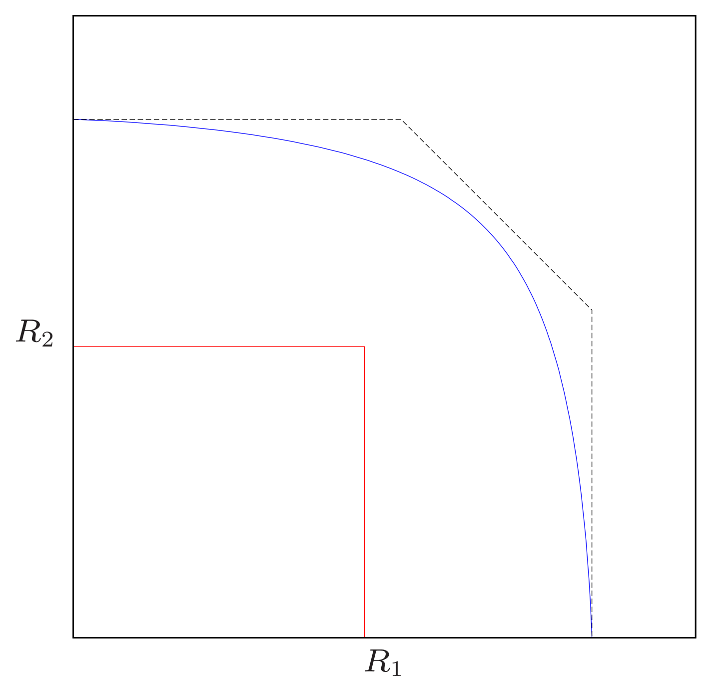
Average transmit power constraint: for all codewords $x_1^n(w_1)$ and $x_2^n(w_2)$,

$$\frac{1}{n}\sum_{j=1}^{n} x_{1j}^2(w_1) \le 1, \quad \frac{1}{n}\sum_{j=1}^{n} x_{2j}^2(w_2) \le 1,$$

highish interference $I = 0.8$



high interference $I = 1.1$



images taken from [A. El Gamal and Y.H. Kim, "Lecture Notes on Network Information Theory," http://arxiv.org/abs/1001.3404.]

DM-IC: Sato's outer bound

- Let $\mathcal{R}(\tilde{p}(y_1, y_2|x_1, x_2))$ be the union over all $p(q)p(x_1|q)p(x_2|q)\tilde{p}(y_1, y_2|x_1, x_2)$ of

$$R_1 \leq I(X_1; Y_1|X_2, Q)$$
$$R_2 \leq I(X_2; Y_2|X_1, Q)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2|Q)\}.$$

Then the intersection of the sets $\mathcal{R}(\tilde{p}(y_1, y_2|x_1, x_2))$ over all $\tilde{p}(y_1, y_2|x_1, x_2)$ with the same marginals as $p(y_1, y_2|x_1, x_2)$ is an outer bound for the DM-IC.

Give user 2 signal as side information at Rx 1

Give user 1 signal as side information at Rx 2

Let both Rxs cooperate in decoding both messages

Handout 1: proof of Sato's outer bound

Handout 1: proof of Sato's outer bound

$$
\begin{aligned}
nR_1 &= H(W_1) \\
&\overset{(a)}{=} H(W_1|W_2) \\
&\overset{(b)}{=} H(W_1|X_2^n) \\
&\overset{(c)}{=} H(W_1|Y_1^n, X_2^n) + I(W_1; Y_1^n|X_2^n) \\
&\overset{(d)}{\leq} n\epsilon_1^n + I(W_1; Y_1^n|X_2^n) \\
&\overset{(e)}{=} n\epsilon_1^n + I(X_1^n; Y_1^n|X_2^n) \\
&\overset{(f)}{\leq} n\epsilon_1^n + \sum_{j=1}^{n} I(X_{1j}; Y_{1j}|X_{2j}) \\
&\overset{(g)}{=} n\epsilon_1^n + nI(X_1; Y_1|X_2, Q),
\end{aligned}
$$

single-letterization is the issue!

Handout 1: proof of Sato's outer bound

$$
\begin{aligned}
I(X_1^n;Y_1^n|X_2^n) &\overset{(h)}{=} \sum_{j=1}^{n} I(X_{1j};Y_1^n|X_2^n, X_{11},\cdots X_{1(j-1)}) \\
&\overset{(i)}{\leq} \sum_{j=1}^{n} I(X_{1j};Y_{1j}|X_2^n, X_{11},\cdots X_{1(j-1)}) \\
&\overset{(j)}{=} \sum_{j=1}^{n} H(Y_{1j}|X_2^n, X_{11},\cdots X_{1(j-1)}) - H(Y_{1j}|X_2^n, X_{11},\cdots X_{1(j-1)}, X_{1j}) \\
&\overset{(k)}{\leq} \sum_{j=1}^{n} H(Y_{1j}|X_{2j}) - H(Y_{1j}|X_2^n, X_{11},\cdots X_{1(j-1)}, X_{1j}) \\
&\overset{(l)}{=} \sum_{j=1}^{n} H(Y_{1j}|X_{2j}) - H(Y_{1j}|X_{2j}, X_{1j})
\end{aligned}
$$

memoryless-ness of the channel!

negative term has all inputs in conditioning

Are these ever *tight*? (*inner = outer*)

Treat interference as noise

(Coded) time-sharing

Simultaneous decoding

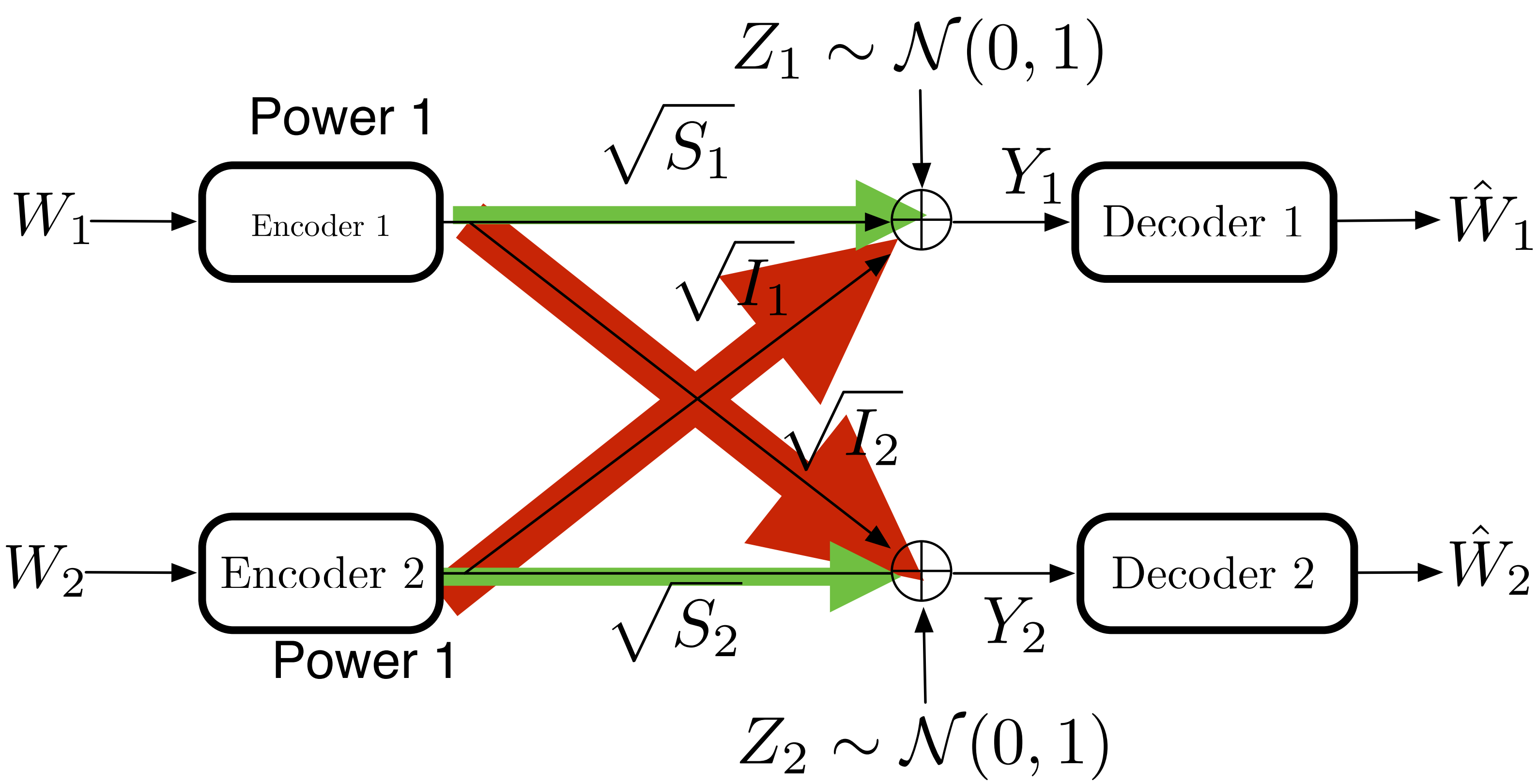
(Successive interference cancellation)

Single-user outer

Sato outer

A DM-IC is said to have *very strong interference* if

$$I(X_1;Y_1|X_2) \leq I(X_1;Y_2)$$
$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1) \qquad \forall p(x_1)p(x_2)$$



A DM-IC is said to have *very strong interference* if

$$I(X_1; Y_1 | X_2) \leq I(X_1; Y_2) \qquad \forall p(x_1)p(x_2)$$
$$I(X_2; Y_2 | X_1) \leq I(X_2; Y_1)$$

Theorem (capacity region under very strong interference). The capacity region of the DM=IC under very strong interference is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq I(X_1; Y_1 | X_2, Q)$$
$$R_2 \leq I(X_2; Y_2 | X_1, Q)$$

for some $p(q, x_1, x_2) = p(q)p(x_1|q)p(x_2|q)$.

Achievability? Successive interference cancellation!

Decode interference FIRST, then desired!

At Rx 1: $R_2 \leq I(X_2; Y_1)$ then $R_1 \leq I(X_1; Y_1 | X_2)$

At Rx 2: $R_1 \leq I(X_1; Y_2)$ then $R_2 \leq I(X_2; Y_2 | X_1)$

A DM-IC is said to have *very strong interference* if

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2) \qquad \forall p(x_1)p(x_2)$$
$$I(X_2;Y_2|X_1) \le I(X_2;Y_1)$$

Theorem (capacity region under very strong interference). The capacity region of the DM=IC under very strong interference is the set of rate pairs (R_1, R_2) such that

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for some $p(q,x_1,x_2) = p(q)p(x_1|q)p(x_2|q)$.

Converse? Basic genie outer bounds (or Sato)

A DM-IC is said to have *very strong interference* if

$$I(X_1;Y_1|X_2) \leq I(X_1;Y_2) \qquad \forall p(x_1)p(x_2)$$
$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1)$$

A DM-IC is said to have *strong interference* if

$$I(X_1;Y_1|X_2) \leq I(X_1;Y_2|X_2) \qquad \forall p(x_1)p(x_2)$$
$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1)$$

Very strong interference $\rightarrow$ *strong interference*

Strong interference $\not\rightarrow$ *very strong interference*

Capacity of DM-IC under *strong interference*

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2|X_2)$$
$$I(X_2;Y_2|X_1) \le I(X_2;Y_1|X_1) \qquad \forall p(x_1)p(x_2)$$

Theorem (capacity region in strong interference). The capacity region of the interference channel $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1,y_2|x_1,x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$in strong interference is the set of rate pairs (R_1, R_2) such that

$$R_1 \le I(X_1;Y_1|X_2,Q) \quad (1)$$
$$R_2 \le I(X_2;Y_2|X_1,Q) \quad (2)$$
$$R_1 + R_2 \le \min\{I(X_1,X_2;Y_1|Q), I(X_1,X_2;Y_2|Q)\} \quad (3)$$

for some $p(q,x_1,x_2) = p(q)p(x_1|q)p(x_2|q)$ where $|\mathcal{Q}| \le 4$.

[M. H. M. Costa and A. El Gamal, "The capacity region of the discrete memoryless interference channel with strong interference," IEEE Trans. Inf. Theory, vol. 33, no. 5, pp. 710–711, 1987.]

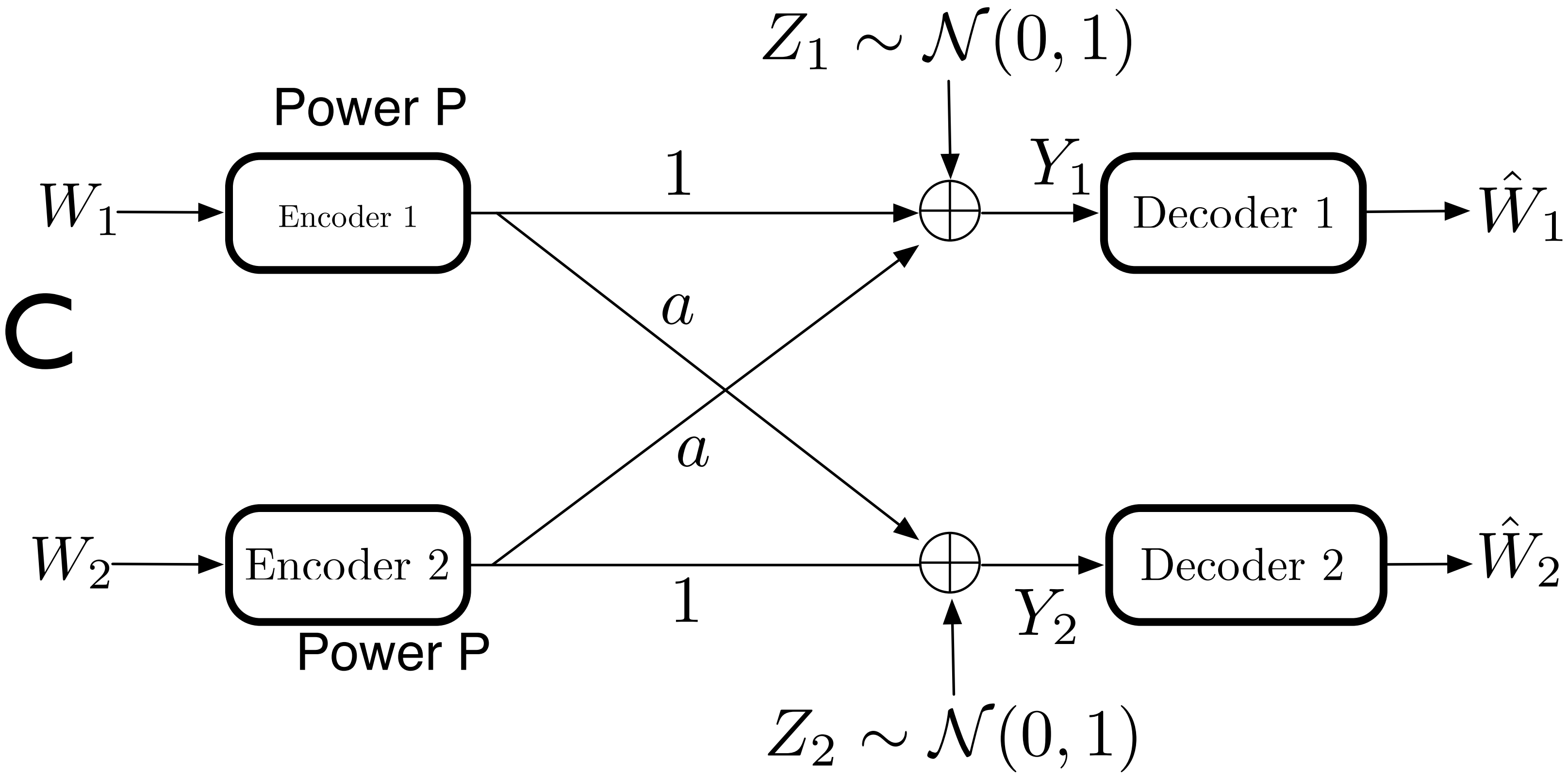
Achievability? Simultaneous decoding inner bound

Converse? See handout

$$
\begin{aligned}
n(R_1+R_2) &= H(W_1)+H(W_2)\\
&\stackrel{(a)}{\leq} I(W_1;Y_1^n)+I(W_2;Y_2^n)+n\epsilon_n\\
&\stackrel{(b)}{\leq} I(X_1^n;Y_1^n)+I(X_2^n;Y_2^n)+n\epsilon_n\\
&\stackrel{(c)}{\leq} I(X_1^n;Y_1^n|X_2^n)+I(X_2^n;Y_2^n)+n\epsilon_n\\
&\stackrel{(d)}{\leq} I(X_1^n;Y_2^n|X_2^n)+I(X_2^n;Y_2^n)+n\epsilon_n\\
&\stackrel{(e)}{=} I(X_1^n,X_2^n;Y_2^n)+n\epsilon_n\\
&\stackrel{(f)}{\leq} \sum_{i]1}^{n} I(X_{1i},X_{2i};Y_{2i})+n\epsilon_n\\
&\stackrel{(g)}{=} nI(X_1,X_2;Y_2|Q)+n\epsilon_n
\end{aligned}
$$

strong interference! one output!

Symmetric Gaussian IC



Very strong interference: $a^2 \geq 1 + P$

capacity region know ... ce fully first

Decode ALL interference

Strong in ... $a^2 \geq 1$

...apacity region known, jointly decode interference and message

Weak interference: $a^2 \leq 1$ *Costa's corner point known, LATER*

Very weak interf... $P) \leq 1$

Decode NO interference

s... known, Gaussian inputs+TIN

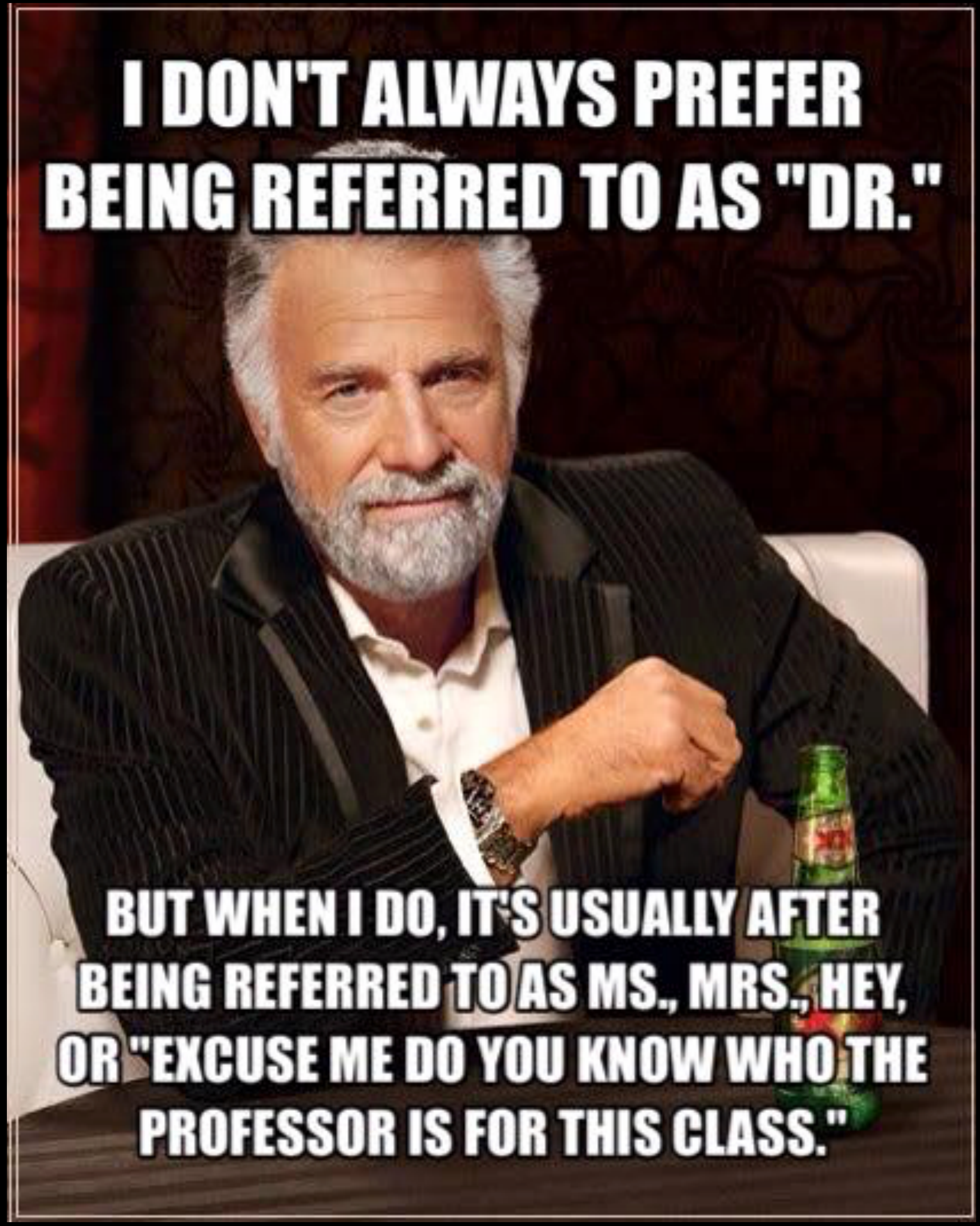
[V.S. Annapureddy and V.V. Veeravalli, *Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region*, Information Theory, IEEE Transactions on 55 (2009), no. 7, 3032 –3050.]

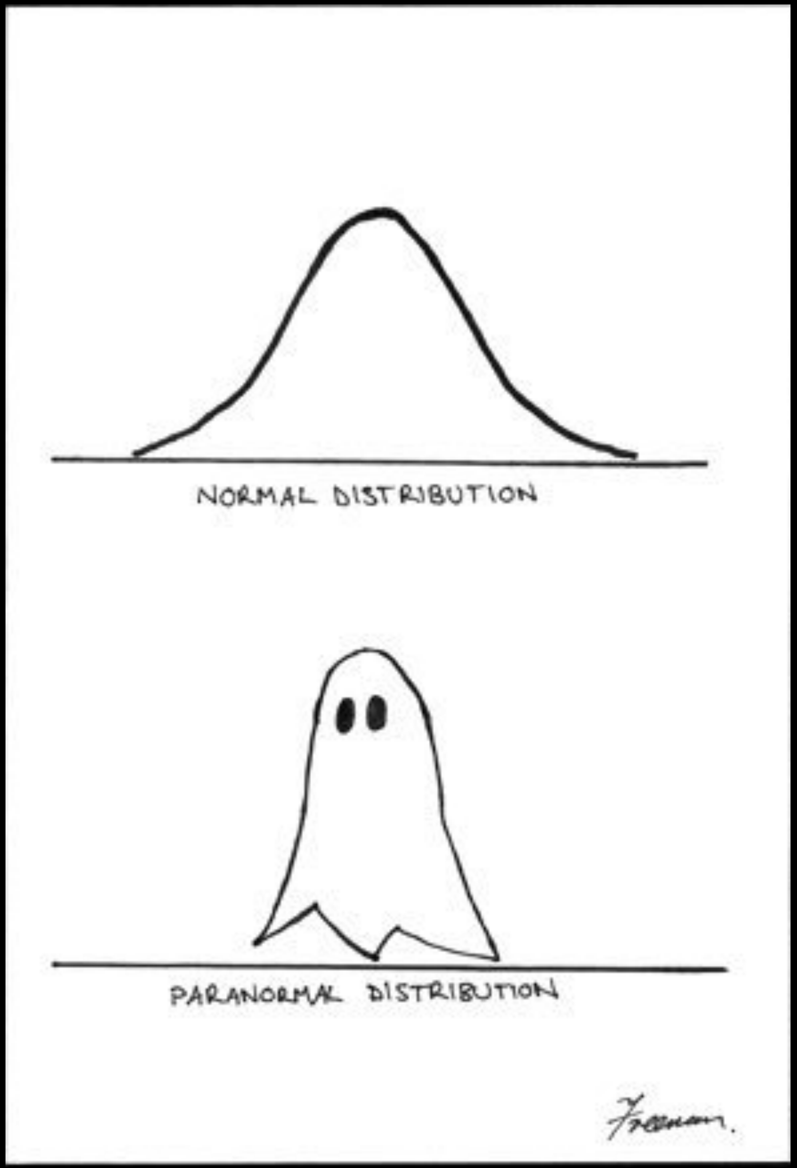
[A.S. Motahari and A.K. Khandani, C*apacity bounds for the gaussian interference channel, Information Theory, I*EEE Transactions on 55 *(2009), no. 2, 620 –643.]*

[Xiaohu Shang, G. Kramer, and Biao Chen, *A new outer bound and the noisy-interference sum-rate capacity for gaussian interference channels*, Information Theory, IEEE Transactions on 55 (2009), no. 2, 689 –699.]

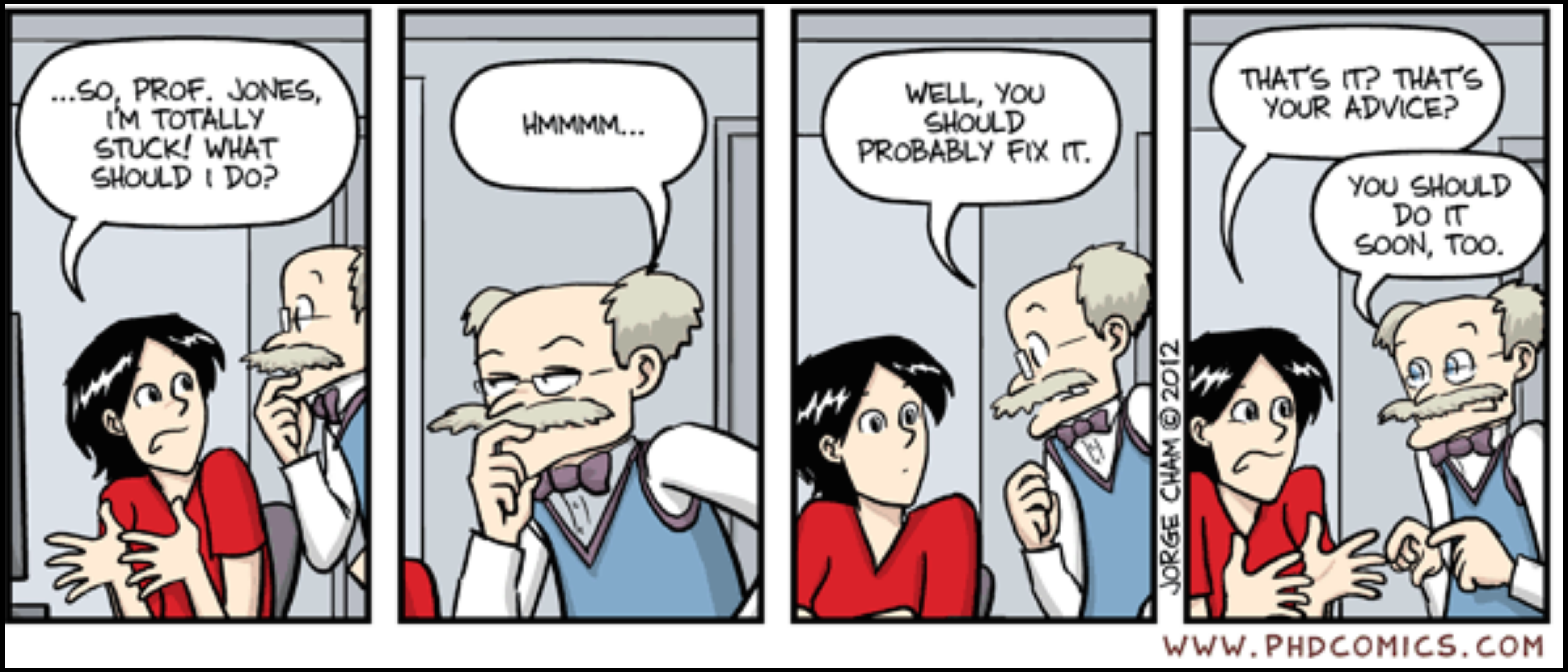
Decoding all or nothing (of the interference), the logical next step is…..

Forced jokes


I DON'T ALWAYS PREFER BEING REFERRED TO AS "DR."
BUT WHEN I DO, IT'S USUALLY AFTER BEING REFERRED TO AS MS., MRS., HEY, OR "EXCUSE ME DO YOU KNOW WHO THE PROFESSOR IS FOR THIS CLASS."


NORMAL DISTRIBUTION
PARANORMAL DISTRIBUTION


REJECT
READING REVIEWS
FIND A NEW JOURNAL
REFORMAT REFS
PROPOSE REVIEWERS
RESUBMIT
MINOR REVISION
ACCEPT


...SO, PROF. JONES, I'M TOTALLY STUCK! WHAT SHOULD I DO?
HMMMM...
WELL, YOU SHOULD PROBABLY FIX IT.
JORGE CHAM © 2012
THAT'S IT? THAT'S YOUR ADVICE?
YOU SHOULD DO IT SOON, TOO.
WWW.PHDCOMICS.COM

Decoding all or nothing (of the interference), the logical next step is…..

DMC: Han+Kobayashi inner bound

+

Largest single-letter achievable rate region

[T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," IEEE Trans. Inf. Theory, vol. 27, no. 1, pp. 49–60, 1981.]

Achieves capacity when we know it

[H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, "On the Han–Kobayashi region for the interference channel," IEEE Trans. Inf. Theory, vol. 54, no. 7, pp. 3188–3195, July 2008.]

Thought to perhaps be the capacity region in general, but NO!

[C. Nair, L. Xia, M. Yazdanpanah, ``**Sub-optimality of the Han-and-Kobayashi Achievable Region for Interference Channels,"** *Proc. of ISIT,* 2015.]

more later… for now, let us understand this important region

DMC: Han+Kobayashi inner bound

+

[T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," IEEE Trans. Inf. Theory, vol. 27, no. 1, pp. 49–60, 1981.]

Theorem (Han+Kobayashi inner bound). A rate pair (R_1, R_2) is achievable for a DM-IC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ if it satisfies

$$R_1 \leq I(X_1; Y_1|U_2, Q) \quad (1)$$

$$R_2 \leq I(X_2; Y_2|U_1, Q) \quad (2)$$

$$R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_2; Y_2|U_1, U_2, Q) \quad (3)$$

$$R_1 + R_2 \leq I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|Q) \quad (4)$$

$$R_1 + R_2 \leq I(X_1, U_2; Y_1|U_1, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (5)$$

$$2R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (6)$$

$$R_1 + 2R_2 \leq I(X_2, U_1; Y_2|Q) + I(X_2; Y_2|U_1, U_2, Q) + I(X_1, U_2; Y_1|U_1, Q) \quad (7)$$

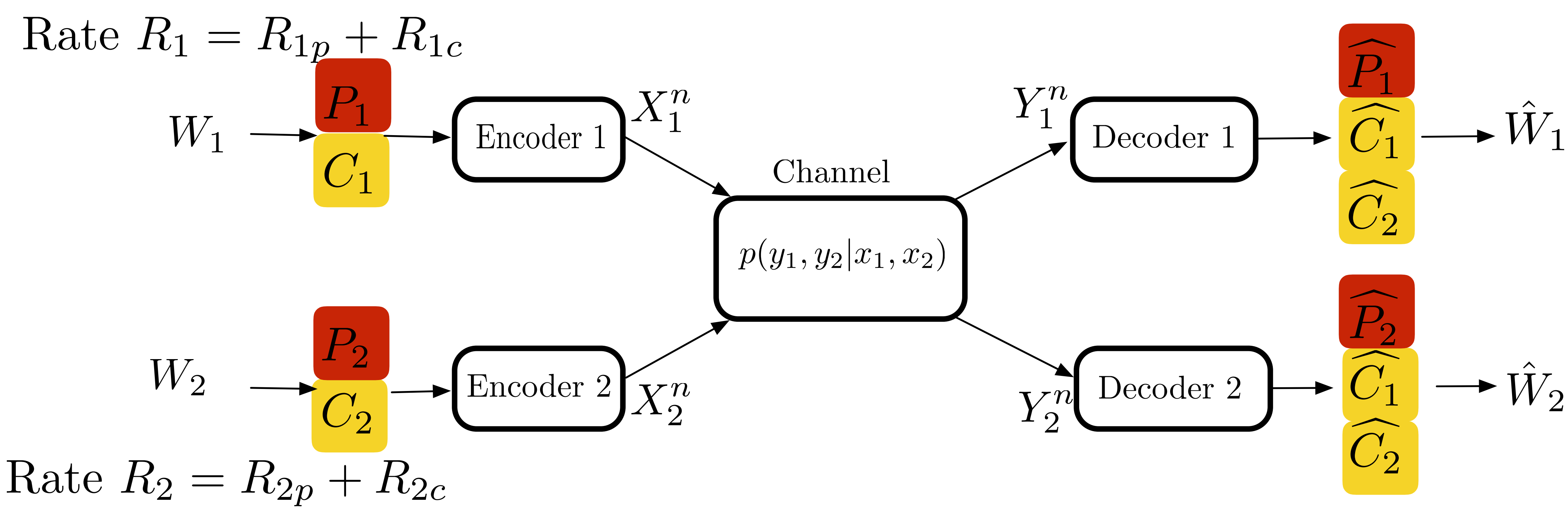
for some $p(q, u_1, u_2, x_1, x_2) = p(q)p(u_1, x_1|q)p(u_2, x_2|q)$ where $|\mathcal{U}_1| \leq |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \leq |\mathcal{X}_2| + 4$, and $|\mathcal{Q}| \leq 7$.

Handout 3: proof of H+K

Outline:

1) Split message into "public" and "private" parts
2) At each Tx, superpose public over private
3) At each Rx, decode both public messages and the desired private message
4) Rate region looks like two simultaneous 3 user MAC channels, one at each receiver
5) Fourier-Motzkin eliminate to put in terms of (R_1, R_2)

1) Split message into "public" and "private" parts

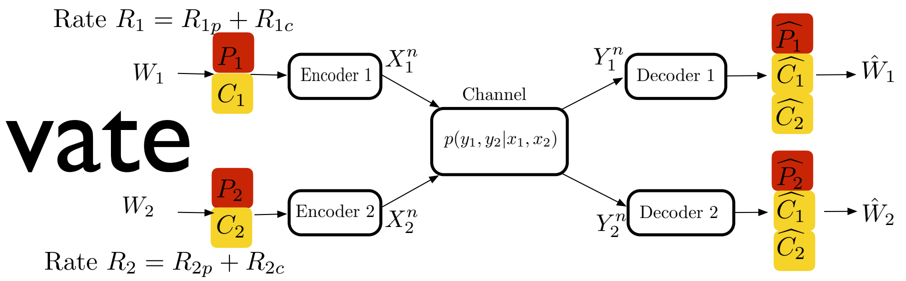


"Private" = decoded only by intended

"Public" = common = decoded by everyone

Idea: carefully split so can decode part of the interference

2) At each Tx, superpose public over private



Codebook generation: Fix $p(q)p(u_1, x_1|q)p(u_2, x_2|q)$

Generate a sequence $q^n \sim \prod_{i=1}^n p_Q(p_i)$

Tx 1 codebook: randomly and conditional independently generate $2^{nR_{1c}}$ sequences $u_1^n(w_{1c}), w_{1c} \in [1 : 2^{nR_{1c}}]$, each according to $\prod_{i=1}^n p_{U_1|Q}(u_{1i}|q_i)$. For each $u_1^n(w_{1c})$, randomly and conditionally independently generate $2^{nR_{1p}}$ sequences $x_1^n(w_{1c}, w_{1p}), w_{1p} \in [1 : 2^{nR_{1p}}]$, each according to $\prod_{i=1}^n p_{X_1|U_1,Q}(x_{1i}|u_{1i}(w_{1c}), q_i)$

(similarly for Tx 2 codebook)

Encoding: to send $w_1 = (w_{1c}, w_{1p})$, encoder 1 transmits $x_1^n(w_{1c}, w_{1p})$ (similarly for encoder 2)

Decoding: upon receiving y_1^n, decoder 1 finds the unique message pair $(\widehat{w_{1c}}, \widehat{w_{1p}})$ such that $(q^n, u_1^n(\widehat{u_{1c}}), u_2^n(w_{2c}), x_1^n(\widehat{w_{1c}}, \widehat{w_{1p}}), y_1^n)$ are jointly typical for some $w_{2c} \in [1 : 2^{nR_{2c}}]$. If no unique pair exists, the decoder declares an error. Similarly for decoder 2.

3) At each Rx, decode both public messages and the desired private message

Possible errors:

	w_{1c}	w_{2c}	w_{1p}	Joint pmf
1	1	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n, u_2^n)$
2	1	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n, u_2^n)$
3	$*$	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
4	$*$	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
5	1	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n)$
6	$*$	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
7	$*$	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
8	1	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n)$

Count them and get probability from the packing lemma:

taken from [A. El Gamal and Y.H. Kim, "Lecture Notes on Network Information Theory," http://arxiv.org/abs/1001.3404.]

3) At each Rx, decode both public messages and the desired private message

Packing Lemma [3]. Let $(U, X, Y) \sim p(u, x, y)$. Let $(\tilde{U^n}, \tilde{Y^n}) \sim p(\tilde{u^n}, \tilde{y^n})$ be a pair of arbitrarily distributed random sequences (not necessarily according to $\prod_{I=1}^{n} p_{U,Y}(\tilde{u}_i, \tilde{y}_i)$). Let $X^n(m), m \in \mathcal{A}$, where $|\mathcal{A}| \leq 2^{nR}$, be random sequences, each distributed according to $\prod_{i=1}^{n} p_{X|U}(x_i|\tilde{u}_1)$. Assume that $X^n(m), m \in \mathcal{A}$, is pairwise conditionally independent of $\tilde{Y^n}$ given $\tilde{U^n}$, but is arbitrarily dependent on other $X^n(m)$ sequences. Then, there exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\Pr\{(\tilde{U^n}, X^n(m), \tilde{Y^n}) \in \mathcal{T}_\epsilon^n\} \to 0 \text{ as } n \to \infty \text{ if } R < I(X;Y|U) - \delta(\epsilon),$$

where $\mathcal{T}_\epsilon^{(n)}$ is defined as the typical set

$$\mathcal{T}_\epsilon^{(n)} = \mathcal{T}_\epsilon^{(n)}(U, X, Y) := \{(u^n, x^n, y^n) : |\pi(u, x, y|u^n, x^n, y^n) - p(u, x, y)| \leq \epsilon \cdot p(u, x, y)\},$$

where

$$\pi(u, x, y|u^n, x^n, y^n) = \frac{|\{i : (u_i, x_i, y_i) = (u, x, y)|}{n} \text{ for } (u, x, y) \in \mathcal{U} \times \mathcal{X} \times \mathcal{Y}$$

U is "correct" and jointly distributed with output Y

X is "incorrect" of rate R

Prob. that (X,U,Y) are jointly typical vanishes if

$$R < I(X;Y|U)$$

[3] A. El Gamal and Y.-H. Kim, Network Information Theory. Cambridge University Press, 2011.

3) At each Rx, decode both public messages and the desired private message

Possible errors:

	w_{1c}	w_{2c}	w_{1p}	Joint pmf
1	1	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n, u_2^n)$
2	1	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n, u_2^n)$
3	$*$	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
4	$*$	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
5	1	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n)$
6	$*$	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
7	$*$	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
8	1	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n)$

Count them and get probability from the packing lemma:

$$2^{nR_{1p}} \cdot 2^{-n(I(X_1;Y_1|U_1,U_2,Q)-\delta(\epsilon))}$$

taken from [A. El Gamal and Y.H. Kim, “Lecture Notes on Network Information Theory,” http://arxiv.org/abs/1001.3404.]

4) Rate region looks like two simultaneous 3 user MAC channels, one at each receiver

	w_{1c}	w_{2c}	w_{1p}	Joint pmf
1	1	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n, u_2^n)$
2	1	1	*	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n, u_2^n)$
3	*	1	*	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
4	*	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_2^n)$
5	1	*	*	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid u_1^n)$
6	*	*	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
7	*	*	*	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
8	1	*	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n \mid x_1^n)$

$$R_{1p} \leq I(X_1; Y_1 | U_1, U_2, Q)$$
$$R_{1p} + R_{1c} \leq I(X_1; Y_1 | U_2, Q)$$
$$R_{1p} + R_{2c} \leq I(X_1, U_2; Y_1 | U_1, Q)$$
$$R_{1p} + R_{1c} + R_{2c} \leq I(X_1, U_2; Y_1 | Q)$$

symmetry at user 2

$$R_{2p} \leq I(X_1; Y_2 | U_1, U_2, Q)$$
$$R_{2p} + R_{2c} \leq I(X_2; Y_2 | U_1, Q)$$
$$R_{2p} + R_{1c} \leq I(X_2, U_1; Y_2 | U_2, Q)$$
$$R_{2p} + R_{2c} + R_{1c} \leq I(X_2, U_1; Y_2 | Q)$$

taken from [A. El Gamal and Y.H. Kim, "Lecture Notes on Network Information Theory," http://arxiv.org/abs/1001.3404.]

5) Fourier-Motzkin eliminate to put in terms of (R_1, R_2)

$$
\begin{aligned}
R_{1p} &\leq I(X_1; Y_1 | U_1, U_2, Q) \\
R_{1p} + R_{1c} &\leq I(X_1; Y_1 | U_2, Q) \\
R_{1p} + R_{2c} &\leq I(X_1, U_2; Y_1 | U_1, Q) \\
R_{1p} + R_{1c} + R_{2c} &\leq I(X_1, U_2; Y_1 | Q) \\
R_{2p} &\leq I(X_1; Y_2 | U_1, U_2, Q) \\
R_{2p} + R_{2c} &\leq I(X_2; Y_2 | U_1, Q) \\
R_{2p} + R_{1c} &\leq I(X_2, U_1; Y_2 | U_2, Q) \\
R_{2p} + R_{2c} + R_{1c} &\leq I(X_2, U_1; Y_2 | Q)
\end{aligned}
$$

OLD

$$
\begin{aligned}
R_1 &\leq I(X_1; Y_1 | U_2, Q) \\
R_2 &\leq I(X_2; Y_2 | U_1, Q) \\
R_1 + R_2 &\leq I(X_1, U_2; Y_1 | Q) + I(X_2; Y_2 | U_1, U_2, Q) \\
R_1 + R_2 &\leq I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | Q) \\
R_1 + R_2 &\leq I(X_1, U_2; Y_1 | U_1, Q) + I(X_2, U_1; Y_2 | U_2, Q) \\
2R_1 + R_2 &\leq I(X_1, U_2; Y_1 | Q) + I(X_1; Y_1 | U_1, U_2, Q) + I(X_2, U_1; Y_2 | U_2, Q) \\
R_1 + 2R_2 &\leq I(X_2, U_1; Y_2 | Q) + I(X_2; Y_2 | U_1, U_2, Q) + I(X_1, U_2; Y_1 | U_1, Q)
\end{aligned}
$$

NEW

taken from [A. El Gamal and Y.H. Kim, "Lecture Notes on Network Information Theory," http://arxiv.org/abs/1001.3404.]

Comments on H+K

Theorem (Han+Kobayashi inner bound). A rate pair (R_1, R_2) is achievable for a DM-IC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ if it satisfies

$$R_1 \leq I(X_1; Y_1|U_2, Q) \tag{1}$$
$$R_2 \leq I(X_2; Y_2|U_1, Q) \tag{2}$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_2; Y_2|U_1, U_2, Q) \tag{3}$$
$$R_1 + R_2 \leq I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|Q) \tag{4}$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|U_1, Q) + I(X_2, U_1; Y_2|U_2, Q) \tag{5}$$
$$2R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|U_2, Q) \tag{6}$$
$$R_1 + 2R_2 \leq I(X_2, U_1; Y_2|Q) + I(X_2; Y_2|U_1, U_2, Q) + I(X_1, U_2; Y_1|U_1, Q) \tag{7}$$

for some $p(q, u_1, u_2, x_1, x_2) = p(q)p(u_1, x_1|q)p(u_2, x_2|q)$ where $|\mathcal{U}_1| \leq |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \leq |\mathcal{X}_2| + 4$, and $|\mathcal{Q}| \leq 7$.

1) Key difficulty?

Ask Mojtaba Vaezi

Comments on H+K

Theorem (Han+Kobayashi inner bound). A rate pair (R_1, R_2) is achievable for a DM-IC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ if it satisfies

$$R_1 \leq I(X_1; Y_1|U_2, Q) \quad (1)$$
$$R_2 \leq I(X_2; Y_2|U_1, Q) \quad (2)$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_2; Y_2|U_1, U_2, Q) \quad (3)$$
$$R_1 + R_2 \leq I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|Q) \quad (4)$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|U_1, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (5)$$
$$2R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (6)$$
$$R_1 + 2R_2 \leq I(X_2, U_1; Y_2|Q) + I(X_2; Y_2|U_1, U_2, Q) + I(X_1, U_2; Y_1|U_1, Q) \quad (7)$$

for some $p(q, u_1, u_2, x_1, x_2) = p(q)p(u_1, x_1|q)p(u_2, x_2|q)$ where $|\mathcal{U}_1| \leq |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \leq |\mathcal{X}_2| + 4$, and $|\mathcal{Q}| \leq 7$.

1) Key difficulty?

2) Tight for a few classes of channels, up next

Comments on H+K

Theorem (Han+Kobayashi inner bound). A rate pair (R_1, R_2) is achievable for a DM-IC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ if it satisfies

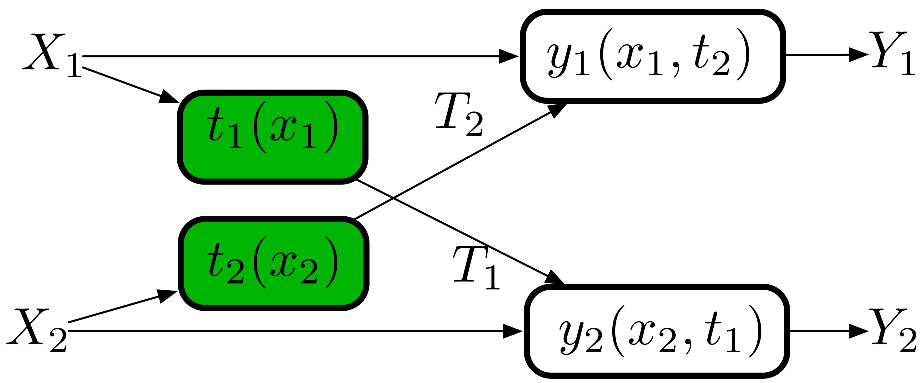
$$R_1 \leq I(X_1; Y_1|U_2, Q) \quad (1)$$
$$R_2 \leq I(X_2; Y_2|U_1, Q) \quad (2)$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_2; Y_2|U_1, U_2, Q) \quad (3)$$
$$R_1 + R_2 \leq I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|Q) \quad (4)$$
$$R_1 + R_2 \leq I(X_1, U_2; Y_1|U_1, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (5)$$
$$2R_1 + R_2 \leq I(X_1, U_2; Y_1|Q) + I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|U_2, Q) \quad (6)$$
$$R_1 + 2R_2 \leq I(X_2, U_1; Y_2|Q) + I(X_2; Y_2|U_1, U_2, Q) + I(X_1, U_2; Y_1|U_1, Q) \quad (7)$$

for some $p(q, u_1, u_2, x_1, x_2) = p(q)p(u_1, x_1|q)p(u_2, x_2|q)$ where $|\mathcal{U}_1| \leq |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \leq |\mathcal{X}_2| + 4$, and $|\mathcal{Q}| \leq 7$.

1) Key difficulty?
2) Tight for a few classes of channels, up next
3) As of 2015 and the great work of Chandra Nair, NOT tight in general!

Does H+K ever achieve capacity?

Class of deterministic ICs



deterministic

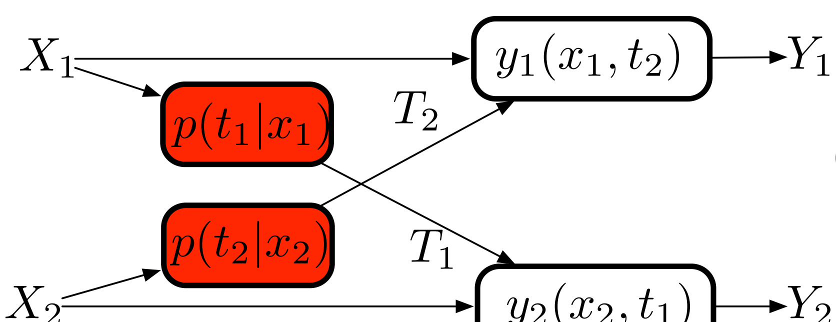
have capacity in general

- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

[A. El Gamal and M. H. M. Costa, "The capacity region of a class of deterministic interference channels," IEEE Trans. Inf. Theory, vol. 28, no. 2, pp. 343–346, 1982.]

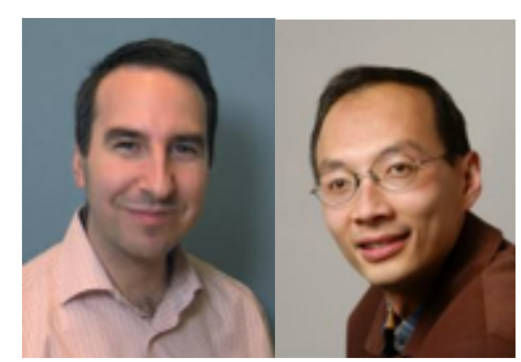
recovers / generalizes

Class of semi-deterministic ICs



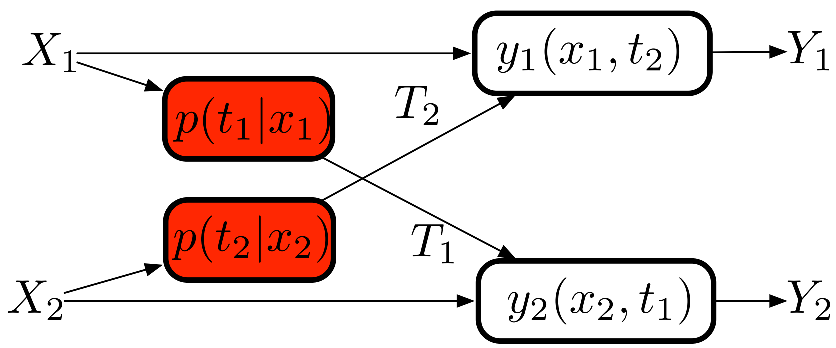
probabilistic

constant gap to capacity



.[I. E. Telatar and D. N. C. Tse, "Bounds on the capacity region of a class of interference channels," in Proc. IEEE International Symposium on Information Theory, Nice, France, June 2007.]

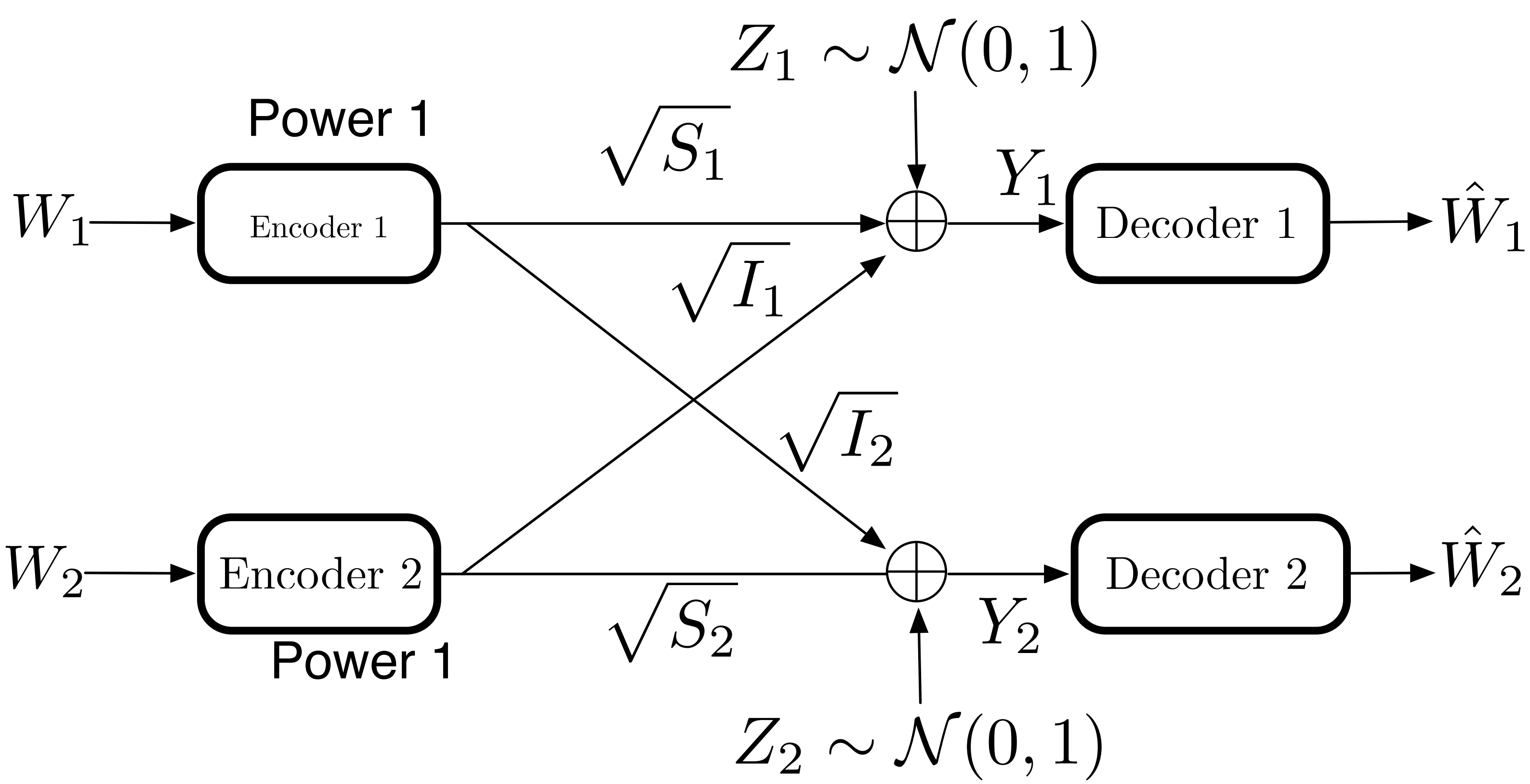
Class of semi-deterministic ICs



- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

probabilistic

Gaussian is a special case!



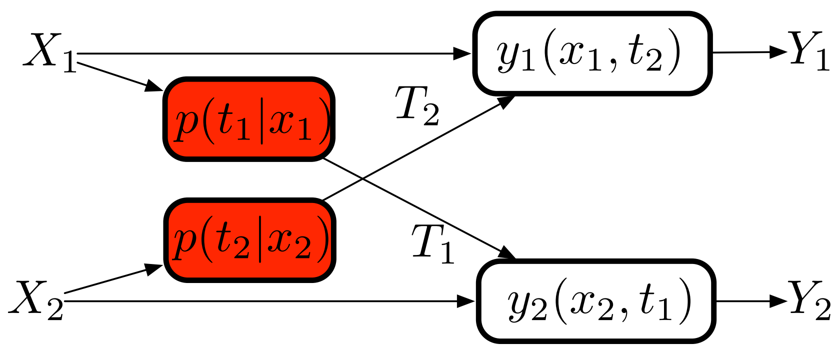
$$T_2 = \sqrt{I_1} X_2 + Z_1$$

$$Y_1 = \sqrt{S_1} X_1 + T_2$$

$$T_1 = \sqrt{I_2} X_1 + Z_2$$

$$Y_2 = \sqrt{S_2} X_2 + T_1$$

Class of semi-deterministic ICs

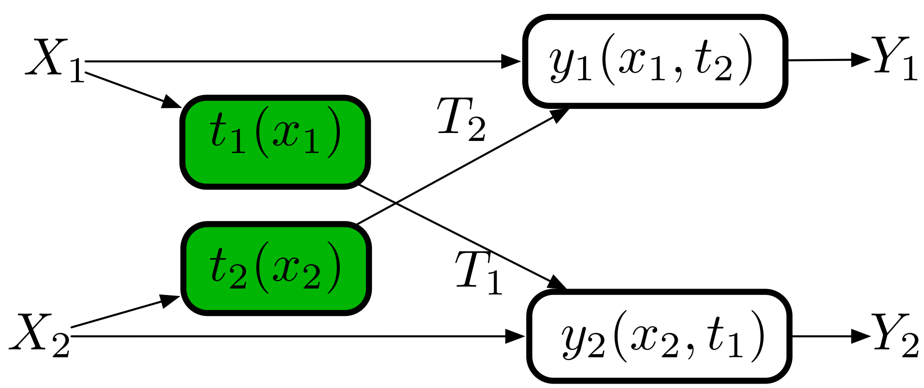


- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

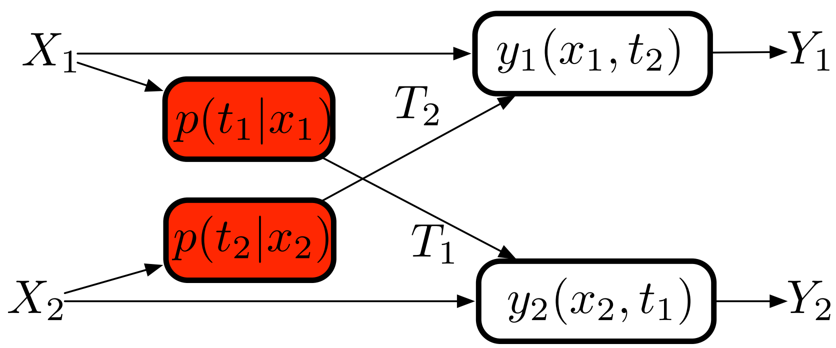
probabilistic

Gaussian is a special case!

Deterministic is a special case!



Inner bound: class of semi-deterministic ICs



- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

probabilistic

Theorem (inner bound of semi-deterministic IC) The following rate pairs (R_1, R_2) are achievable (Han+Kobayashi scheme under restriction $p(u_1, u_2|q, x_1, x_2) = p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$:

$$R_1 \le H(Y_1|U_2,Q) - H(T_2|U_2,Q) \quad (1)$$

$$R_2 \le H(Y_2|U_1,Q) - H(T_1|U_1,Q) \quad (2)$$

$$R_1 + R_2 \le H(Y_1|Q) + H(Y_2|U_1,U_2,Q) - H(T_1|U_1,Q) - H(T_2|U_2,Q) \quad (3)$$

$$R_1 + R_2 \le H(Y_1|U_1,U_2,Q) + H(Y_2|Q) - H(T_1|U_1,Q) - H(T_2|U_2,Q) \quad (4)$$

$$R_1 + R_2 \le H(Y_1|U_1,Q) + H(Y_2|U_2,Q) - H(T_1|U_1,Q) - H(T_2|U_2,Q) \quad (5)$$

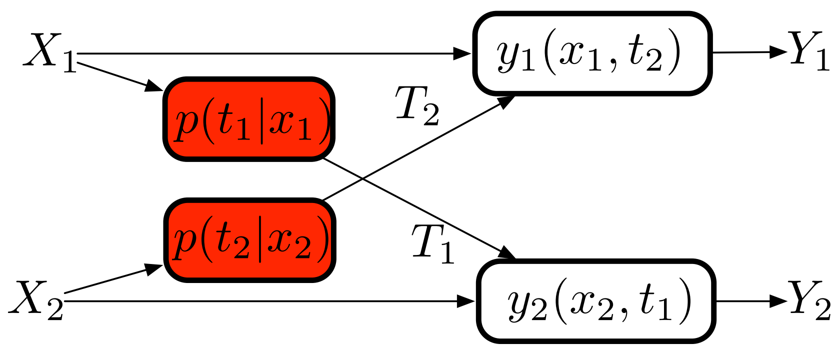
$$2R_1 + R_2 \le H(Y_1|Q) + H(Y_1|U_1,X_2,Q) + H(Y_2|U_2,Q) - H(T_1|U_1,Q) - 2H(T_2|U_2,Q) \quad (6)$$

$$R_1 + 2R_2 \le H(Y_2|Q) + H(Y_2|U_1,U_2,Q) + H(Y_1|U_1,Q) - 2H(T_1|U_1,Q) - H(T_2|U_2,Q) \quad (7)$$

Han+Kobayashi

for some $p(q, x_1, x_2) = p(q)p(x_1|q)p(x_2|q)$ and $p(u_1, u_2|q, x_1, x_2) = p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$.

Outer bound: class of semi-deterministic ICs



- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

probabilistic

$\mathcal{R}_O(Q, X_1, X_2)$

Theorem (outer bound of semi-deterministic IC) Every achievable rate pair (R_1, R_2) must satisfy

NEAT TRICK! Clever side information

$$R_1 \le H(Y_1|X_2, Q) - H(T_2|X_2) \quad (1)$$

$$R_2 \le H(Y_2|X_1, Q) - H(T_1|X_1) \quad (2)$$

$R_1 + R_2 \le H(Y_1|Q) + H(Y_2|U_2$ [illegible] $|X_1) - H(T_2|X_2)$ (3)

$R_1 + R_2 \le H(Y_1|U_1, X$ [illegible] $|Q) - H(T_1|X_1) - H(T_2|X_2)$ (4)

$R_1 + R_2 \le H$ [illegible] $H(Y_2|U_2, Q) - H(T_1|X$ [illegible] $) - H(T_2|X_2)$ (5)

$2R_1 + R_2 \le H$ [illegible] $|Q) + H(Y_1|U_1, X_2, Q$ [illegible] $Q) - H(T_1|X_1) - 2H(T_2|X_2)$ (6)

Handout!

$$R_1 + 2R_2 \le H(Y_2|Q) + H(Y_2|U_2, X_1, Q) + H(Y_1|U_1, Q) - 2H(T_1|X_1) - H(T_2|X_2) \quad (7)$$

for some $p(q, x_1, x_2) = p(q)p(x_1|q)p(x_2|q)$ and $p(u_1, u_2|q, x_1, x_2) = p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$.

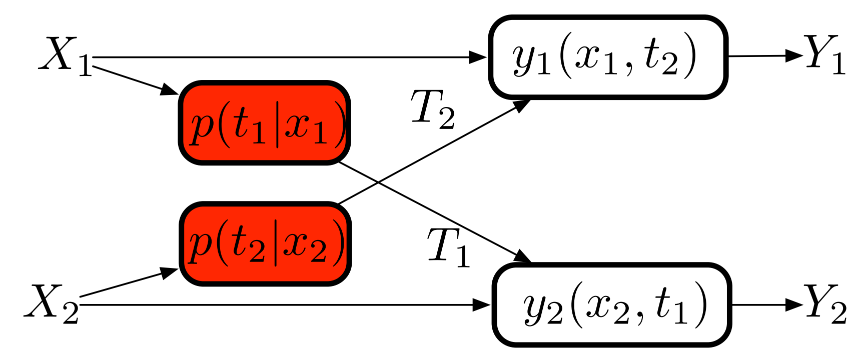
Handout 4: outer bound semi-deterministic

Outline:

1) Showcases major difficulty in converses: single-letterization (if this is desired…)
2) Neat trick of combining many multi-letter terms

Handout 4: outer bound semi-deterministic

Define random variables U_1^n and U_2^n such that U_{ji} is jointly distributed with X_{ji} according to $p_{T_j|X_j}(u|x_{ji})$, conditionally independent of T_{ji} given X_{ji} for every $j = 1, 2$ and every $i \in [1:n]$.

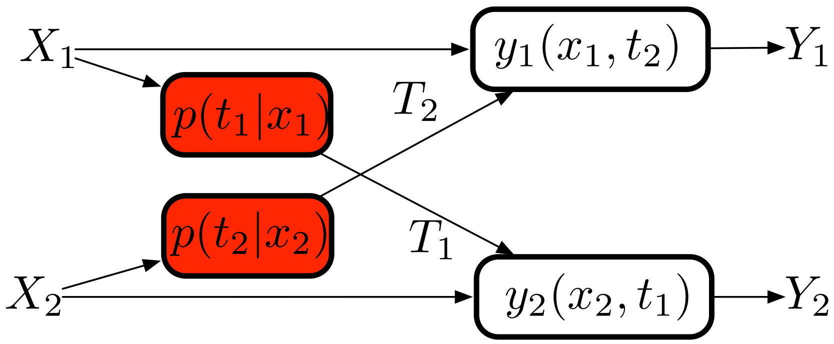


$$
\begin{aligned}
nR_j = H(W_j) &= I(W_j; Y_j^n) + H(W_j|Y_j^n) \\
&\le I(W_j; Y_j^n) + n\epsilon_n \\
&\le I(X_j^n; Y_j^n) + n\epsilon_n
\end{aligned}
$$

Bound A1:

$$
\begin{aligned}
nR_1 &\le I(X_1^n; Y_1^n) \\
&= H(Y_1^n) - H(Y_1^n|X_1^n) \\
&= H(Y_1^n) - H(T_2^n|X_1^n) \\
&= H(Y_1^n) - H(T_2^n) \\
&\le \sum_{i=1}^{n} H(Y_{1i}) - \boxed{H(T_2^n)}
\end{aligned}
$$

Handout 4: outer bound semi-deterministic



Bound B1: (genie at Rx 1 of U_1^n, X_2^n)

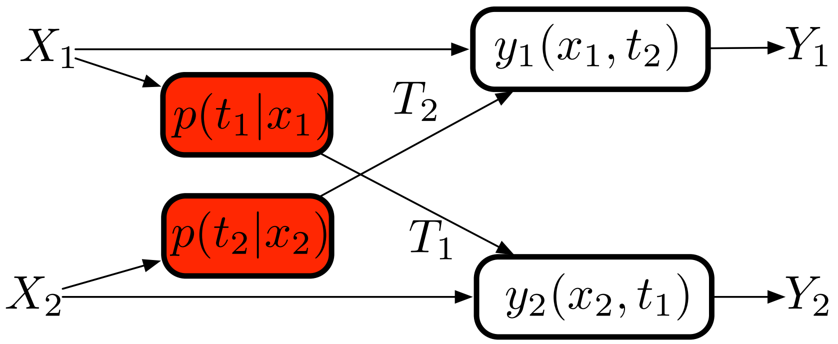
$$
\begin{aligned}
nR_1 &\le I(X_1^n; Y_1^n, U_1^n, X_2^n) \\
&= I(X_1^n; U_1^n) + I(X_1^n; X_2^n | U_1^n) + I(X_1^n; Y_1^n | U_1^n, X_2^n) \\
&= H(U_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n, X_2^n) - H(Y_1^n|X_1^n, U_1^n, X_2^n) \\
&\overset{(a)}{=} H(T_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n, X_2^n) - H(T_2^n|X_2^n) \\
&\le \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i}, X_{2i}) - \sum_{i=1}^{n} H(T_{2i}|X_{2i})
\end{aligned}
$$

Bound C1: (genie at Rx 1 of U_1^n)

why do some terms single-letterize but not others?

$$
\begin{aligned}
nR_1 &\le I(X_1^n; Y_1^n, U_1^n) \\
&= I(X_1^n; U_1^n) + I(X_1^n; Y_1^n | U_1^n) \\
&= H(U_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n) - H(Y_1^n|X_1^n, U_1^n) \\
&= H(T_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n) - H(T_2^n) \\
&\le \boxed{H(T_1^n)} - \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i})
\end{aligned}
$$

Handout 4: outer bound semi-deterministic

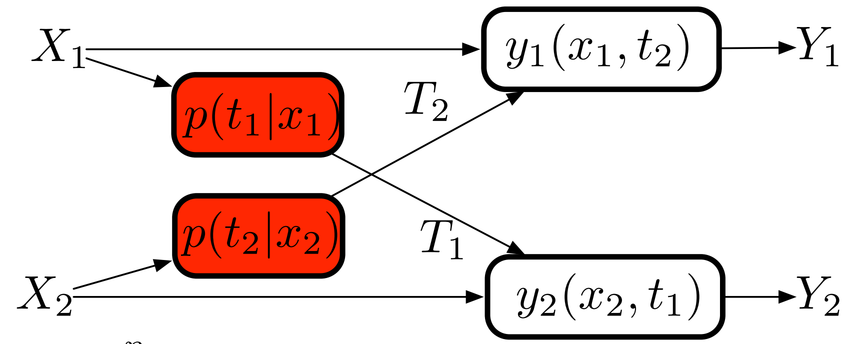


Bound D1: (genie at Rx 1 of X_2^n)

$$
\begin{aligned}
nR_1 &\leq I(X_1^n; Y_1^n, X_2^n) \\
&= I(X_1^n; X_2^n) + I(X_1^n; Y_1^n | X_2^n) \\
&= H(Y_1^n | X_2^n) - H(Y_1^n | X_1^n, X_2^n) \\
&= H(Y_1^n | X_2^n) - H(T_2^n | X_2^n) \\
&\leq \sum_{i=1}^{n} H(Y_{1i} | X_{2i}) - \sum_{i=1}^{n} H(T_{2i} | X_{2i})
\end{aligned}
$$

by symmetry, obtain analogous bounds at Rx 2

Handout 4: outer bound semi-deterministic



Bound A1: $$nR_1 \le \sum_{i=1}^{n} H(Y_{1i}) - \boxed{H(T_2^n)}$$

Bound B1: $$nR_1 \le \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i},X_{2i}) - \sum_{i=1}^{n} H(T_{2i}|X_{2i})$$

Bound C1: $$nR_1 \le \boxed{H(T_1^n)} - \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i})$$

Bound D1: $$nR_1 \le \sum_{i=1}^{n} H(Y_{1i}|X_{2i}) - \sum_{i=1}^{n} H(T_{2i}|X_{2i})$$

Bound A2: $$nR_2 \le \sum_{i=1}^{n} H(Y_{2i}) - \boxed{H(T_1^n)}$$

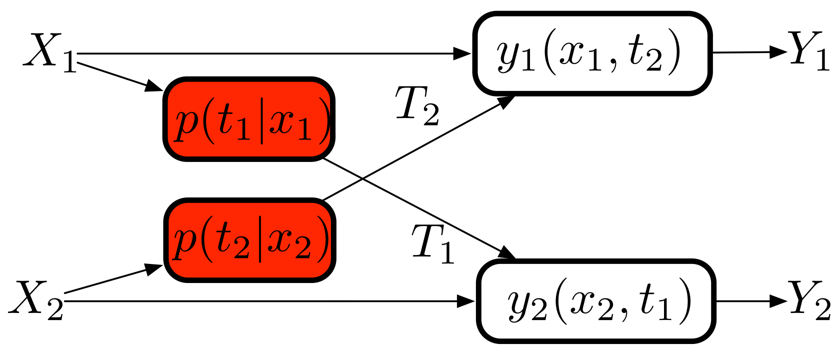
Bound B2: $$nR_2 \le \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{2i}|X_{2i}) + \sum_{i=1}^{n} H(Y_{2i}|U_{2i},X_{1i}) - \sum_{i=1}^{n} H(T_{1i}|X_{1i})$$

Bound C2: $$nR_2 \le \boxed{H(T_2^n)} - \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{2i}|X_{2i}) + \sum_{i=1}^{n} H(Y_{2i}|U_{2i})$$

Bound D2: $$nR_2 \le \sum_{i=1}^{n} H(Y_{2i}|X_{1i}) - \sum_{i=1}^{n} H(T_{1i}|X_{1i})$$

Combine these in different ways and use Q time-sharing

Outer bound: class of semi-deterministic ICs



- for every x_1, $y_1(x_1,t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2,t_1)$ is a one-to-one function of t_1

probabilistic

$\mathcal{R}_O(Q,X_1,X_2)$

Theorem (outer bound of semi-deterministic IC) Every achievable rate pair (R_1,R_2) must satisfy

$$R_1 \le H(Y_1|X_2,Q) - H(T_2|X_2) \quad (1)$$

$$R_2 \le H(Y_2|X_1,Q) - H(T_1|X_1) \quad (2)$$

$$R_1+R_2 \le H(Y_1|Q) + H(Y_2|U_2,X_1,Q) - H(T_1|X_1) - H(T_2|X_2) \quad (3)$$

$$R_1+R_2 \le H(Y_1|U_1,X_2,Q) + H(Y_2|Q) - H(T_1|X_1) - H(T_2|X_2) \quad (4)$$

$$R_1+R_2 \le H(Y_1|U_1,Q) + H(Y_2|U_2,Q) - H(T_1|X_1) - H(T_2|X_2) \quad (5)$$

$$2R_1+R_2 \le H(Y_1|Q) + H(Y_1|U_1,X_2,Q) + H(Y_2|U_2,Q) - H(T_1|X_1) - 2H(T_2|X_2) \quad (6)$$

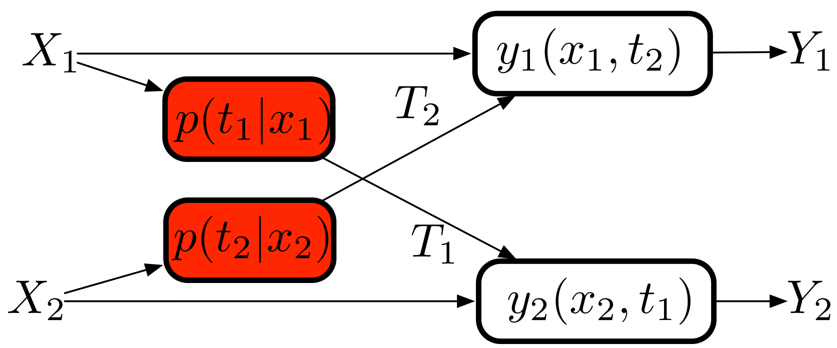
$$R_1+2R_2 \le H(Y_2|Q) + H(Y_2|U_2,X_1,Q) + H(Y_1|U_1,Q) - 2H(T_1|X_1) - H(T_2|X_2) \quad (7)$$

for some $p(q,x_1,x_2) = p(q)p(x_1|q)p(x_2|q)$ and $p(u_1,u_2|q,x_1,x_2) = p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$.

NEAT TRICK! Clever side information

Handout!

GAP: Class of semi-deterministic ICs



- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

probabilistic

Compare H+K + new outer directly

Theorem (gap for class … **eterminstic IC)** If $(R_1, R_2) \in \mathcal{R}_O(Q, X_1, X_2)$ then $(R_1$ … $X_2; T_2|U_2, Q), R_2 - I(X_1; T_1|U_1, Q))$ is achievable.

.[I. E. Telatar and D. N. C. Tse, "Bounds on the capacity region of a class of interference channels," in Proc. IEEE International Symposium on Information Theory, Nice, France, June 2007.]

Bound gap for Gaussian

Theorem (gap for Gaussian IC) If (R_1, R_2) … uter bound $\mathcal{R}_O^{\text{AWGN}}$ then $(R_1 - 1/2, R_2 - 1/2)$ is achievable.

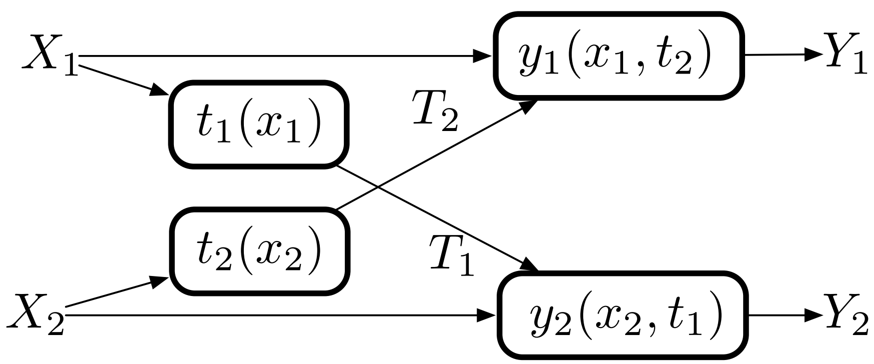
[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

When X to T is deterministic, then U=T and I(X;T|U,Q)=0

Theorem (gap for class of deterministic ICs) If … eterministic function of x_i, and the one-to-one constr… then the inner and outer bounds match and we hav… region of a class of deterministic ICs.

[A. El Gamal and M. H. M. Costa, "The capacity region of a class of deterministic interference channels," IEEE Trans. Inf. Theory, vol. 28, no. 2, pp. 343–346, 1982.]

Capacity: class of deterministic ICs



- for every x_1, $y_1(x_1, t_2)$ is a one-to-one function of t_2
- for every x_2, $y_2(x_2, t_1)$ is a one-to-one function of t_1

Theorem (capacity of class of deterministic IC) The capacity region of the class of deterministic interference channels is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq H(Y_1|T_2, Q) \quad (1)$$

$$R_2 \leq H(Y_2|T_1, Q) \quad (2)$$

$$R_1 + R_2 \leq H(Y_1|Q) + H(Y_2|T_1, T_2, Q) \quad (3)$$

$$R_1 + R_2 \leq H(Y_1|T_1, T_2, Q) + H(Y_2|Q) \quad (4)$$

$$R_1 + R_2 \leq H(Y_1|T_2, Q) + H(Y_2|T_1, Q) \quad (5)$$

$$2R_1 + R_2 \leq H(Y_1|Q) + H(Y_1|T_1, T_2, Q) + H(Y_2|T_2, Q) \quad (6)$$

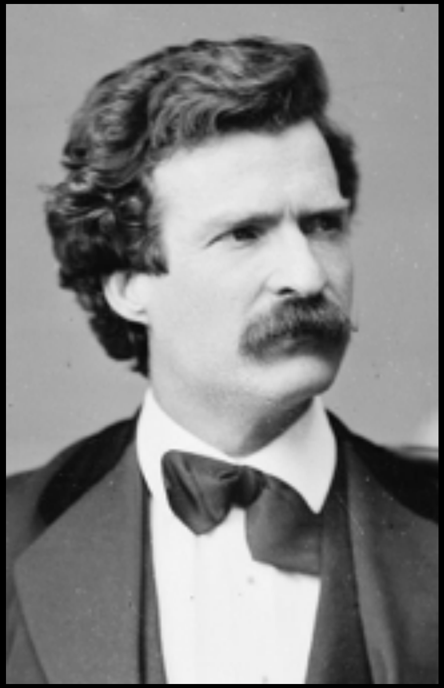
$$R_1 + 2R_2 \leq H(Y_2|Q) + H(Y_2|T_1, T_2, Q) + H(Y + 1|T_1, Q) \quad (7)$$

for some $p(q)p(x_1|q)p(x_2|q)$.

Forced interference quotes

“Do not let the things that you can’t do interfere with the things that you can do”

\- John Wooden



"I have never let schooling interfere with my education"

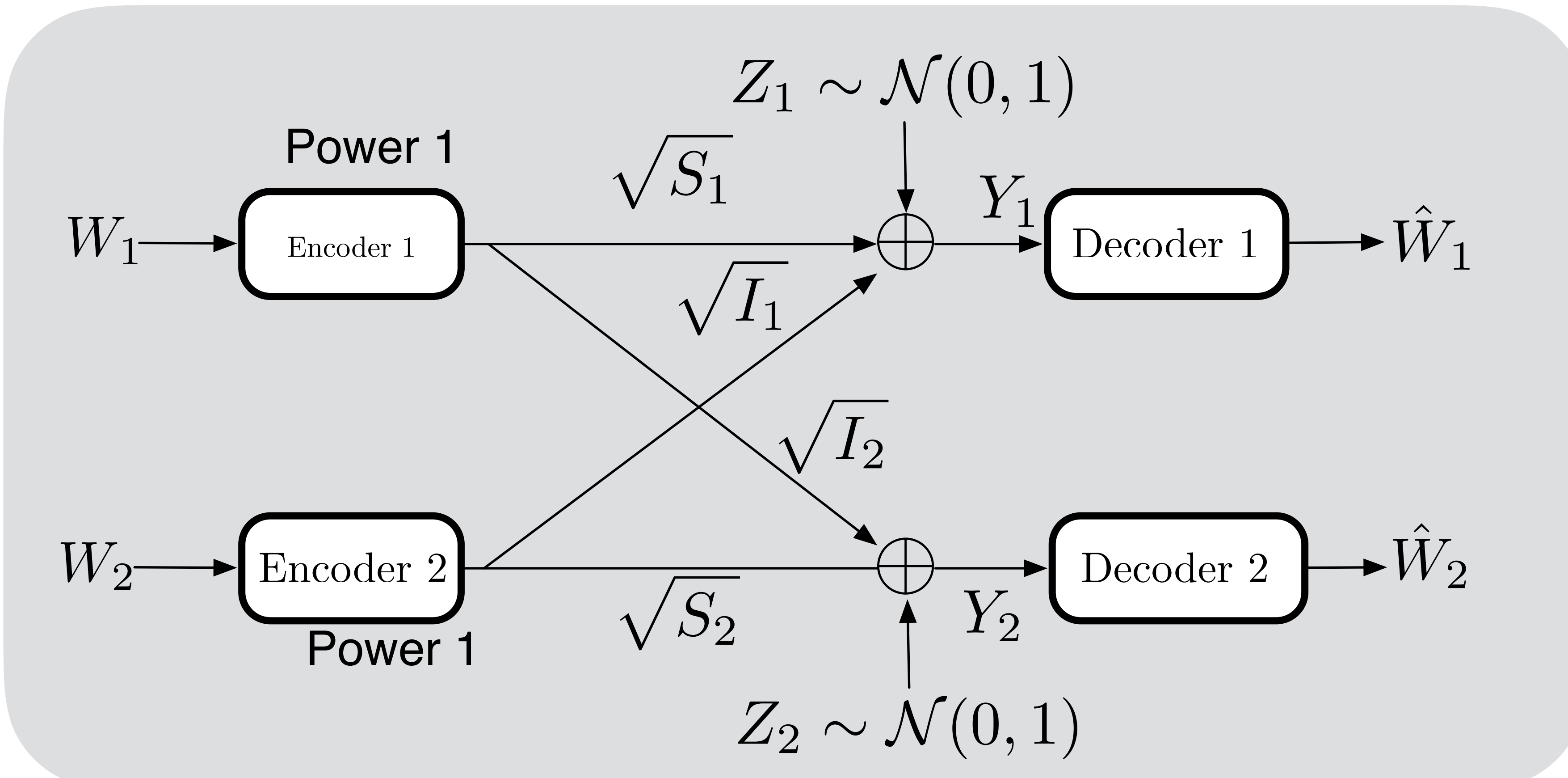
\- Mark Twain

Does H+K ever achieve capacity?

almost…..

The Gaussian IC

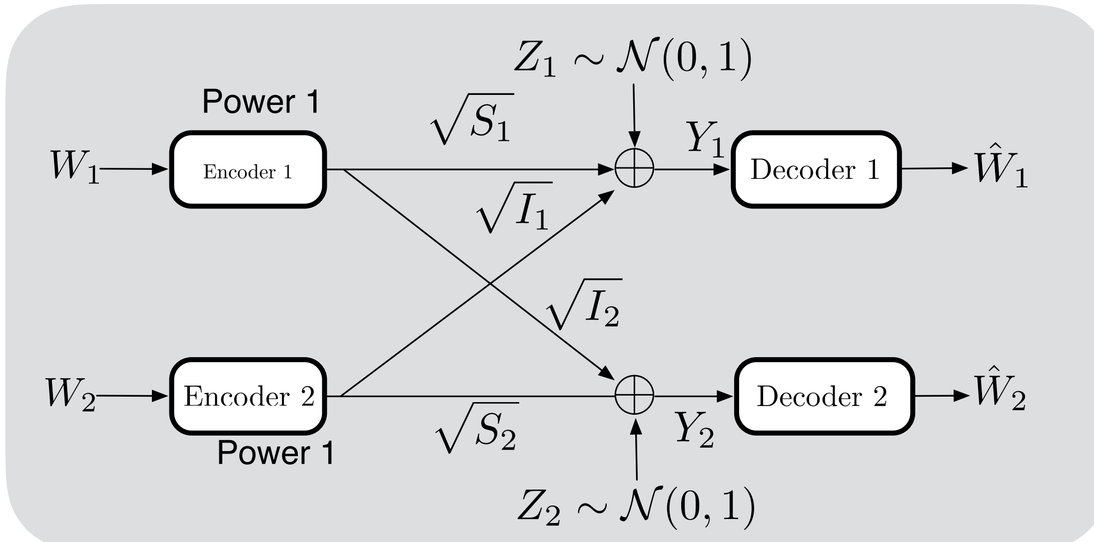
The AWGN-IC



of practical relevance in wireless systems:
cellular, wireless local area networks (WiFi),
ad hoc networks (wireless sensors or nodes)

Excellent survey of results on Gaussian IC in Introduction of [R. Bustin, H.V. Poor, and S. Shamai "The Effect of Maximal Rate Codes on the Interfering Message Rate," http://arxiv.org/abs/1404.6690]

AWGN: H+K achieves capacity to within 1/2 bit



Theorem (gap for Gaussian IC) If (R_1, R_2) is in the outer bound $\mathcal{R}_O^{\text{AWGN}}$ then $(R_1 - 1/2, R_2 - 1/2)$ is achievable.

Etkin, Tse, Wang show how to pick Gaussian inputs in H+K scheme

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

depends on the regime of operation

AWGN: the "W" curve for the GDoF

highlights effect of interference rather than noise

$$\mathcal{D}(\alpha) := \left\{ (d_1, d_2) \in \mathbb{R}_+^2 : d_i := \lim_{\substack{\text{inr} = \text{snr}^\alpha, \\ \text{snr} \to \infty}} \frac{R_i}{\frac{1}{2}\log(1+\text{snr})}, i \in [1:2], \ (R_1, R_2) \text{ is achievable} \right\}.$$

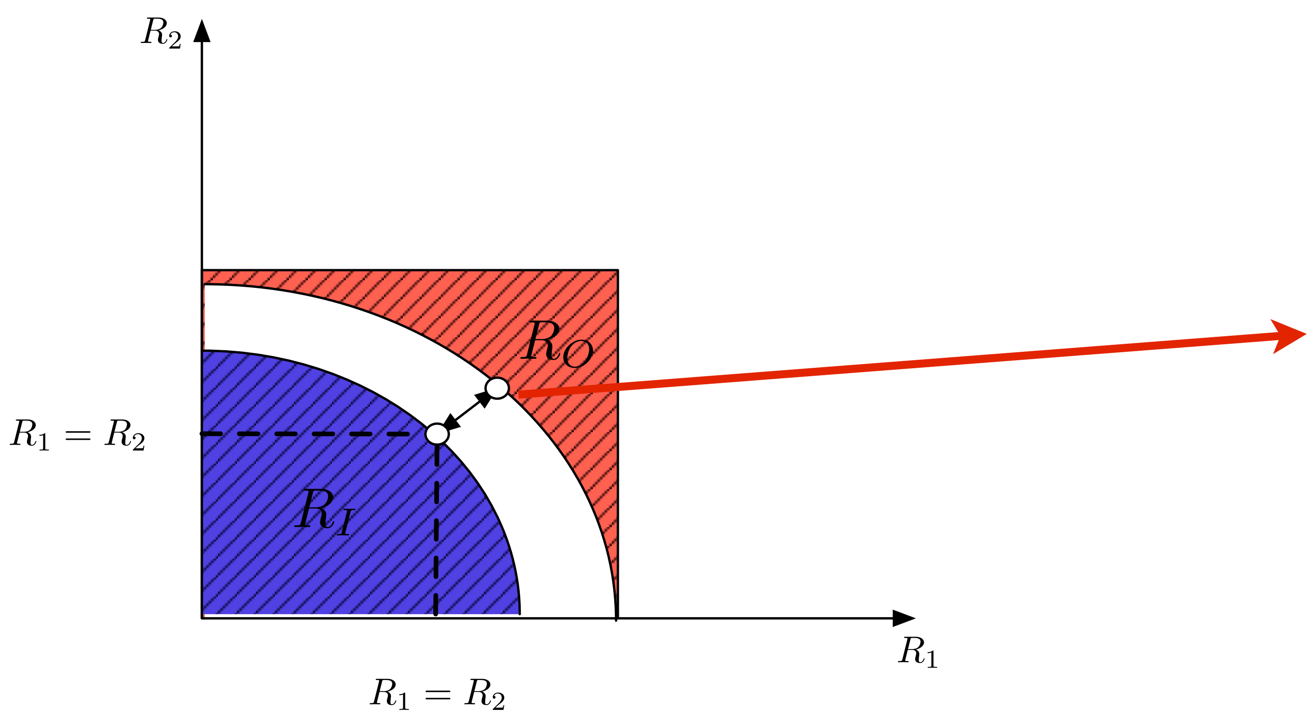
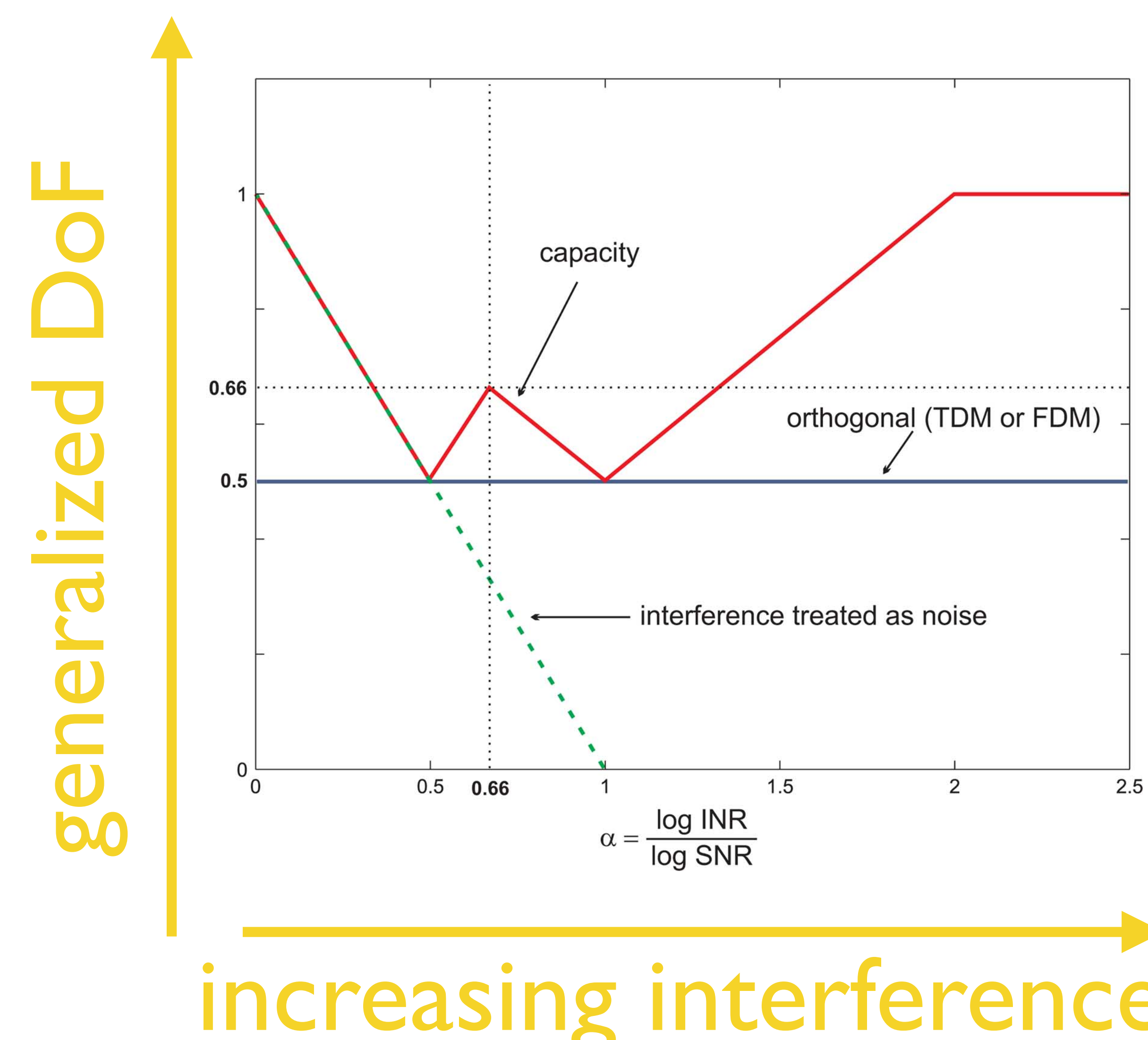


image taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

"W" curve for R_1=R_2

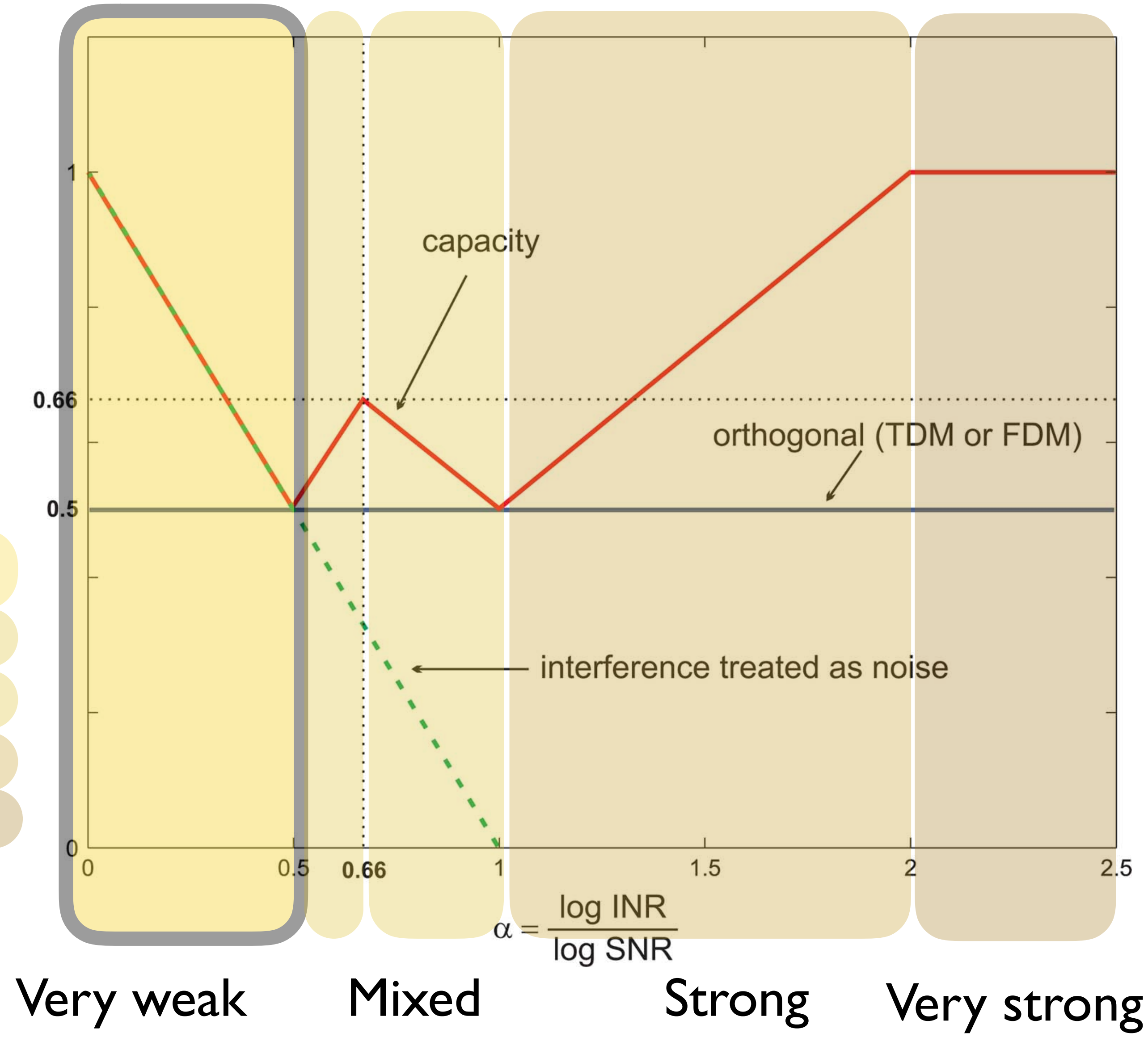


AWGN: the "W" curve for the GDoF

image + formula taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

$$C_{\text{sym}} \approx \begin{cases} \log\left(\frac{\mathsf{SNR}}{\mathsf{INR}}\right) & \log \mathsf{INR} < \frac{1}{2}\log \mathsf{SNR} \\ \log \mathsf{INR} & \frac{1}{2}\log \mathsf{SNR} < \log \mathsf{INR} < \frac{2}{3}\log \mathsf{SNR} \\ \log \frac{\mathsf{SNR}}{\sqrt{\mathsf{INR}}} & \frac{2}{3}\log \mathsf{SNR} < \log \mathsf{INR} < \log \mathsf{SNR} \\ \log \sqrt{\mathsf{INR}} & \log \mathsf{SNR} < \log \mathsf{INR} < 2\log \mathsf{SNR} \\ \log \mathsf{SNR} & \log \mathsf{INR} > 2\log \mathsf{SNR} \end{cases}$$



Regimes

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

[X. Shang, G. Kramer, and B. Chen, "A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels," IEEE Trans. Inf. Theory, vol. 55, no. 2, pp. 689–699, Feb. 2009.]

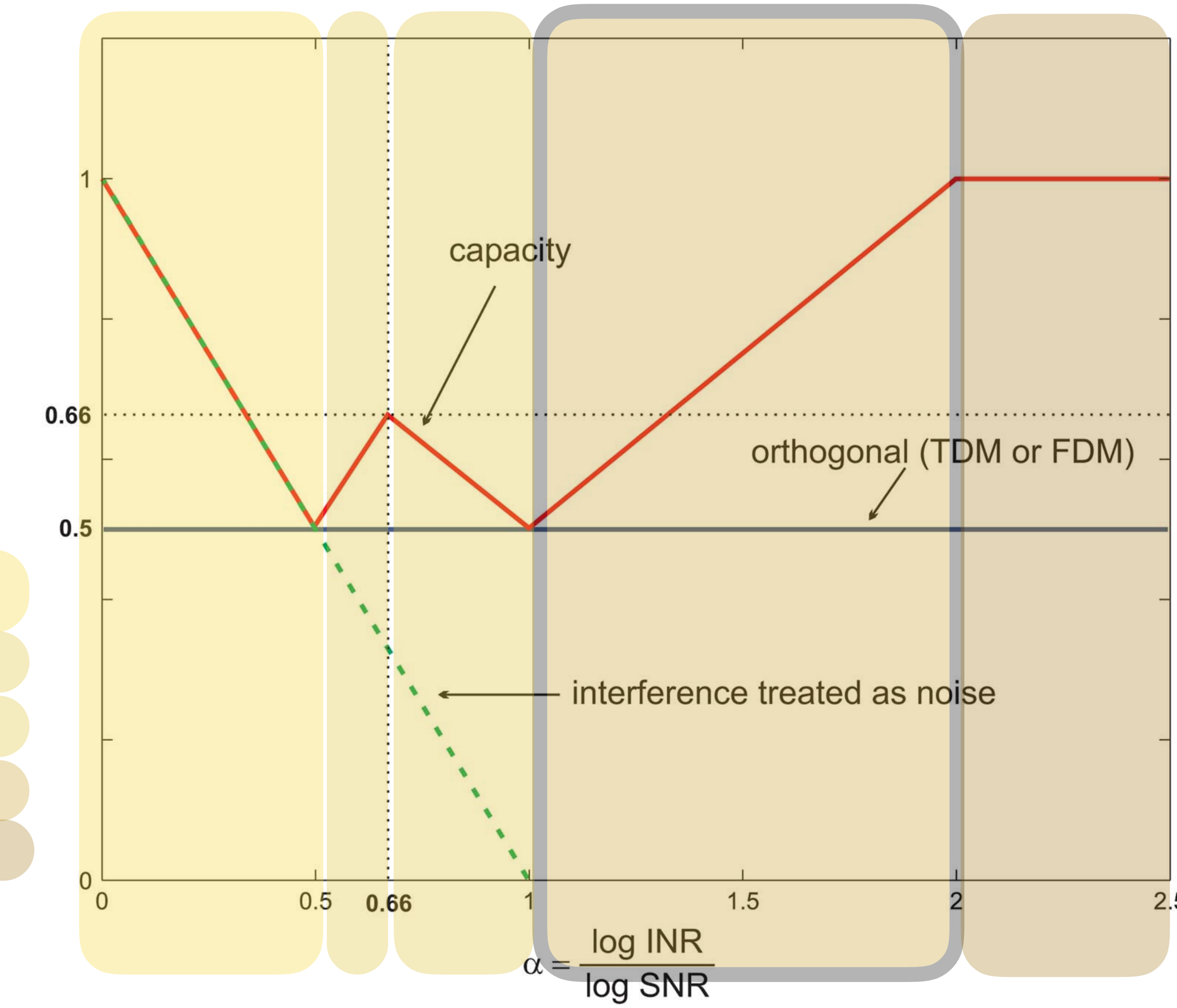
[V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: Sum capacity in the low interference regime and new outer bounds on the capacity region," IEEE Trans. Inf. Theory, vol. 55, no. 7, pp. 3032–3050, July 2009.]

[A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel," IEEE Trans. Inf. Theory, vol. 55, no. 2, pp. 620–643, Feb. 2009.]

AWGN: the "W" curve for the GDoF

image + formula taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]



$$C_{\text{sym}} \approx \begin{cases} \log\left(\frac{\mathsf{SNR}}{\mathsf{INR}}\right) & \log \mathsf{INR} < \frac{1}{2}\log \mathsf{SNR} \\ \log \mathsf{INR} & \frac{1}{2}\log \mathsf{SNR} < \log \mathsf{INR} < \frac{2}{3}\log \mathsf{SNR} \\ \log \frac{\mathsf{SNR}}{\sqrt{\mathsf{INR}}} & \frac{2}{3}\log \mathsf{SNR} < \log \mathsf{INR} < \log \mathsf{SNR} \\ \log \sqrt{\mathsf{INR}} & \log \mathsf{SNR} < \log \mathsf{INR} < 2\log \mathsf{SNR} \\ \log \mathsf{SNR} & \log \mathsf{INR} > 2\log \mathsf{SNR} \end{cases}$$

Regimes

Very weak   Mixed   Strong   Very strong

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

Strong: jointly decoding both messages at both receivers is capacity optimal, capacity known

AWGN: the "W" curve for the GDoF

image + formula taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

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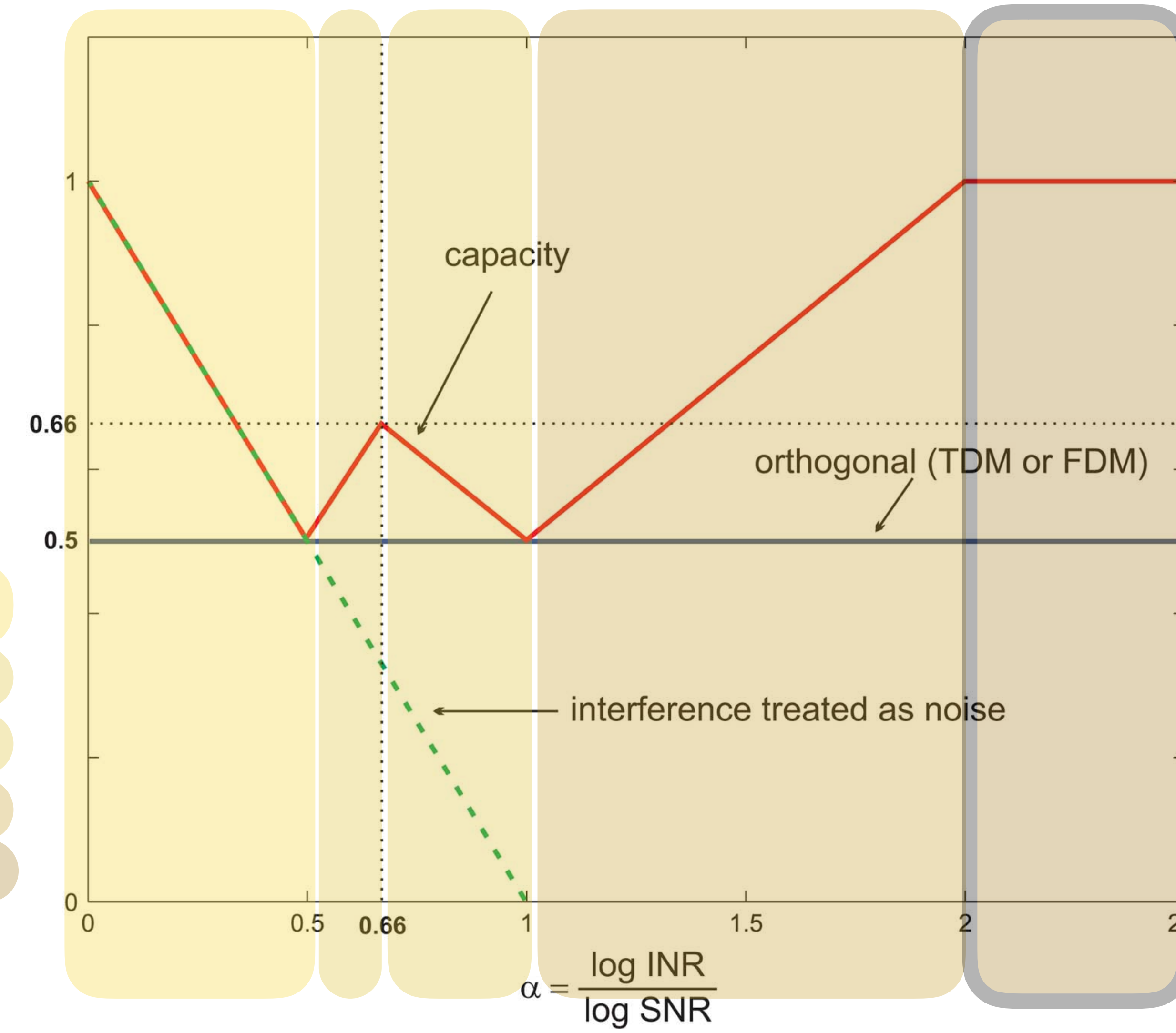

Regimes

Very weak Mixed Strong Very strong

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

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Very strong: first decode interference then desired is capacity optimal, capacity known

AWGN: the "W" curve for the GDoF

image + formula taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

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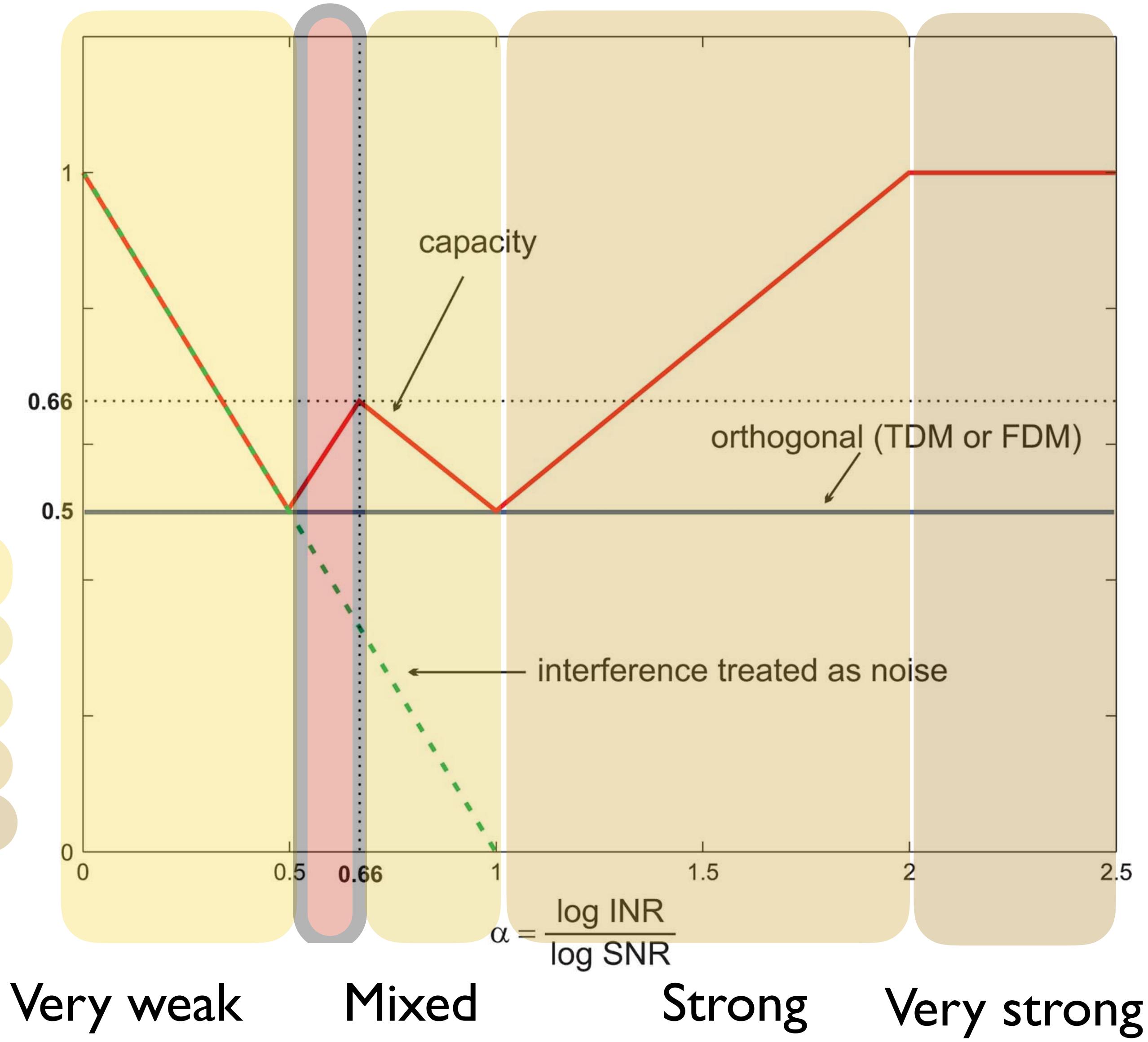

Regimes

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Strong: jointly decode both messages at both receivers is capacity optimal, capacity known

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AWGN: the "W" curve for the GDoF

image + formula taken from

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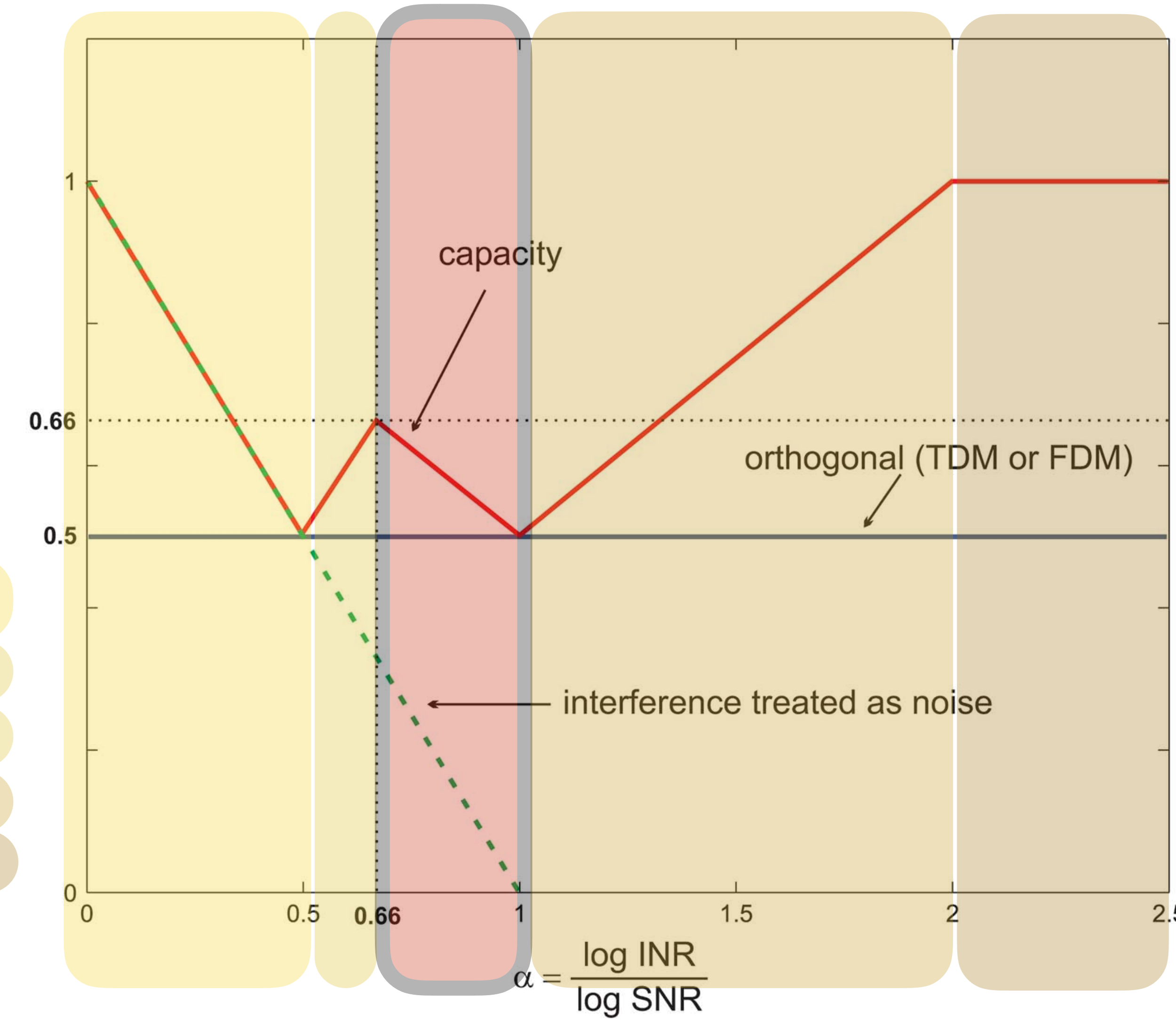

Very weak Mixed Strong Very strong

Regimes

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Mixed 2: partially decode interference H+K is gDoF optimal — larger INR hurts, capacity unknown

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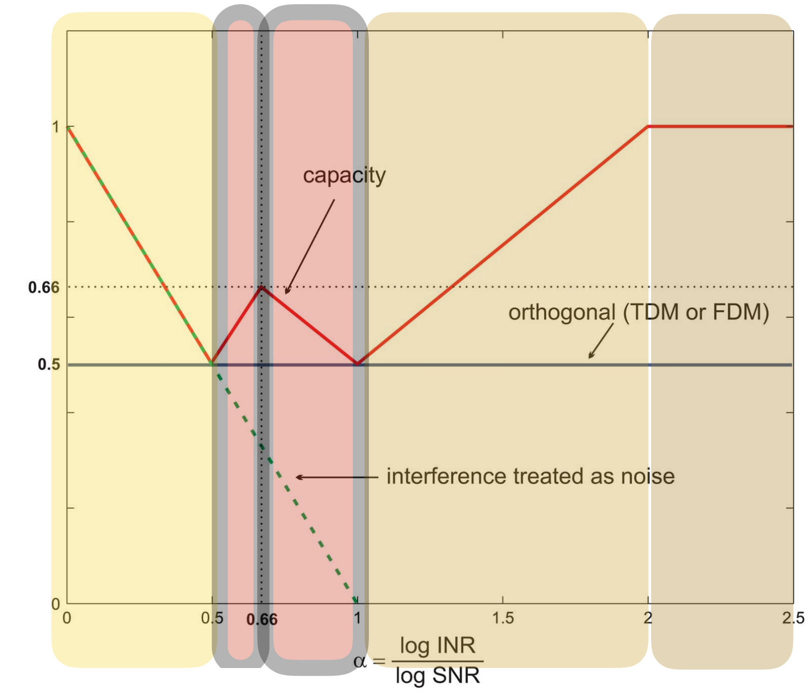
AWGN: Regimes

OPEN



HIGHLY RECOMMEND LOOKING AT DAVID TSE'S SLIDES + IGAL SASON'S PAPERS ON GAUSSIAN ICs FOR FURTHER INSIGHT!

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Mixed 2: partially decode interference H+K is gDoF optimal — larger INR hurts, capacity unknown

Strong: jointly decode both messages at both receivers is capacity optimal, capacity known

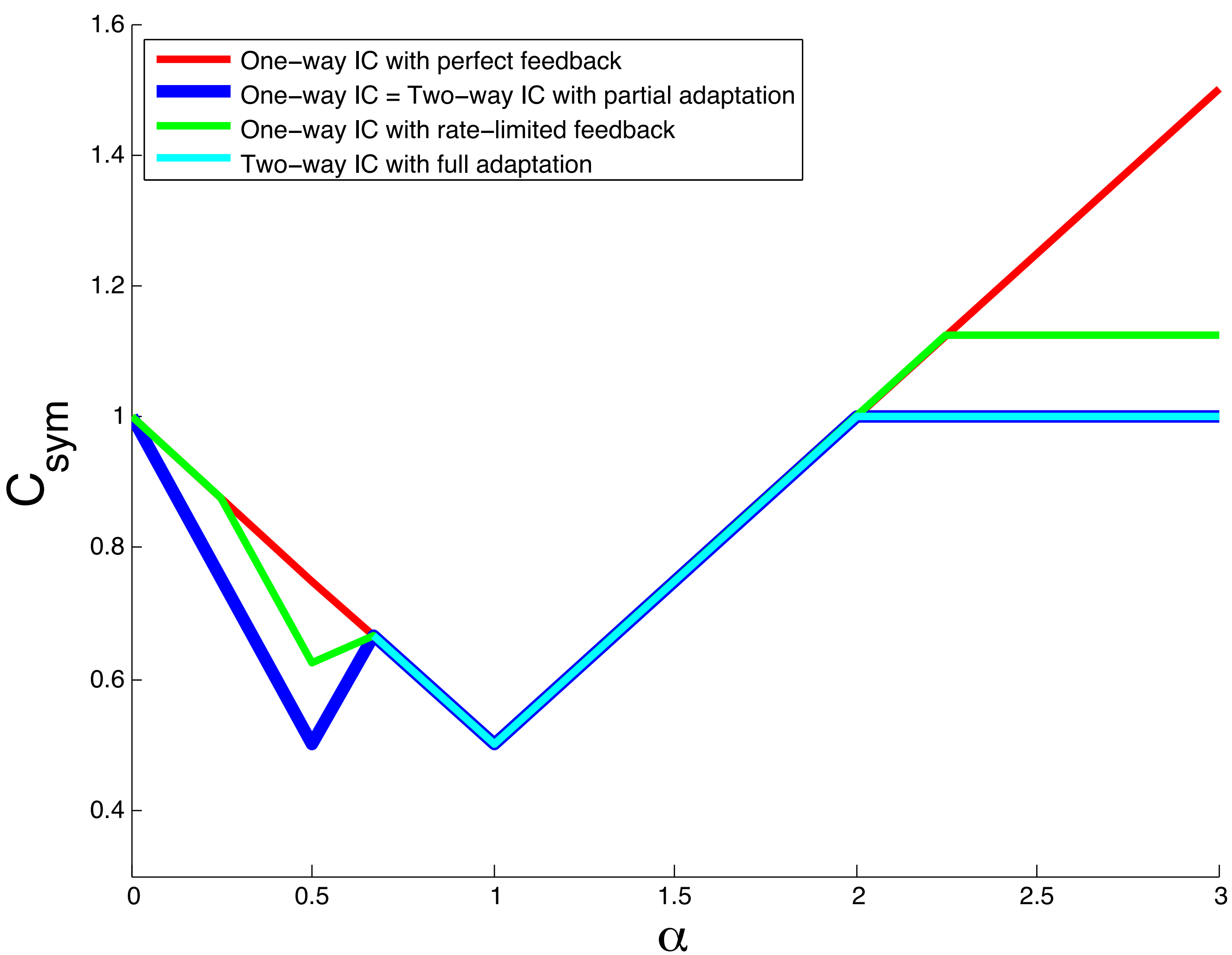
Very strong: first decode interference then desired is capacity optimal, capacity known

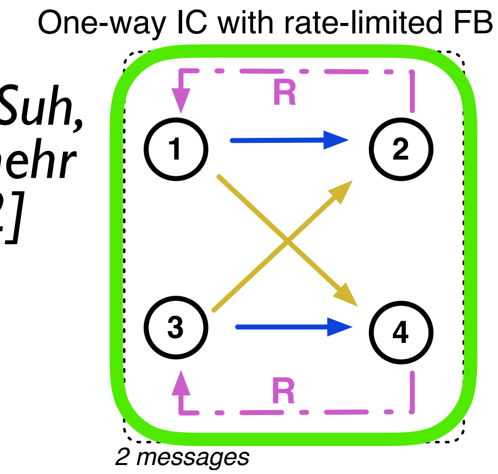
Use Han+Kobayashi scheme with private level set such that received at same level as noise at undesired receiver

Simple, almost optimal but not necessarily the best, in these challenging regimes

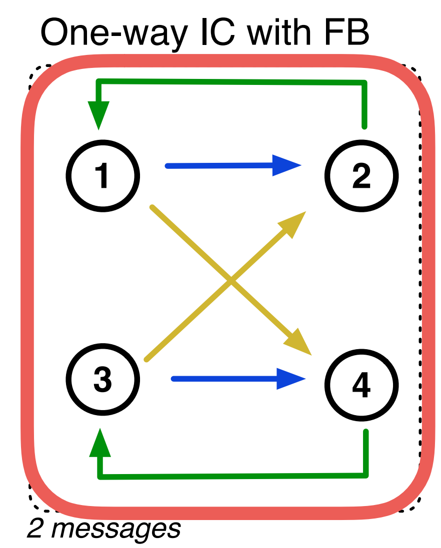
[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.]

Some other GDoF comparisons

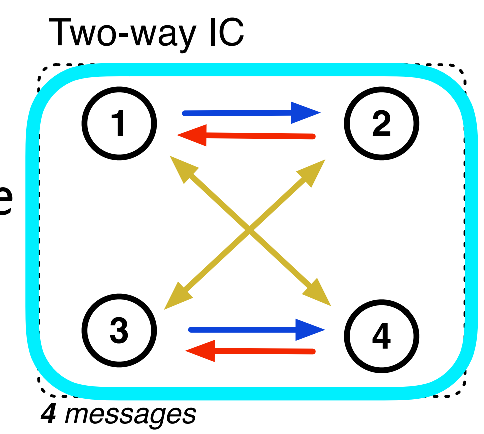




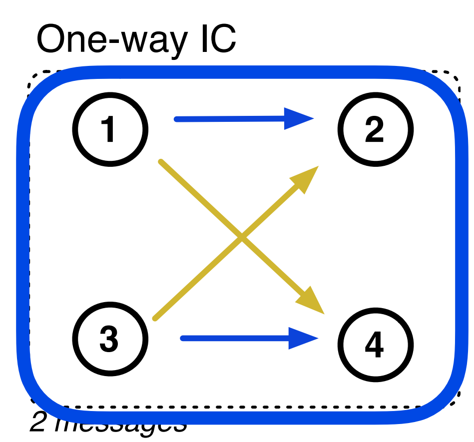
[Vahid, Suh, Avestimehr 2012]



[Suh, Tse 2011]



[Cheng, Devroye 2012]



[Bresler, Tse 2008]

[Etkin, Tse, Wang 2008]

2 recent results

Is H+K capacity achieving in general? NO!

[C. Nair, L. Xia, M. Yazdanpanah, ``**Sub-optimality of the Han-and-Kobayashi Achievable Region for Interference Channels,**" *Proc. of ISIT,* 2015.]

1) Simplified channel model, Z-IC

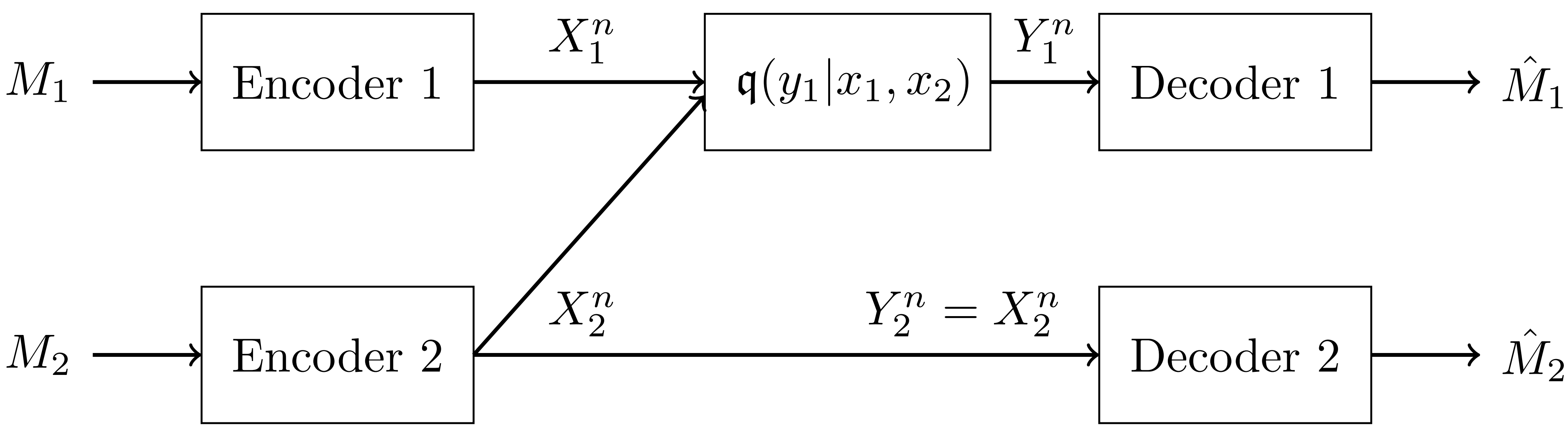


Fig. 2: Discrete memoryless CZI channel

Is H+K capacity achieving in general? NO!

[C. Nair, L. Xia, M. Yazdanpanah, ``**Sub-optimality of the Han-and-Kobayashi Achievable Region for Interference Channels,"** *Proc. of ISIT,* 2015.]

2) Characterize max sum-rate for H+K along the direction $\lambda R_1 + R_2$

Lemma 1. *For a CZI channel, for all* $\lambda > 1$

$$\max_{\mathcal{R}_{hk}}(\lambda R_1 + R_2) = \max_{p_1(x_1)p_2(x_2)} \left\{ I(X_1, X_2; Y_1) + \underset{p_2(x_2)}{\mathfrak{C}} \left[H(X_2) - I(X_2; Y_1|X_1) + (\lambda - 1) I(X_1; Y_1) \right] \right\},$$

where $\underset{x}{\mathfrak{C}}[f(x)]$ *of* $f(x)$ *denotes the upper concave envelope of* $f(x)$ *over* x*. [4]*

[4] Chandra Nair, Upper concave envelopes and auxiliary random variables, International Journal of Advances in Engineering Sciences and Applied Mathematics **5** (2013), no. 1, 12–20 (English).

3) Look at two-letter treating interference as noise region

Proposition 3. *The set of rate pairs satisfying*

$$R_1 = \frac{1}{2} I(X_{11}, X_{12}; Y_{11}, Y_{12}|Q),$$
$$R_2 = \frac{1}{2} H(X_{21}, H_{22}|Q),$$

for some pmf $p(q)p(x_{11}, x_{12}|q)p(x_{21}x_{22}|q)$ *with* $|Q| \leq 2$ *is achievable by the original channel.*

Is H+K capacity achieving in general? NO!

[C. Nair, L. Xia, M. Yazdanpanah, ``**Sub-optimality of the Han-and-Kobayashi Achievable Region for Interference Channels,**" *Proc. of ISIT,* 2015.]

$$\mathsf{q}(y_1|x_1,x_2) = \begin{bmatrix} P(Y_1=0|X_1,X_2=0,0) & P(Y_1=0|X_1,X_2=0,1) \\ P(Y_1=0|X_1,X_2=1,0) & P(Y_1=0|X_1,X_2=1,1) \end{bmatrix}.$$

Tab. 1: Table of counter-examples

λ	channel	$\max_{\mathcal{R}_{hk}}(\lambda R_1 + R_2)$	$\max_{\mathcal{R}_{two}}(\lambda R_1 + R_2)$
2	$\begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix}$	1.107516	1.108141
2.5	$\begin{bmatrix} 0.204581 & 0.364813 \\ 0.030209 & 0.992978 \end{bmatrix}$	1.159383	1.169312
3	$\begin{bmatrix} 0.591419 & 0.865901 \\ 0.004021 & 0.898113 \end{bmatrix}$	1.241521	1.255814
3	$\begin{bmatrix} 0.356166 & 0.073253 \\ 0.985504 & 0.031707 \end{bmatrix}$	1.292172	1.311027
3	$\begin{bmatrix} 0.287272 & 0.459966 \\ 0.113711 & 0.995405 \end{bmatrix}$	1.117253	1.123151
4	$\begin{bmatrix} 0.429804 & 0.147712 \\ 0.948192 & 0.002848 \end{bmatrix}$	1.181392	1.196189
4	$\begin{bmatrix} 0.068730 & 0.443630 \\ 0.011377 & 0.954887 \end{bmatrix}$	1.223409	1.243958
5	$\begin{bmatrix} 0.969199 & 0.564440 \\ 0.954079 & 0.061409 \end{bmatrix}$	1.351229	1.372191
5	$\begin{bmatrix} 0.943226 & 0.447252 \\ 0.950791 & 0.024302 \end{bmatrix}$	1.231254	1.250564
6	$\begin{bmatrix} 0.943292 & 0.045996 \\ 0.589551 & 0.202487 \end{bmatrix}$	1.069405	1.076932
6	$\begin{bmatrix} 0.714431 & 0.019375 \\ 0.955918 & 0.448539 \end{bmatrix}$	1.528508	1.541781
7	$\begin{bmatrix} 0.058449 & 0.558649 \\ 0.194915 & 0.959172 \end{bmatrix}$	1.424974	1.452769
7	$\begin{bmatrix} 0.033312 & 0.876067 \\ 0.286125 & 0.992825 \end{bmatrix}$	1.179438	1.187867
10	$\begin{bmatrix} 0.307723 & 0.874843 \\ 0.032090 & 0.710535 \end{bmatrix}$	1.370830	1.388674
15	$\begin{bmatrix} 0.946802 & 0.311909 \\ 0.730770 & 0.155075 \end{bmatrix}$	1.391596	1.406325
100	$\begin{bmatrix} 0.382410 & 0.081474 \\ 0.584797 & 0.241840 \end{bmatrix}$	3.754016	3.789316
100	$\begin{bmatrix} 0.673979 & 0.194596 \\ 0.781192 & 0.285216 \end{bmatrix}$	1.711938	1.730715

4) Find computable channel for which two-letter TIN outperforms i.i.d. HK $\lambda R_1 + R_2$

Analytical for $\lambda = 2$

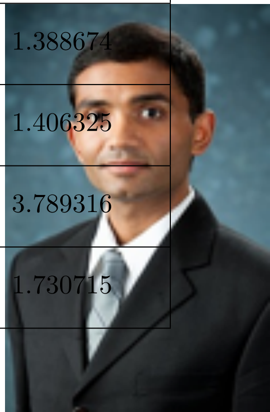
$$(p_0, q_0) = (0.507829413, 0.436538150)$$

$$P((X_{11}, X_{12}) = (0,0)) = p_0$$
$$P((X_{11}, X_{12}) = (1,1)) = 1 - p_0$$

repetition coding!

$$P((X_{21}, X_{22}) = (0,0)) = 0.36 q_0$$
$$P((X_{21}, X_{22}) = (0,1)) = P((X_{21}, X_{22}) = (1,0)) = 0.64 q_0$$
$$P((X_{21}, X_{22}) = (1,1)) = 1 - 1.64 q_0$$

memory



Is H+K capacity achieving in general?

NO!

[C. Nair, L. Xia, M. Yazdanpanah, ``**Sub-optimality of the Han-and-Kobayashi Achievable Region for Interference Channels,**" *Proc. of ISIT,* 2015.]

All known capacity results use H+K….

Intuition:

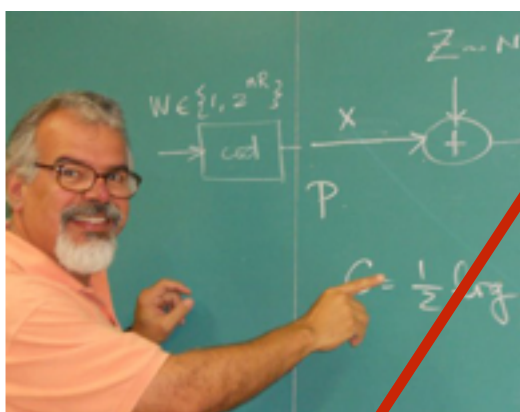
X_2 acts as a state for $X_1 \rightarrow Y_1$ channel

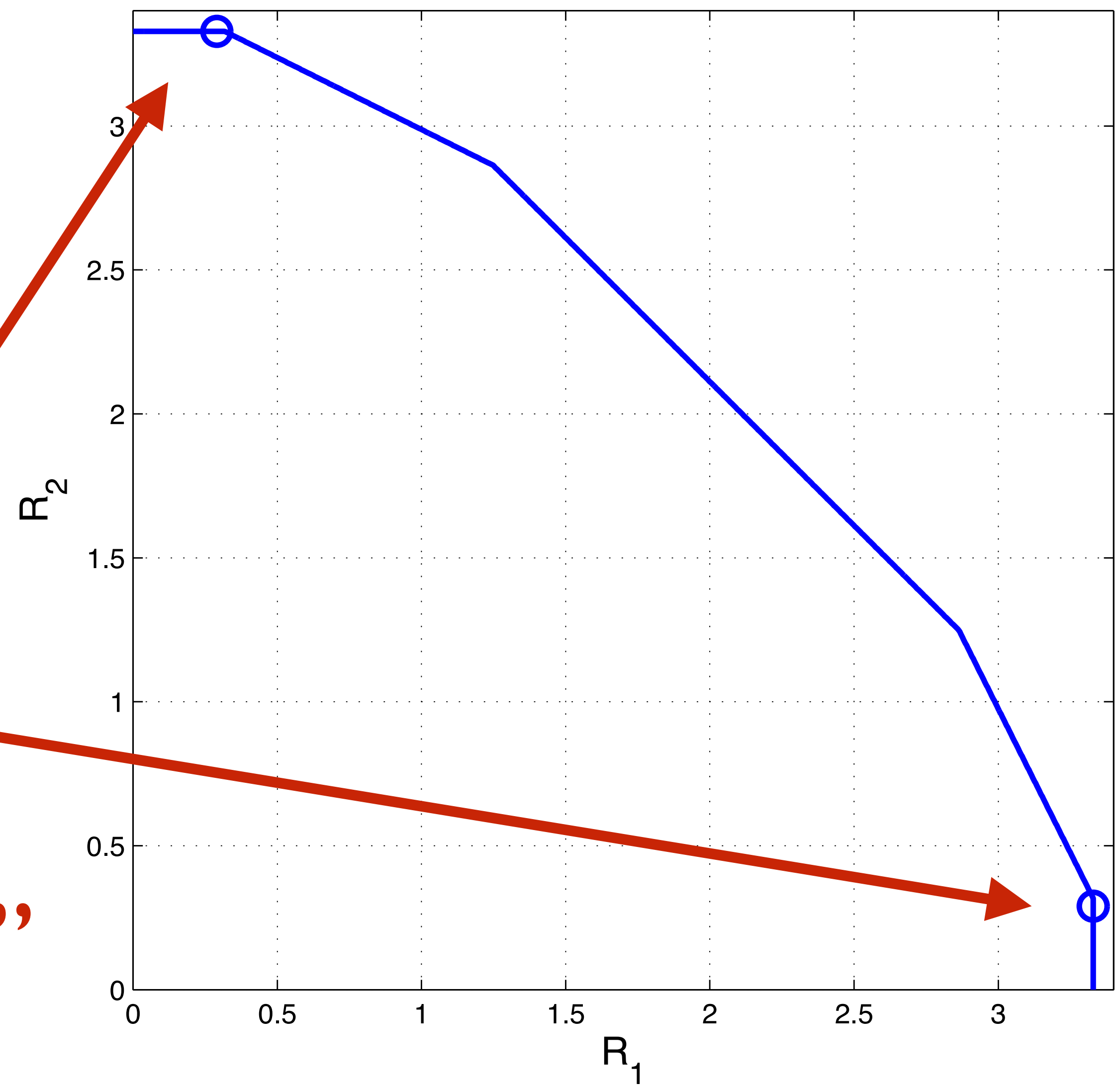
i.i.d. distributions on X_1 are not optimal if state has memory

2 recent results

2) Costa's corner point conjecture

images taken from slides of [I. Sason, ``**On the Corner Points of the Capacity Region of a Two-User Gaussian Interference Channel,"** *Proc. of ISIT,* 2014.]





"Costa's corner points"

Recent result: Costa's corner point conjecture

images taken from slides of [I. Sason, ``**On the Corner Points of the Capacity Region of a Two-User Gaussian Interference Channel,**" *Proc. of ISIT,* 2014.]

Conjecture (Originated by Costa, 1985)

For a two-user GIC with positive cross-link gains, let

$$C_1 \triangleq \frac{1}{2}\log(1+P_1), \quad C_2 \triangleq \frac{1}{2}\log(1+P_2)$$

be the capacities of the single-user AWGN channels, and

$$R_1^* \triangleq \frac{1}{2}\log\left(1+\frac{a_{21}P_1}{1+P_2}\right), \quad R_2^* \triangleq \frac{1}{2}\log\left(1+\frac{a_{12}P_2}{1+P_1}\right).$$

Then, the following is conjectured to hold for reliable communication:

1. If $R_2 \geq C_2 - \varepsilon$, then $R_1 \leq R_1^* + \delta_1(\varepsilon)$ where $\lim_{\varepsilon\to 0}\delta_1(\varepsilon) = 0$.
2. If $R_1 \geq C_1 - \varepsilon$, then $R_2 \leq R_2^* + \delta_2(\varepsilon)$ where $\lim_{\varepsilon\to 0}\delta_2(\varepsilon) = 0$.

Gaussian IC:

$$Y_1 = X_1 + \sqrt{a_{12}}\,X_2 + Z_1$$
$$Y_2 = \sqrt{a_{21}}\,X_1 + X_2 + Z_2$$

Maximal P2P rates

Maximal treat-interference as noise rates

Interpretation of this conjecture for weak GIC

If one user transmits at its maximal possible rate, the other user should decrease its rate such that both decoders can reliably decode its message.

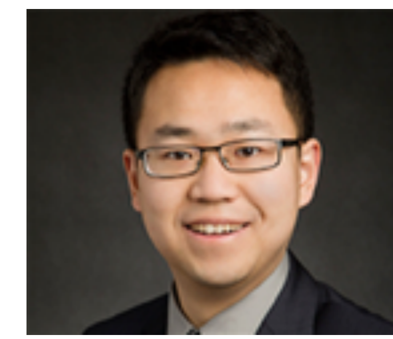
Recent result: Costa's corner point conjecture

Recent progress by Sason:

[I. Sason, ``**On the Corner Points of the Capacity Region of a Two-User Gaussian Interference Channel,"** *Proc. of ISIT,* 2014.]

idea: similar to gDoF analysis, shows asymptotic tightness of new bounds on corner point

And finally proven by:

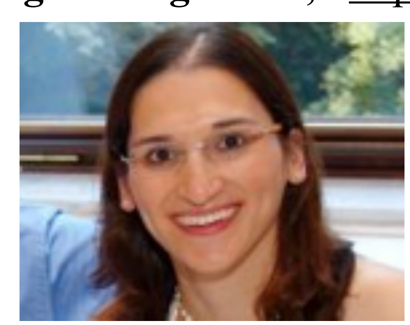


[Y. Polyanskiy and Y. Wu **"Wasserstein continuity of entropy and outer bounds for interference channels,"** http://arxiv:1504.04419]

idea: new converse which relates differential entropies to Wasserstein distances and bounds these using Talagrand's inequality

[R. Bustin, H.V. Poor, and S. Shamai **"The Effect of Maximal Rate Codes on the Interfering Message Rate,"** http://arxiv.org/abs/1404.6690

idea: use properties of the MMSE of good channel codes



BREAK!

Variations, extensions and implications

Alex Dysto

Overview

Daniela Tuninetti

Natasha Devroye

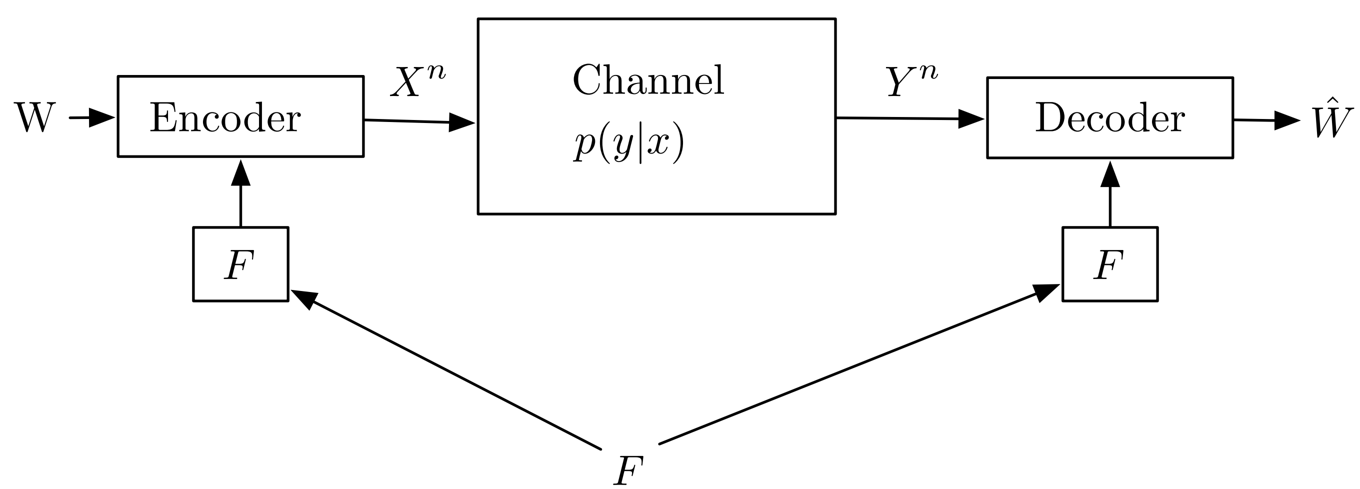
Discrete inputs in Gaussian interference channel: “good” codes and “good” interferers

some slides taken from Alex Dytso’s Ph.D. defense, May 2016

Variations of the IC

ICs with lack of codebook knowledge

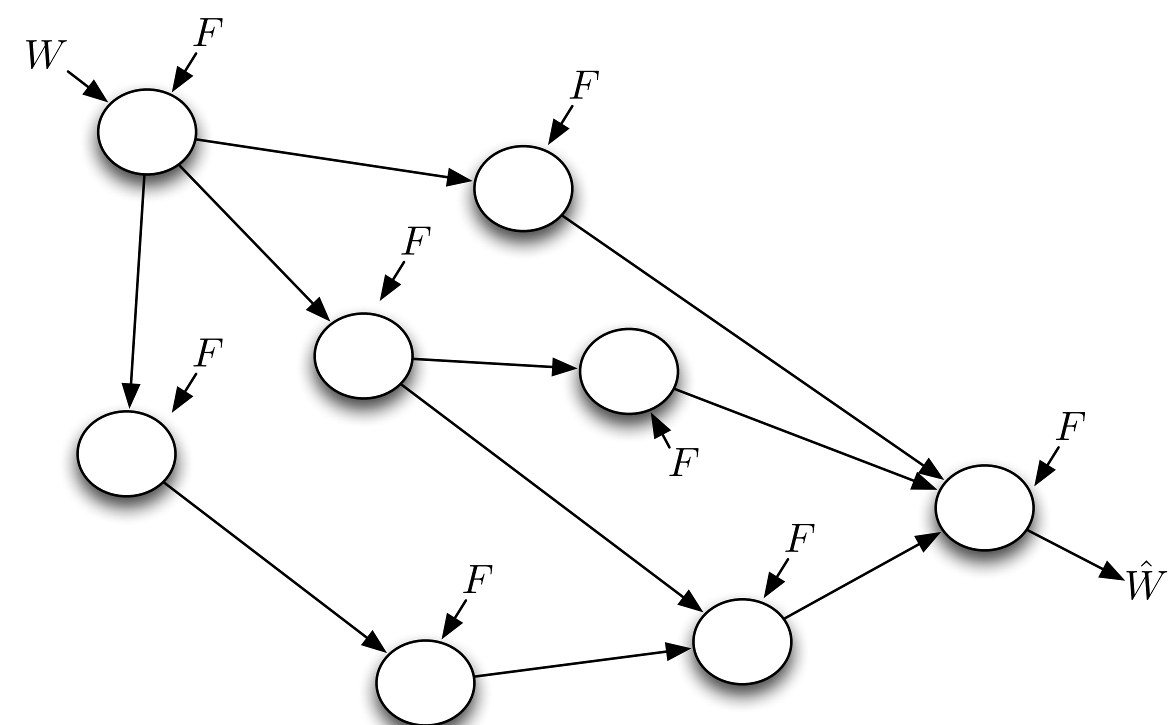
"F" is the codebook, known to all Tx,Rx



$$\left(\begin{array}{l} W \to X^n \\ \hline 1 \to X_1, X_2, \ldots, X_n \\ 2 \to X_1, X_2, \ldots, X_n \\ . \\ . \\ . \\ |W| \to X_1, X_2, \ldots, X_n \end{array} \right)$$

ICs with lack of codebook knowledge

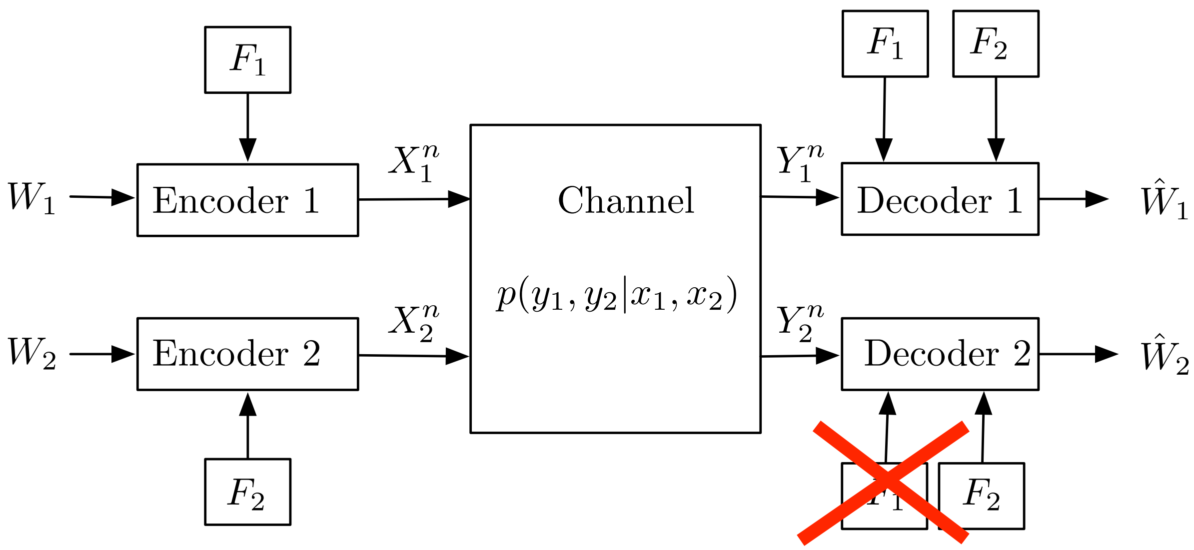
- in networks, often assume nodes know all codebooks of ALL other nodes



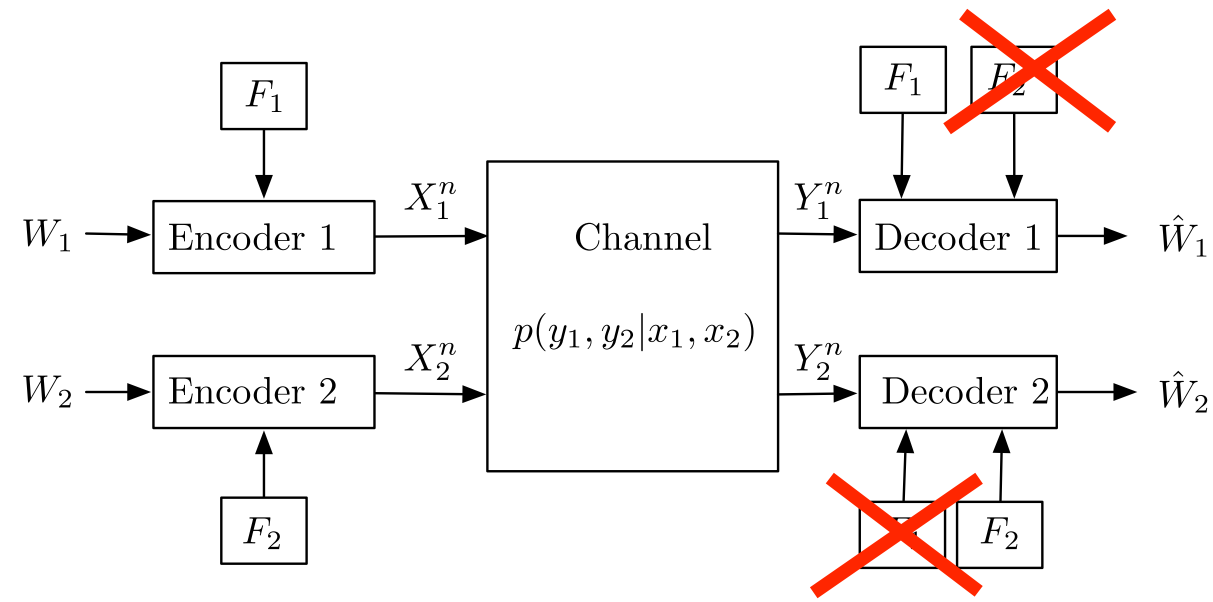
- this may be unrealistic sometimes….

ICs with lack of codebook knowledge

Our motivation:



IC with one oblivious Rx



IC with two oblivious Rx

A. Dytso, N. Devroye, and D. Tuninetti, "*On the capacity of interference channels with partial codebook knowledge*," ISIT 2013

A. Dytso, D. Tuninetti and N. Devroye, ``On the Two-User Interference Channel With Lack of Knowledge of the Interference Codebook at One Receiver,'' IEEE Transactions on Information Theory, Vol. 61, No. 3, pp. 1256-1276, March 2015.

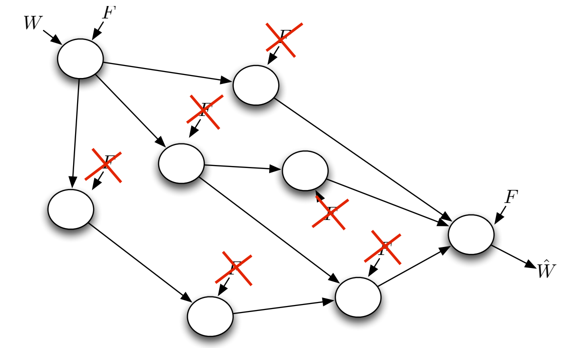
A. Dytso, D. Tuninetti and N. Devroye. "*On Gaussian Interference Channels with Mixed Gaussian and Discrete Inputs*," ISIT 2014

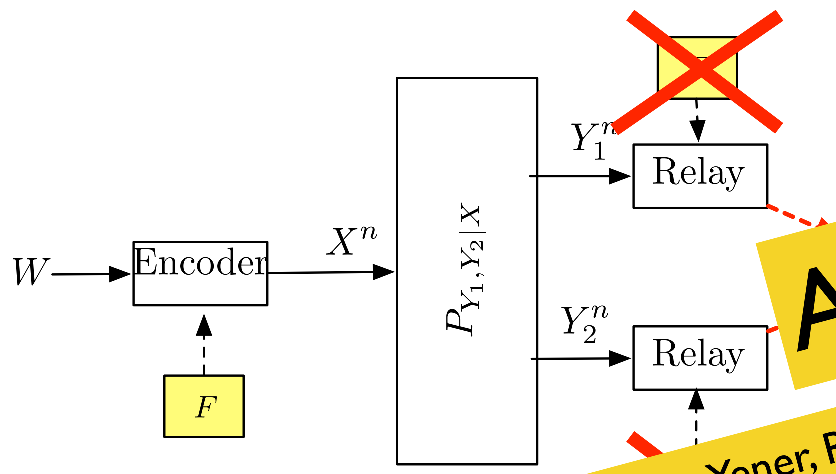
A. Dytso, D. Tuninetti and N. Devroye "*Interference as Noise: Friend of Foe?*" IEEE Trans. on Info Theory, June 2016.

Past work: lack of codebooks leads to non-Gaussians outperforming Gaussians

A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, "*Communication via decentralized processing*," IT July 2008.

Gaussian noise: example where BPSK outperforms Gaussian inputs



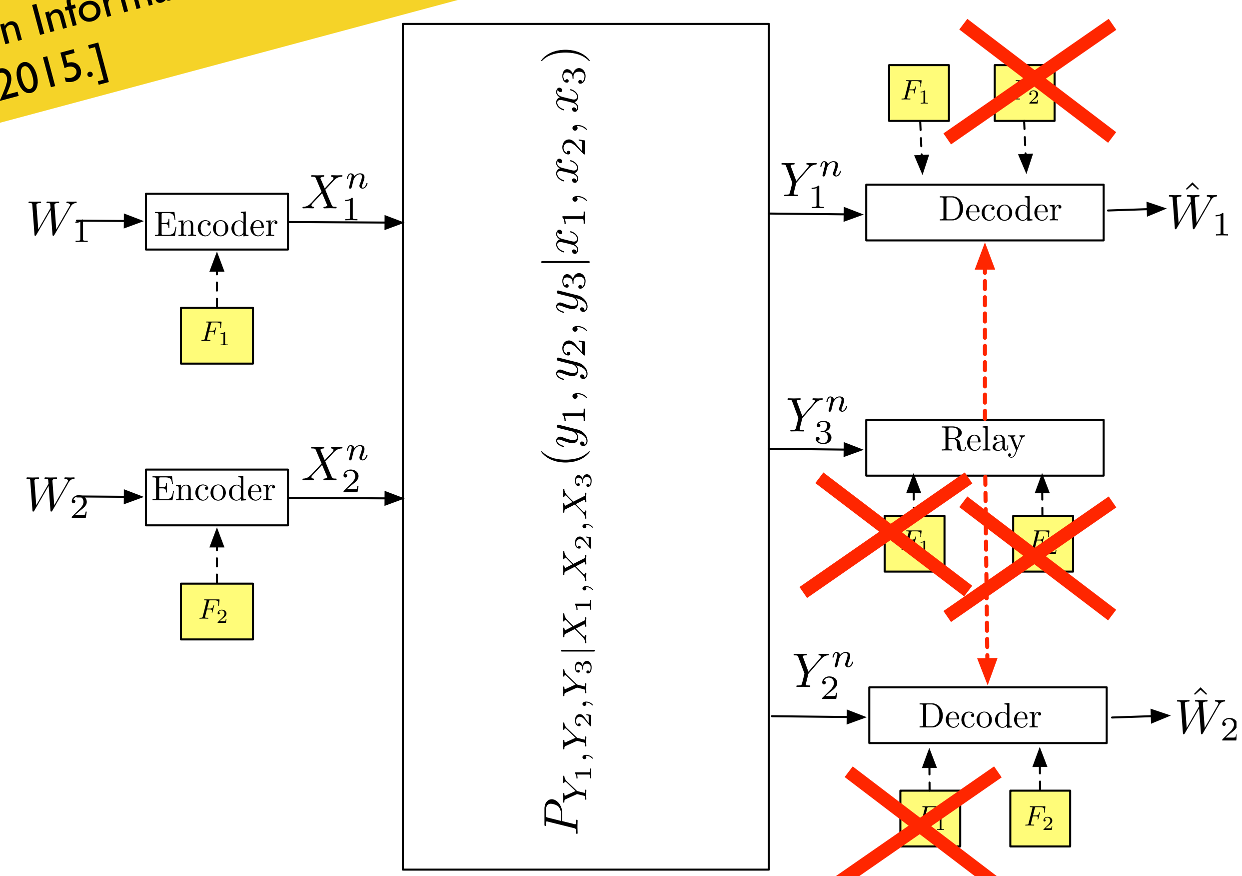


A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, "*Communication via decentralized processing*," IT July 2008.

Upper and lower bo… …e for deterministic channels
Gaussian … …wn
3. G… …utperforms Gaussian inputs

Ask Aylin

[Ye Tian and Aylin Yener, Relaying for Multiuser Networks in the Absence of Codebook Information, IEEE Transactions on Information Theory, 61(3), pp. 1247-1256, Mar. 2015.]



O. Simeone, E. Erkip, and S. Sh…, "*On codebook information for interference relay channels with out-of-band relaying*," IT May 2011.

1. Primitive relay channel: capacity with compress forward
2. IC+R+Oblivious receivers: capacity with compress forward and TIN
3. Gaussian noise: optimizing input unknown

$$\mathcal{C}^{\text{IC-OR}} = \bigcup_{P_Q P_{X_1|Q} P_{X_2|Q}} \left\{ \begin{array}{l} R_1 \leq I(X_1;Y_1|Q) \\ R_2 \leq I(X_2;Y_2|Q) \end{array} \right\}$$

Discrete inputs in Gaussian channels — deeper?

Other supporting arguments

- E. Abbe and L. Zheng, *"A coordinate system for Gaussian networks,"* IT 2012.
- E. Calvo, J. Fonollosa, and J. Vidal, *"On the totally asynchronous interference channel with single-user receivers,"* ISIT 2009

- No gDoF Gain
- Discrete input conclusions are simulation based

Questions

- loss in performance due to lack of codebook knowledge? due to lack of synchronization?

- are there inputs that outperform Gaussians in the AWGN IC under these conditions?

- can we show analytical gains?

How we tackle discrete inputs for G-IC

- best inner bound for Gaussian IC is the complex H+K scheme

- simpler scheme — Treating Interference as Noise with no Time Sharing:

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1 X_2} = P_{X_1} P_{X_2}} \left\{ \begin{array}{c} 0 \le R_1 \le I(X_1; Y_1) \\ 0 \le R_2 \le I(X_2; Y_2) \end{array} \right\}$$

- we show discrete inputs in TINnoTS performs well!

- neat, general tools to bound minimum distance of sum-sets, and [illegible]al information achieved by discrete RVs in Gaussian nois[illegible]y

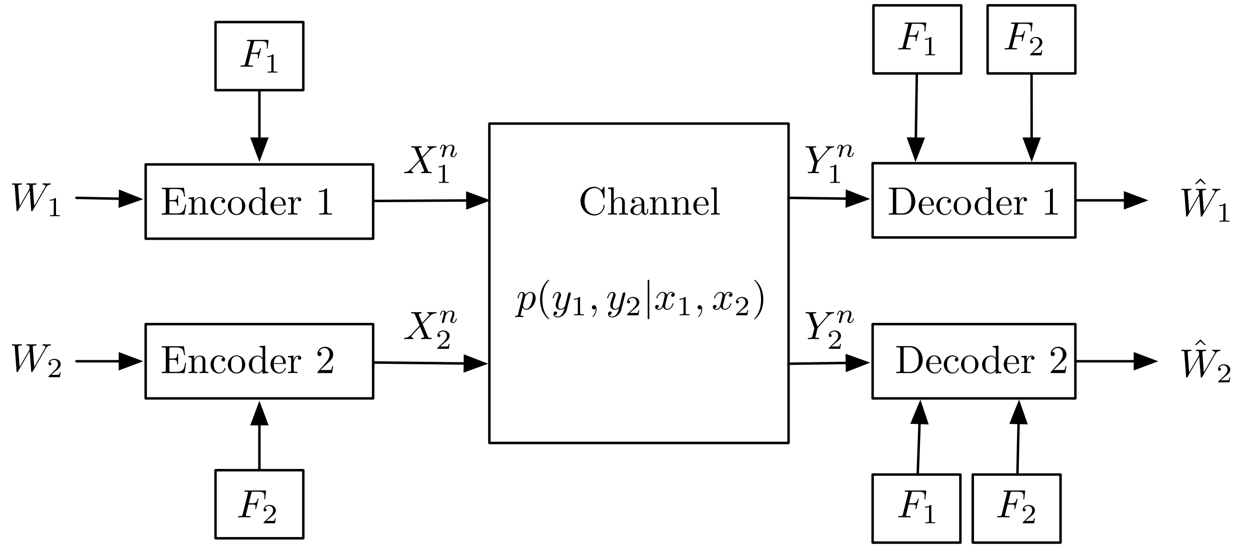
Ask Henry

Similar results as

S. Li, Y.-C. Huang, T. Liu, and H.D. Pfister, “On the limits of treating interference as noise in the two-user Gaussian symmetric interference channel,” ISIT 2015.

Capacity is actually known.... sort of

Interference Channel (IC)



$$C = \lim_{n\to\infty} \mathrm{co}\left(\bigcup_{\mathrm{P_{X_1^n X_2^n} = P_{X_1^n} P_{X_2^n}}} \left\{ (\mathrm{R}_1, \mathrm{R}_2) : \begin{array}{l} 0 \le R_1 \le \frac{1}{n} I(X_1^n; Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n; Y_2^n) \end{array} \right\} \right)$$

Uncomputable

Complexity $|\mathcal{X}_1 \times \mathcal{X}_2|^n$

Treating interference as noise inner bound

Capacity: $\mathcal{C} = \lim_{n\to\infty} \mathrm{co}\left(\bigcup_{P_{X_1^n X_2^n} = P_{X_1^n} P_{X_2^n}} \left\{ \begin{array}{l} 0 \le R_1 \le \frac{1}{n} I(X_1^n; Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n; Y_2^n) \end{array} \right\}\right)$

R. Ahlswede, "Multi-way communication channels," in Proc. IEEE Int. Symp. Inf. Theory, March 1973, pp. 23–52.

↓ i.i.d. inputs

Treat Interference as Noise Inner Bound:

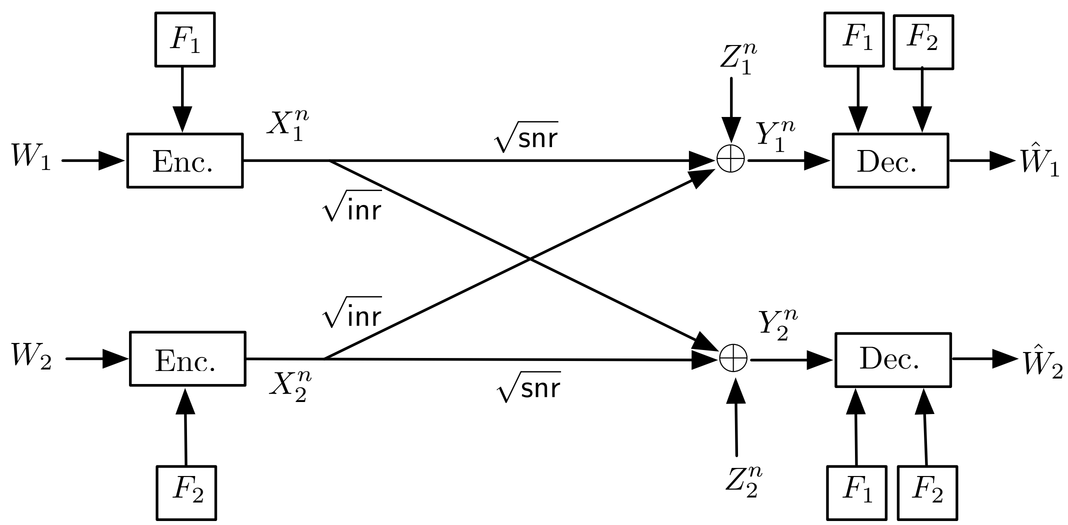
$$\mathcal{R}_{\mathrm{in}}^{\mathrm{TIN+TS}} = \mathrm{co}\left(\bigcup_{P_{X_1 X_2} = P_{X_1} P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1; Y_1) \\ 0 \le R_2 \le I(X_2; Y_2) \end{array} \right\}\right)$$ With Time Sharing

$$\mathcal{R}_{\mathrm{in}}^{\mathrm{TINnoTS}} = \bigcup_{P_{X_1 X_2} = P_{X_1} P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1; Y_1) \\ 0 \le R_2 \le I(X_2; Y_2) \end{array} \right\}$$ No Time Sharing

How far away is TINnoTS from capacity?

Is it really "treating interference as noise"?

Gaussian channels with discrete inputs



$$Z_1, Z_2 \sim \mathcal{N}(0,1)$$

$$Y_1 = \sqrt{\mathsf{snr}}X_1 + \sqrt{\mathsf{inr}}X_2 + Z_1$$
$$Y_2 = \sqrt{\mathsf{inr}}X_1 + \sqrt{\mathsf{snr}}X_2 + Z_2$$

- instead of taking X_1 and X_2 to be Gaussian, take them to be discrete
- difficulty: how to evaluate mutual information expressions with discrete and Gaussian mixtures

Tools for Discrete Inputs

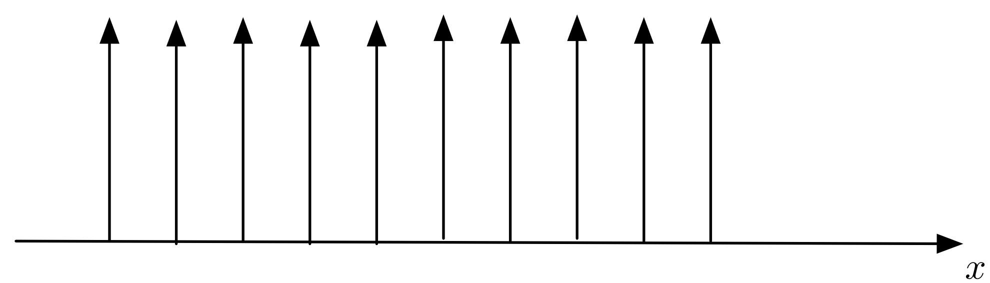
Discrete+mixed inputs

- Discrete input

$$X_D \sim P(X_D) = \sum_{i=1}^{|X|} p_i \delta(x_i)$$

$$X_D \sim \mathrm{PAM(N)}, |\mathrm{X}| = \mathrm{N}, \mathrm{p_i} = \frac{1}{\mathrm{N}} \text{ for all i} \in [1, ..., \mathrm{N}]$$

- PAM input



- Minimum distance

$$d_{\min}(X_D) = \min_{x_i, x_j : i \neq j} \|x_i - x_i\|$$

- Mixed inputs

$$X_{\text{mix}} = \sqrt{1-\delta} X_D + \sqrt{\delta} X_G,$$
$$\delta \in [0, 1],$$
$$X_G \sim \mathcal{N}(0, 1)$$
$$\mathbb{E}[X_D^2] \leq 1$$

Bounds on mutual information

We define:

$$Y = \sqrt{\mathrm{snr}}X + Z,$$
$$Z \sim \mathcal{N}(0,1)$$

$$I(X;Y_{\mathrm{snr}}) = I(X,\mathrm{snr})$$
$$\mathbb{E}\left[(X - \mathbb{E}[X|Y_{\mathrm{snr}}])^2\right] = \mathrm{mmse}(X,\mathrm{snr})$$

Interested in:

$$[H(X_D) - \mathrm{gap}]^+ \leq I(X_D,\mathrm{snr}) \leq H(X_D)$$

Want the tightest version of the "gap" term for a given PMF

Bounds on mutual information

$$[H(X_D) - \text{gap}]^+ \le I(X_D, \text{snr}) \le H(X_D)$$

Ozarow-Wyner-A

$$\text{gap}_{\text{OW-A}} \le \xi \log \frac{1}{\xi} + (1-\xi) \log \frac{1}{1-\xi} + \xi \log(N-1)\ , \ \ \xi := 2Q\left(\frac{\sqrt{\text{snr}} d_{\min(X_D)}}{2}\right)$$

Ozarow-Wyner-B

$$\text{gap}_{\text{OW-B}} \le \frac{1}{2} \log\left(\frac{\pi \mathrm{e}}{6}\right) + \frac{1}{2} \log\left(1 + \frac{12}{\text{snr}\ d^2_{\min(X_D)}}\right)$$

L. Ozarow and A. Wyner, “On the capacity of the Gaussian channel with a finite number of input levels,” IEEE Trans. Inf. Theory, vol. 36, no. 6, pp. 1426–1428, Nov 1990.

DTD-ITA`14-A

$$\left[-\log\left(\sum_{(i,j)\in[1:N]^2} \frac{p_i p_j}{\sqrt{4\pi}} \mathrm{e}^{-\frac{\text{snr}(x_i - x_j)^2}{4}}\right) - \frac{1}{2}\log(2\pi \mathrm{e})\right]^+ \le I(X_D, \text{snr}) \le H(X_D)$$

$$\text{gap}_{\text{ITA}} \le \frac{1}{2} \log\left(\frac{\mathrm{e}}{2}\right) + \log\left(1 + (N-1)\mathrm{e}^{-\frac{\text{snr} d^2_{\min(X_D)}}{4}}\right)$$

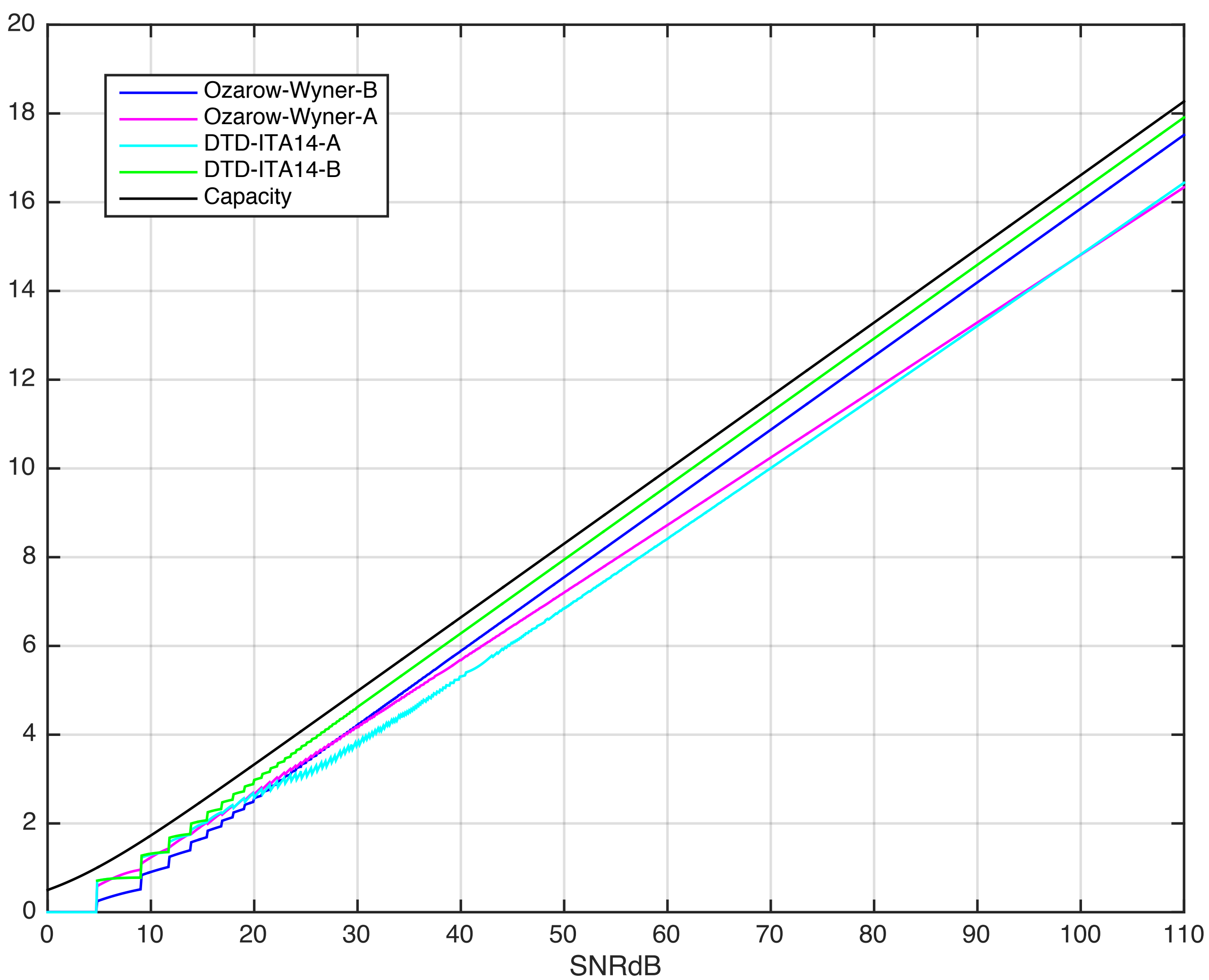
DTD-ITA`14-B

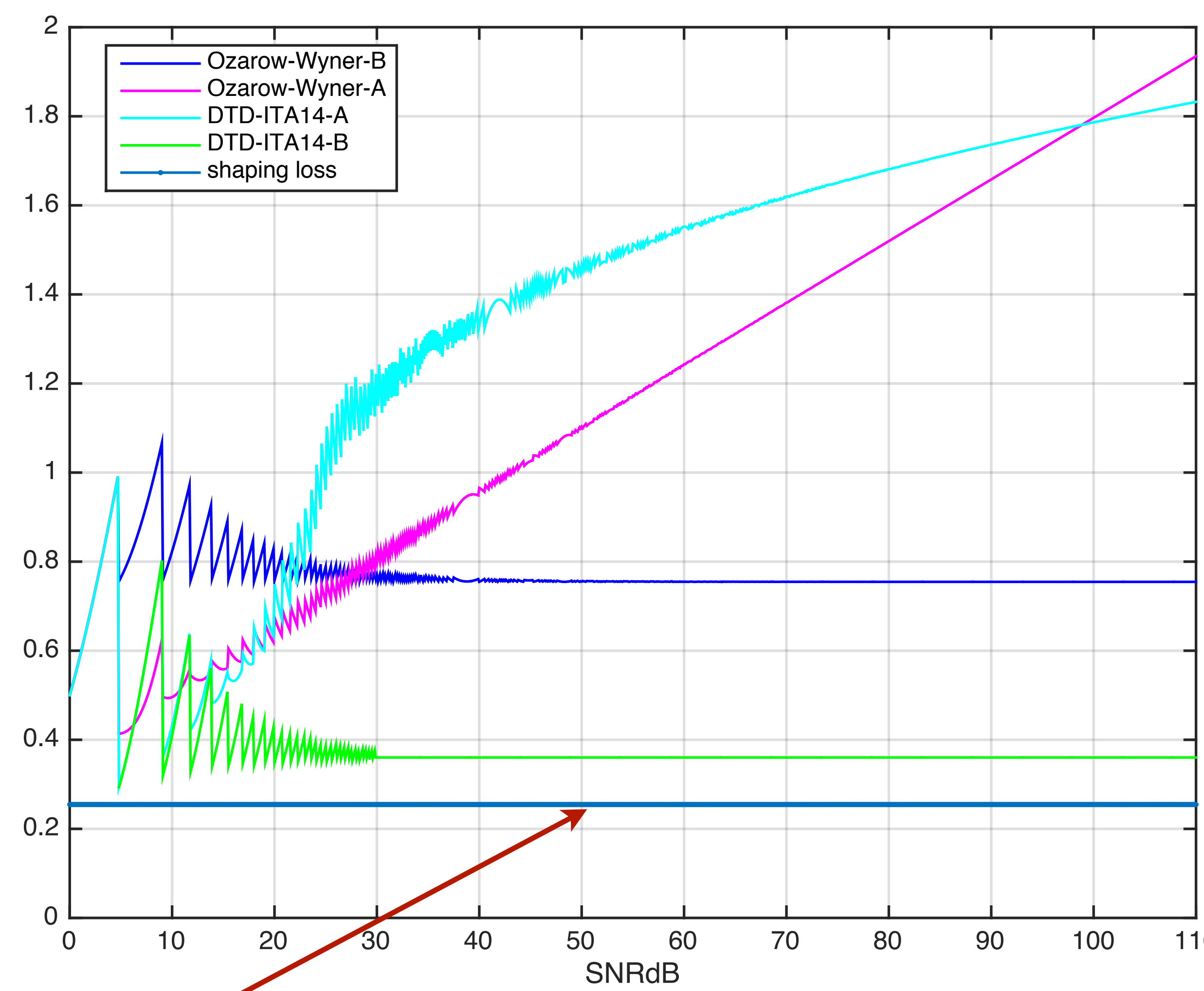
Dytso, A.; Tuninetti, D.; Devroye, N., "On discrete alphabets for the two-user Gaussian interference channel with one receiver lacking knowledge of the interfering codebook," ITA, 2014 , vol., no., pp.1,8, 9-14 Feb. 2014

Comparison of bounds

Input: PAM with number of points

$$N = \lfloor \sqrt{1+\text{snr}} \rfloor \Rightarrow H(X) = \log(N) \approx \frac{1}{2}\log(1+\text{snr})$$





shaping loss of uniform lattice

Why is discrete good? Examples.

1. Point-to-point Gaussian noise Channel

$$Y = \sqrt{\text{snr}}X + Z_G :$$

$$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1)$$

good input

2. Point-to-point Gaussian noise Channel with State

$$Y = \sqrt{\text{snr}}X + hT + Z_G :$$

$$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1),$$

$$T \sim \text{discrete: } |T| = N \text{ and } d^2_{\min(T)} > 0$$

good state / interferer

Channel State is Unknown at Transmitter and Receiver

Discrete is a good input.

1.Point-to-point Gaussian noise Channel

$$Y = \sqrt{\mathrm{snr}}\,X + Z_G :$$

$$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1)$$

Capacity

$$C = \frac{1}{2}\log(1+\mathrm{snr})$$

achieved by Gaussian

with PAM:

$$N = \lfloor \sqrt{1+\mathrm{snr}} \rfloor$$

$$C \geq \frac{1}{2}\log(1+\mathrm{snr}) - \mathrm{gap}$$

$$\mathrm{gap} = \frac{1}{2}\log\left(\frac{4\pi e}{3}\right)$$

Discrete is a good interferer.

2.Point-to-point Gaussian noise Channel with State

$$Y = \sqrt{\text{snr}}X + hT + Z_G :$$

$$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1),$$

$$T \sim \text{discrete: } |T| = N \text{ and } d^2_{\min(T)} > 0$$

Discrete Interference

$$C \geq I(X_G; \sqrt{\text{snr}}X_G + hT + Z_G)$$
$$\geq \frac{1}{2}\log(1+\text{snr}) - \text{gap}$$

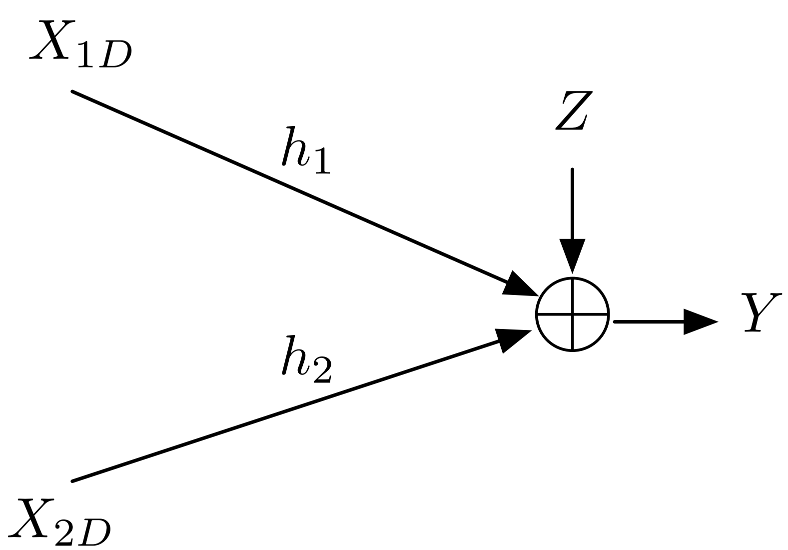
$$\text{gap} = \frac{1}{2}\log\left[\frac{2\pi\text{e}}{12}\left(1 + \frac{12}{d^2_{\min}(T)}\frac{|h|^2\mathcal{E}_T}{|h|^2\mathcal{E}_T + 1 + \text{snr}}\right)\right]$$

Gaussian Interference

$$C \geq I(X_G; \sqrt{\text{snr}}X_G + hT_G + Z_G)$$
$$= \frac{1}{2}\log\left(1 + \frac{\text{snr}}{1 + |h|^2\mathcal{E}_T}\right)$$

Discrete inputs in multi-user channels

More complex in multi-user scenarios



"sum-set"

$$h_1 X_{1D} + h_2 X_{2D} = \{h_1 x_{1D} + h_2 x_{2D} | x_1 \in X_{1D}, x_2 \in X_{2D}\}$$

$$|h_1 X_{1D} + h_2 X_{2D}| = |\{h_1 x_{1D} + h_2 x_{2D} | x_1 \in X_{1D}, x_2 \in X_{2D}\}|$$???

$d_{\min}(h_1 X_{1D} + h_2 X_{2D}) = \min\{|s_i - s_j| : s_i, s_j \in h_1 X_{1D} + h_2 X$ … $j\}$???

Ask Helmut!

[D. Stotz and H. Bölcskei, "Characterizing degrees of freedom through additive combinatorics" IEEE Transactions on Information Theory, (revised version: Apr. 2016), June 2015, submitted.]

New phenomenon

Example, BPSK:

$$X_{1D} = X_{2D} = \{-1, +1\}$$

$$h_1 X_{1D} + h_2 X_{2D} \overset{(h_1=1, h_2=2)}{=} \{3, -1, 1, 3\}$$
$$\overset{(h_1=1, h_2=1)}{=} \{1, 0, -1\}$$

“Cardinality is Sensitive to Channel Gain Values.”

Overall proposition / tool

- cardinality of the sum-set $\{h_x X + h_y Y\}$

Proposition: Let $X \sim \mathsf{PAM}(|X|, d_{\min(X)})$ and $Y \sim \mathsf{PAM}(|Y|, d_{\min(Y)})$. Then for $(h_x, h_y) \in \mathbb{R}^2$

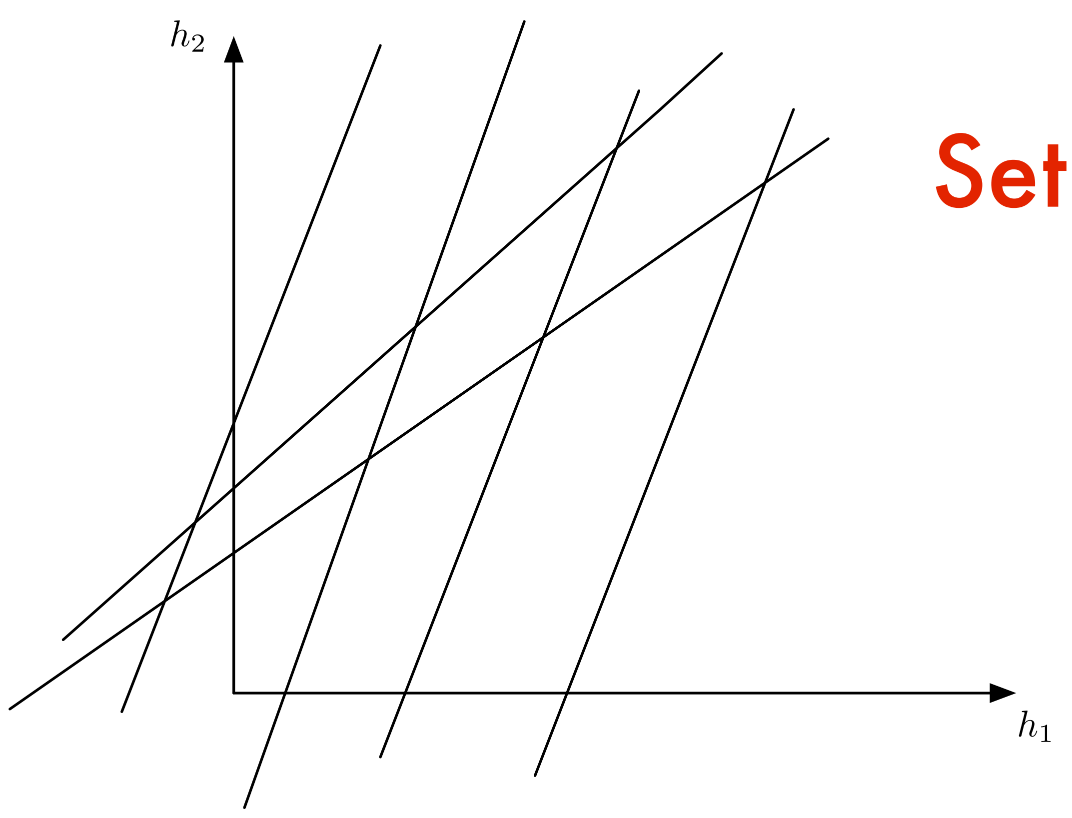
$$|h_x X + h_y Y| = |X||Y| \text{ almost everywhere (a.e.)}, \tag{1}$$

- minimum distance of the sum-set

and $d_{\min(h_x X + h_y Y)} \geq ......?$

Cardinality

$|h_x X + h_y Y| = |X||Y|$ almost everywhere (a.e.)

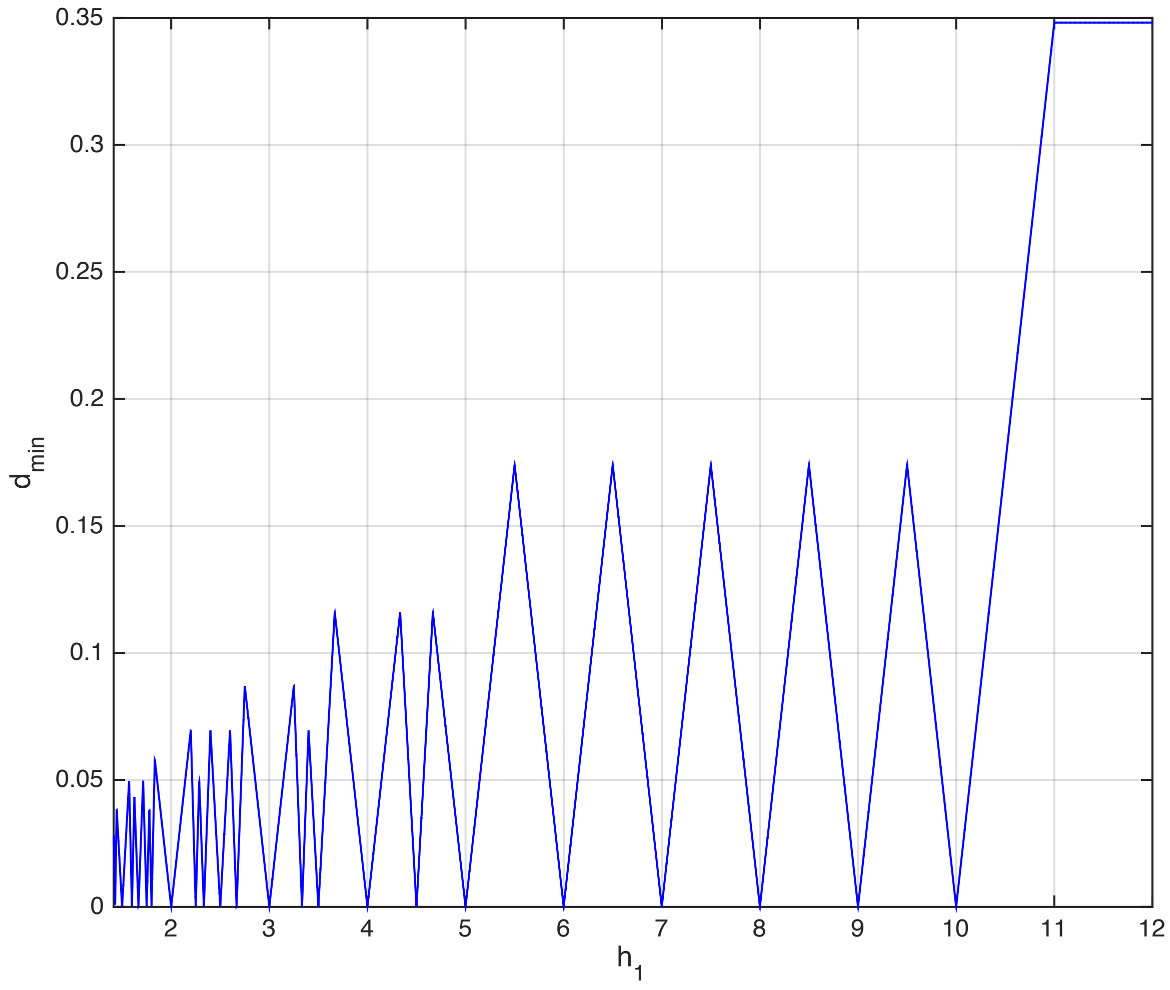


Set Values where cardinality is less

Union of lines has measure 0

Minimum distance

Example:h2=1, N1=N2=10



Very Irregular

Can we even have a lower bound?

$$\text{gap}_{\text{OW-B}} \leq \frac{1}{2}\log\left(\frac{\pi e}{6}\right) + \frac{1}{2}\log\left(1 + \frac{12}{\text{snr}\ d^2_{\min(X_D)}}\right)$$

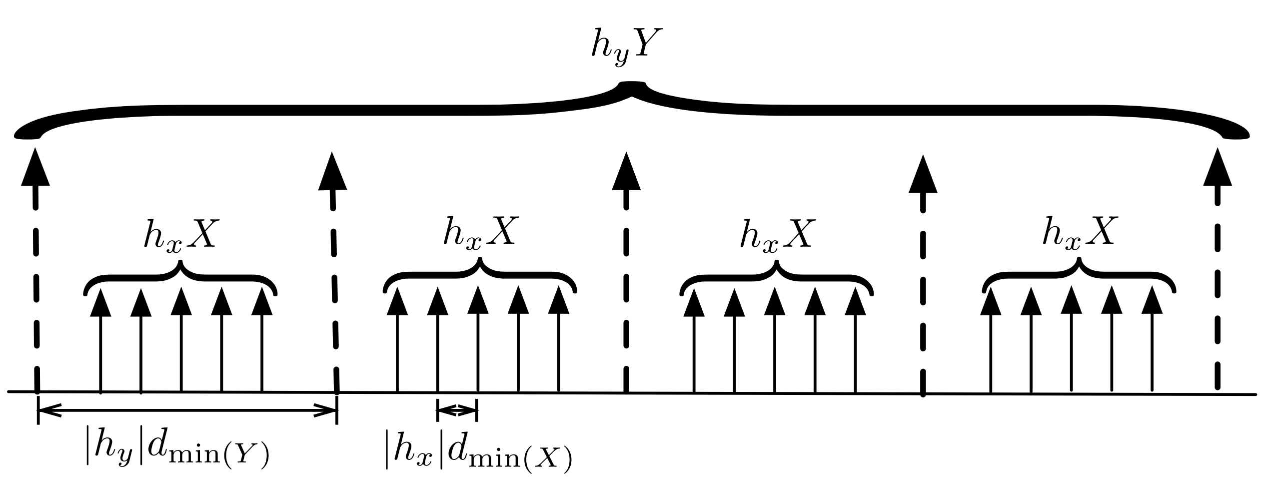
Minimum distance, case 1: no overlap

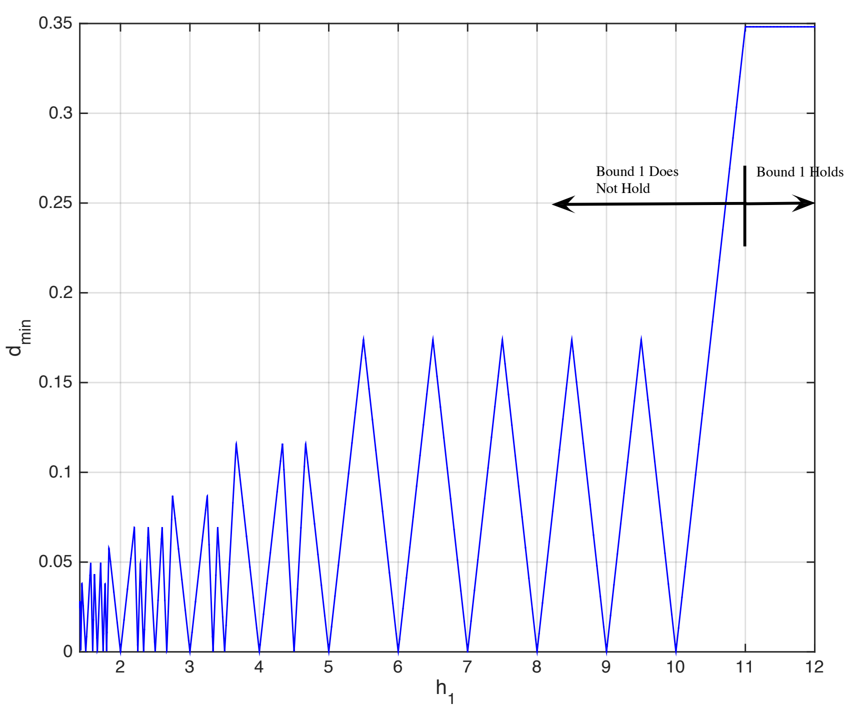
We have

$$d_{\min(h_x X + h_y Y)} = \min\left(|h_x| d_{\min(X)}, |h_y| d_{\min(Y)}\right)$$

under the following conditions

$$\begin{aligned} \text{either} \quad & |Y||h_y| d_{\min(Y)} \leq |h_x| d_{\min(X)}, \\ \text{or} \quad & |X||h_x| d_{\min(X)} \leq |h_y| d_{\min(Y)} \text{ (shown below).} \end{aligned}$$





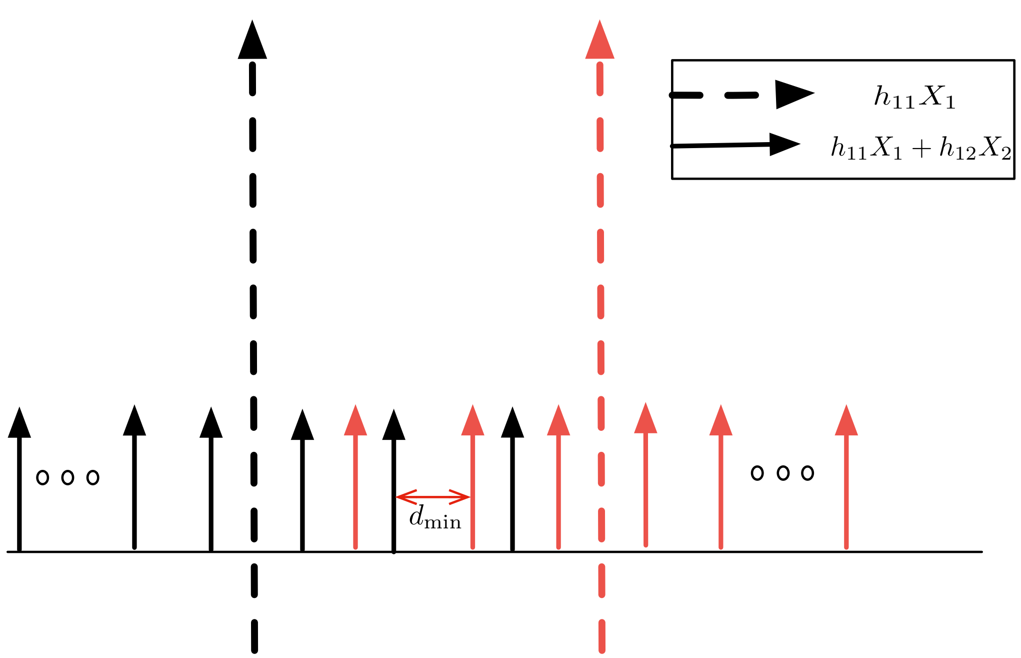
Minimum distance, case 2: with overlap

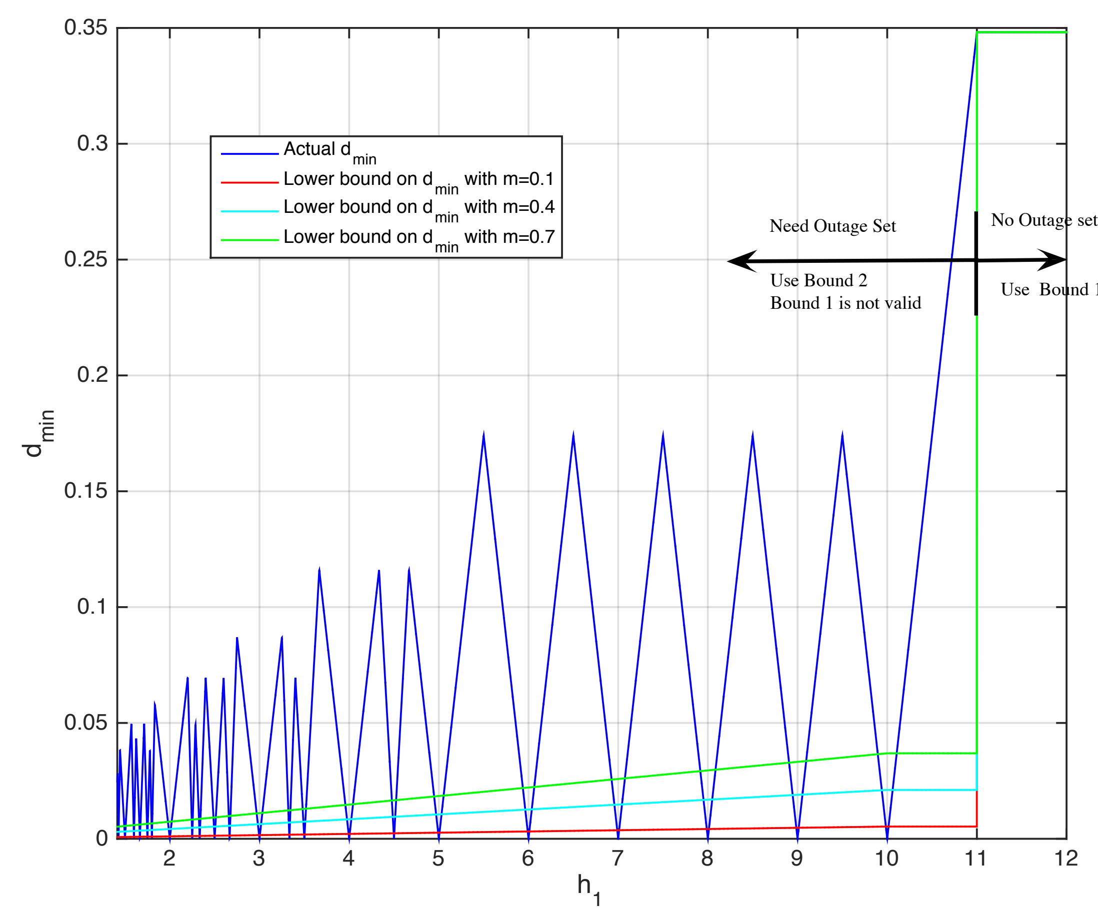
Then, up to a set of (h_x, h_y) of measure no more than γ, we have

$$d_{\min(h_x X + h_y Y)} \geq \kappa_{\gamma,|X|,|Y|} \cdot \min\left(|h_x| d_{\min(X)}, |h_y| d_{\min(Y)}, \xi_{|h_x|,|h_y|,|X|,|Y|}\right),$$

$$\kappa_{\gamma,|X|,|Y|} := \frac{\gamma/2}{1 + \ln(\max(|X|, |Y|))},$$

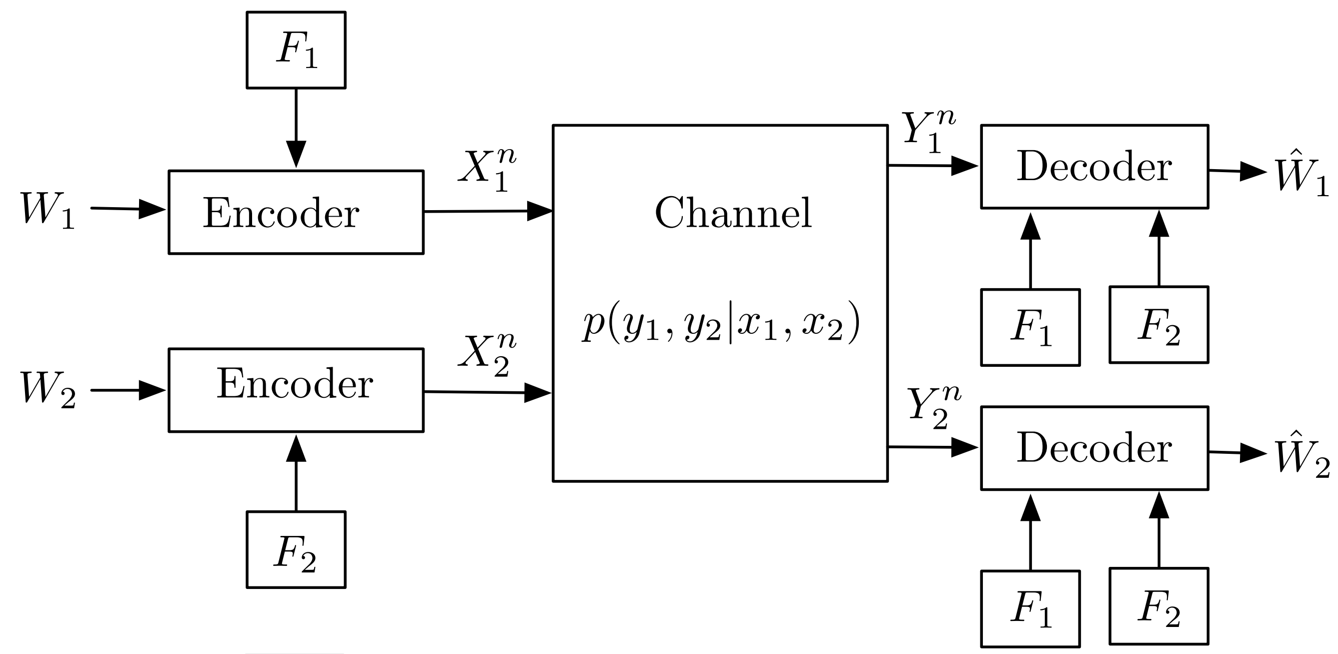
$$\xi_{|h_x|,|h_y|,|X|,|Y|} := \max\left(\frac{|h_x| d_{\min(X)}}{|Y|}, \frac{|h_y| d_{\min(Y)}}{|X|}\right),$$





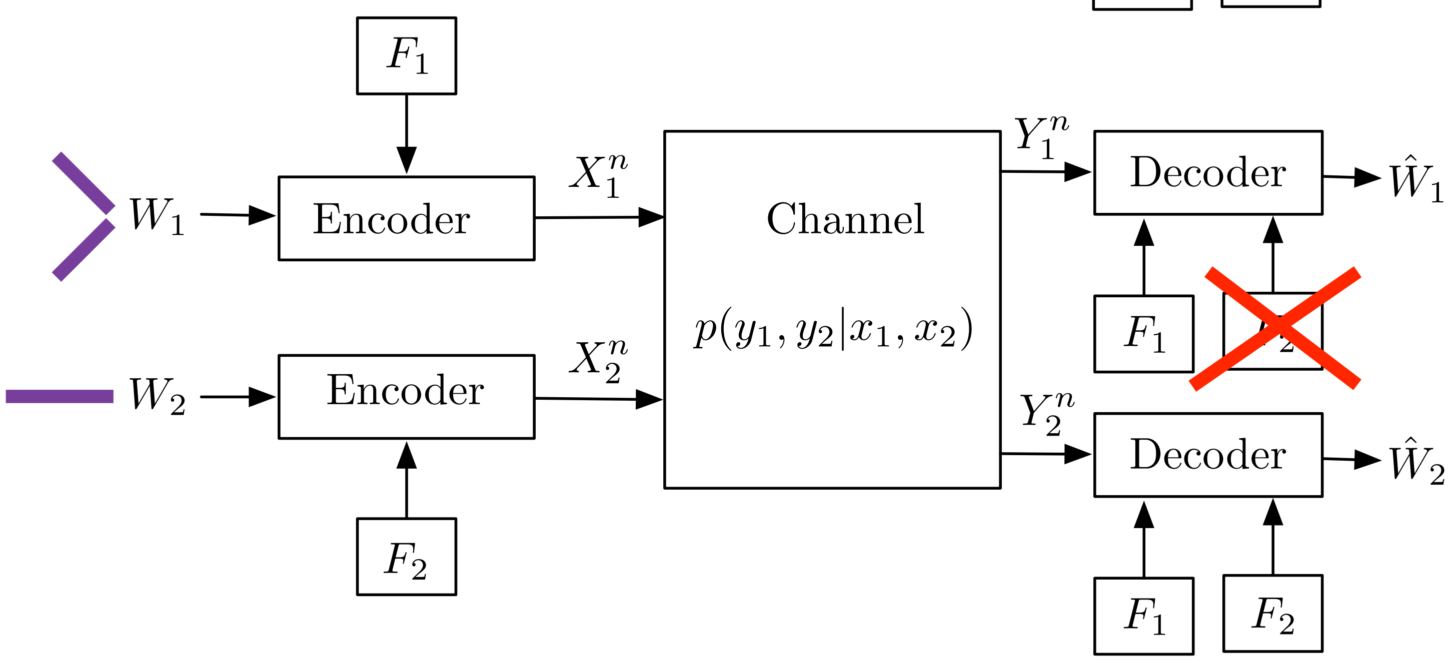
Applications of discrete inputs

Approximate capacity without codebooks



HK+Gaussian Inputs
1/2 bit

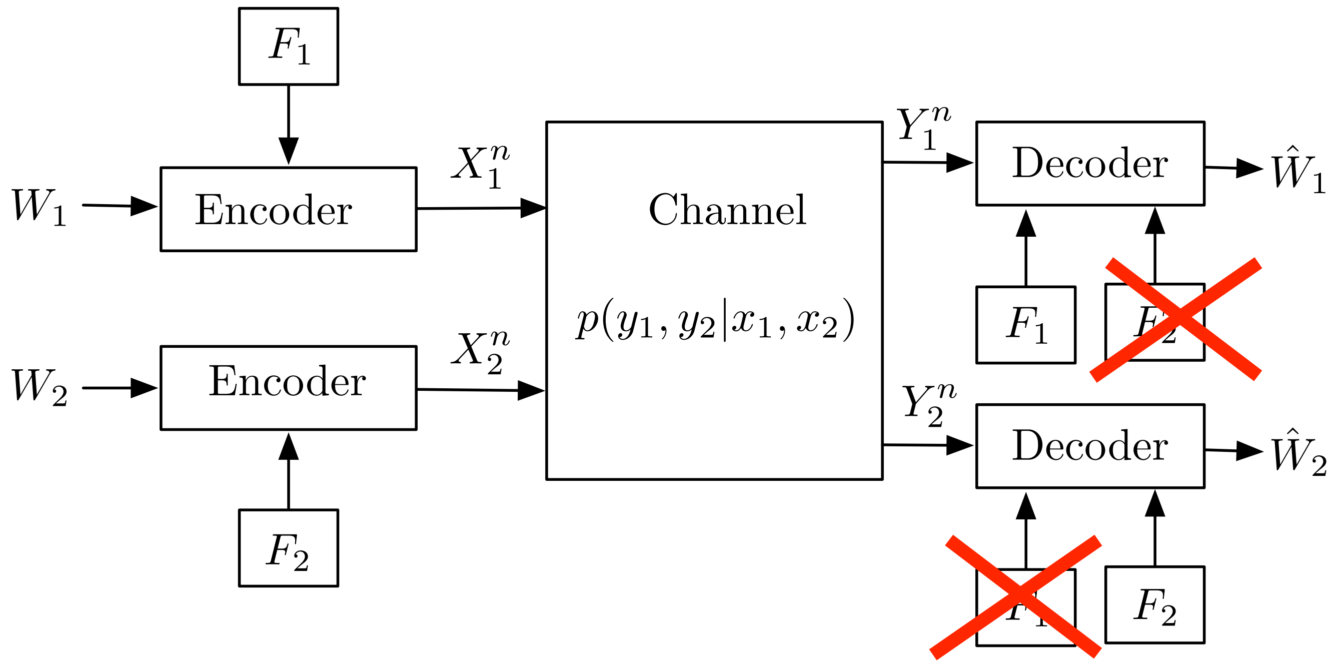
R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.



"One-sided" HK+
Mixed Inputs
3.34 bits

A. Dytso, D. Tuninetti, and N. Devroye, "On the two-user interference channel with lack of knowledge of the interference codebook at one receiver," IEEE Trans. Inf. Theory, vol. 61, no. 3, pp. 1257–1276, March 2015.

Approximate capacity without codebooks



TINnoTS +
Mixed Inputs
constant or log-log gaps

A. Dytso, D. Tuninetti, and N. Devroye, "Interference as Noise: Friend of Foe?" to appear in IEEE Trans. Inf. Theory, 2016. Available on arXiv.

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{N_1, N_2, \delta_1, \delta_2} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1; Y_1) \\ 0 \le R_2 \le I(X_2; Y_2) \end{array} \right\}$$

with

$$\begin{aligned} X_i &= \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG}, \\ &\delta_i \in [0,1], \\ &X_{iD} \sim \text{PAM}\,(N_i)\,, \\ &X_{iG} \sim \mathcal{N}(0,1), \\ &i = 1, 2. \end{aligned}$$

Choice of N_i, δ_i looks like

discrete $\Longleftrightarrow$ public

Gaussian $\Longleftrightarrow$ private

Approximate optimality of TINnoTS in Gaussian-IC

Very Weak	Weak I	Weak II	Strong	Very Strong
$\text{gap} = 1/2$	$\text{gap} = 3.79$	$\text{gap} = O(\log\log(\min(\mathrm{S},\mathrm{I})))$ up to an outage of controllable measure		$\text{gap} = 1.25$

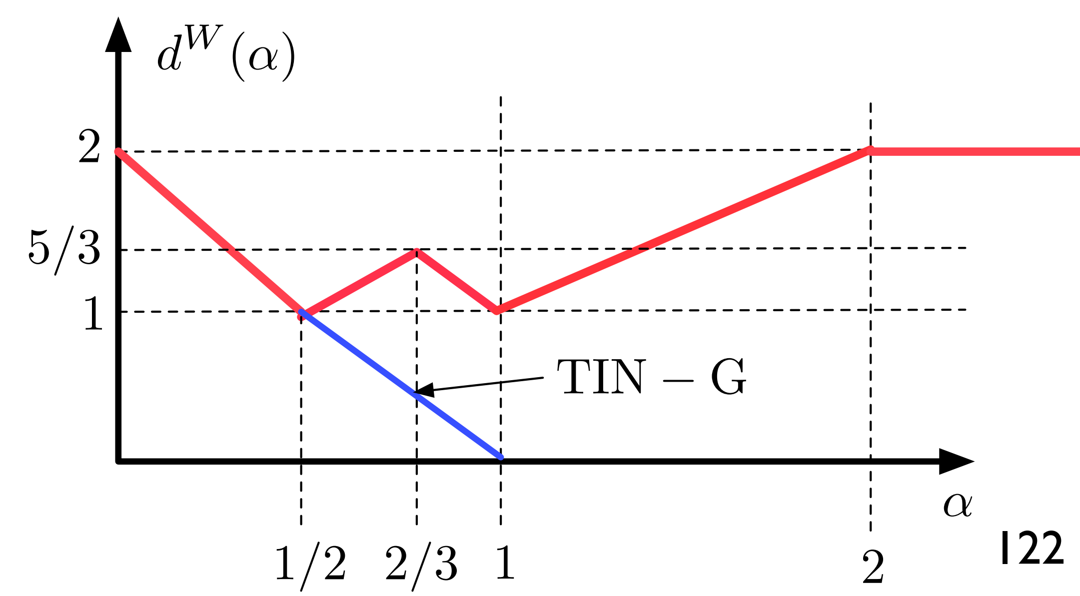
α: $1/2$, $2/3$, 1, 2

Gaussian	Mixed	Mixed	Discrete	Discrete

$$X_i = \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG},\ \ i \in [1:2],$$

$$\alpha = \frac{\text{inrdB}}{\text{snrdB}}$$

DoF gain over Gaussians with TINnoTS!



Approximate optimality of TINnoTS in Gaussian-IC

Very Weak	Weak I	Weak II	Strong	VeryStrong
$\text{gap} = 1/2$	$\text{gap} = 3.79$	$\text{gap} = O(\log\log(\min(\mathrm{S},\mathrm{I})))$ up to an outage of controllable measure		$\text{gap} = 1.25$

α: 0, 1/2, 2/3, 1, 2

Gaussian	Mixed	Mixed	Discrete	Discrete

$$X_i = \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG},\ \ i \in [1:2],$$

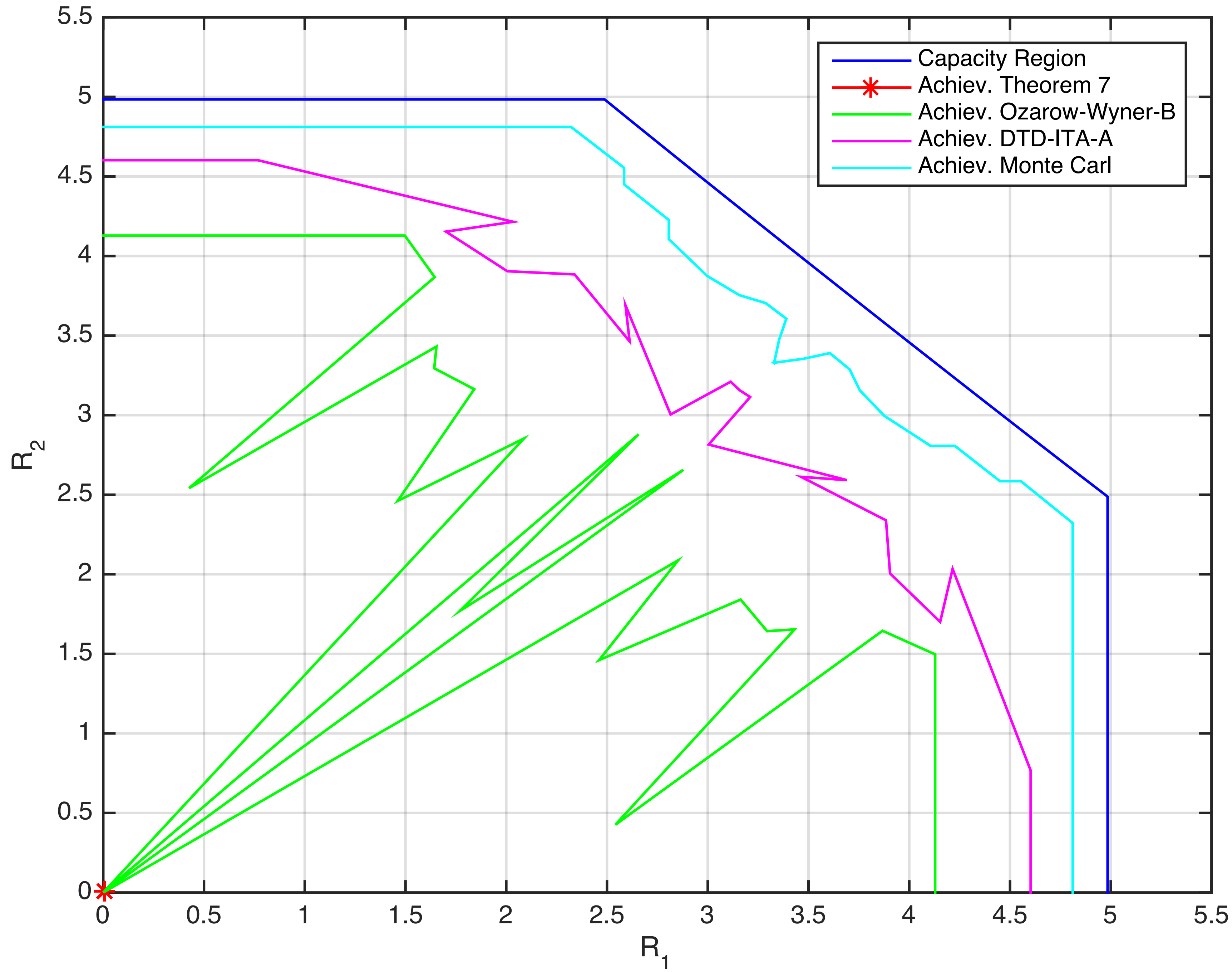
$$\alpha = \frac{\text{inrdB}}{\text{snrdB}}$$

<u>Closed-form</u> expressions for number of points, power splits and gap

Numerical evaluation

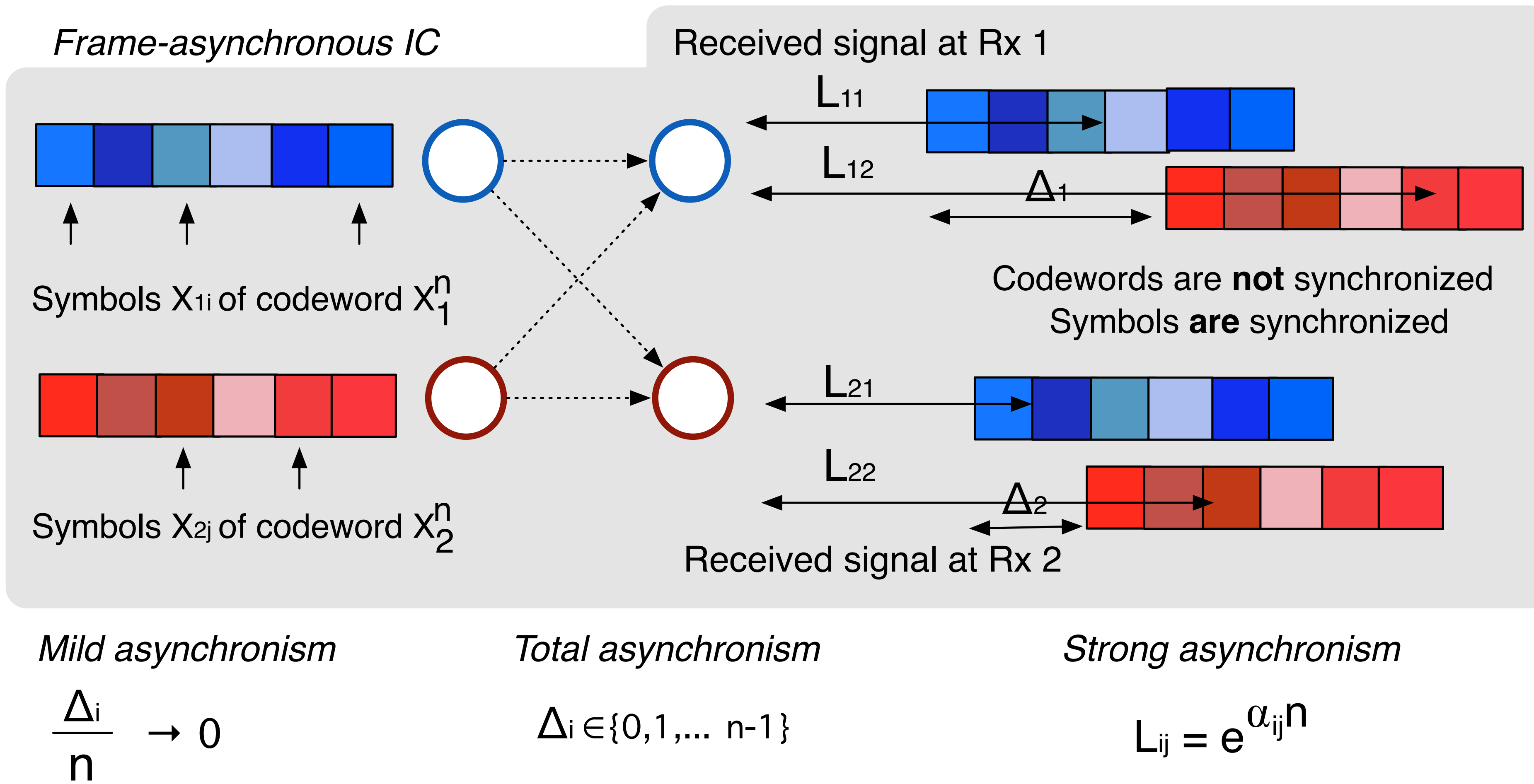
Analytical bounds on gaps are pessimistic!

strong interference, discrete inputs with analytical lower bound is red region!



Approximate capacity of ICs with lack of synchronization

- in networks, often assume all nodes are synchronized



- this may be unrealistic sometimes....

Approximate capacity of ICs with lack of synchronization

Treat Interference as Noise without Time Sharing Inner Bound:

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1 X_2} = P_{X_1} P_{X_2}} \left\{ \begin{array}{l} 0 \leq R_1 \leq I(X_1; Y_1) \\ 0 \leq R_2 \leq I(X_2; Y_2) \end{array} \right\}$$

No Time Sharing

- this is achievable by asynchronous G-IC, so our approximate gap to capacity results apply even without synchronization!

Key ideas + open problems

- use non-Gaussian inputs: good inputs, good interferers
- general tools on bounding dmin, mutual information applicable elsewhere?
- mixed inputs hence approximately optimal for the block asynchronous G-IC and the codebook oblivious G-IC
- **OPEN:** better constellation than PAM? What about higher dimensions?
- **OPEN:** can we develop a smart set of multi-letter discrete inputs and evaluate these in the capacity achieving expression for the G-IC?

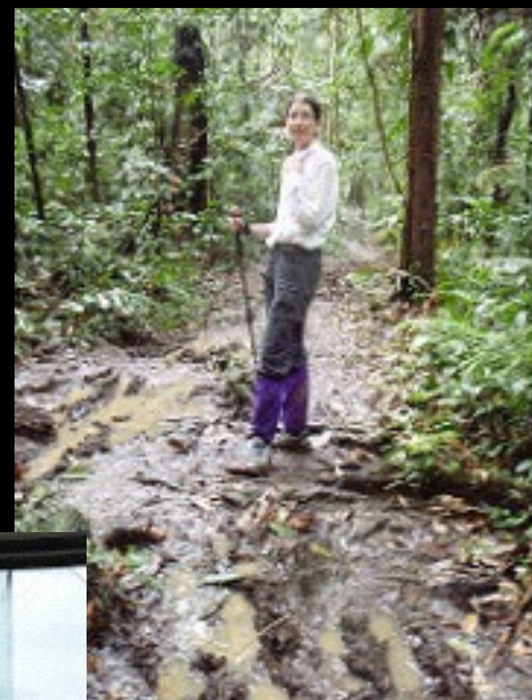
Capacity: $$\mathcal{C} = \lim_{n\to\infty} \text{co}\left(\bigcup_{P_{X_1^n X_2^n}=P_{X_1^n}P_{X_2^n}} \left\{ \begin{array}{l} 0 \le R_1 \le \frac{1}{n} I(X_1^n;Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n;Y_2^n) \end{array} \right\}\right)$$

R. Ahlswede, "Multi-way communication channels," in Proc. IEEE Int. Symp. Inf. Theory, March 1973, pp. 23–52.

Forced travel pics

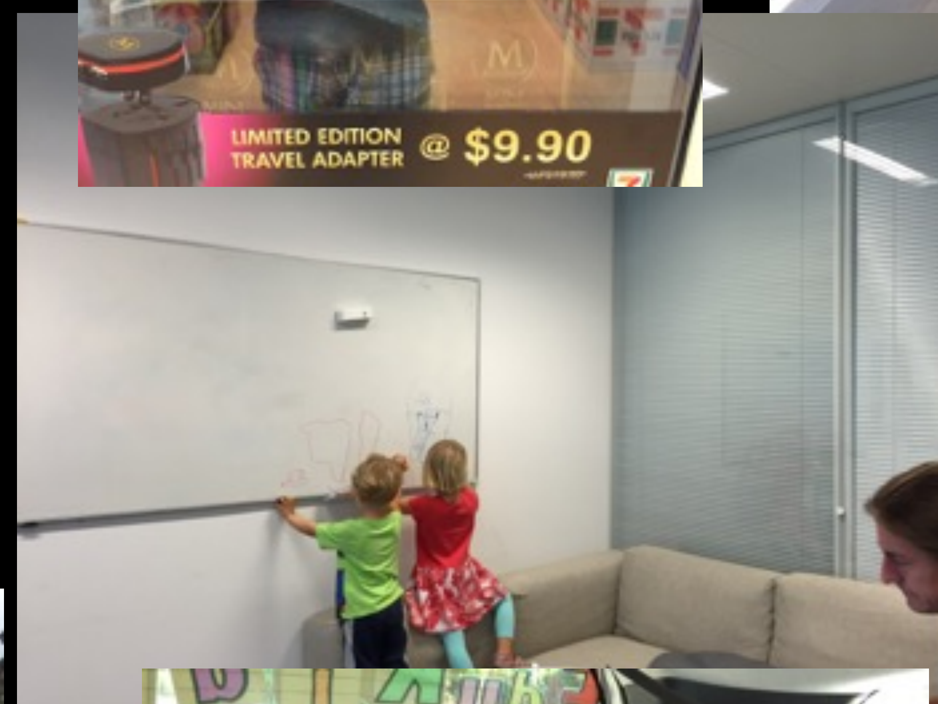
unsolicited MENTORING advice:
TRAVEL!

Post-Ph.D., pre tenure track



On the tenure track

On sabbatical



Practical codes for interference channels?

Polar codes over interference channels?


[L. Wang and E. Sasoglu, "Polar coding for interference networks," in *Proc. IEEE Int. Symp. Inf. Theory*, Honolulu, Hawaii 2014.]

propose a polar coding scheme able to achieve the H+K region for the IC

based on ideas in

[E. Arıkan, "Polar coding for the slepian-wolf problem based on mono- tone chain rules," in *Proc. IEEE Int. Symp. Inf. Theory*, Cambridge, MA, 2012, pp. 566–570.]

[S. H. Hassani and R. L. Urbanke, "Universal polar codes," 2013. [Online]. Available: http://arxiv.org/abs/1307.7223]

Informal statement by Ruediger Urbanke: "any region achievable by i.i.d. inputs usually can be shown to be achievable by polar codes"

Point to point codes for interference networks?

Young-Han Kim's group has worked extensively on this, see excellent slides: http://circuit.ucsd.edu/~yhk/pdfs/swcm.pdf

Other thoughts:
Ask Henry+Krishna?

Main ideas: high performance, lo

Block coding (relaying, feedback)

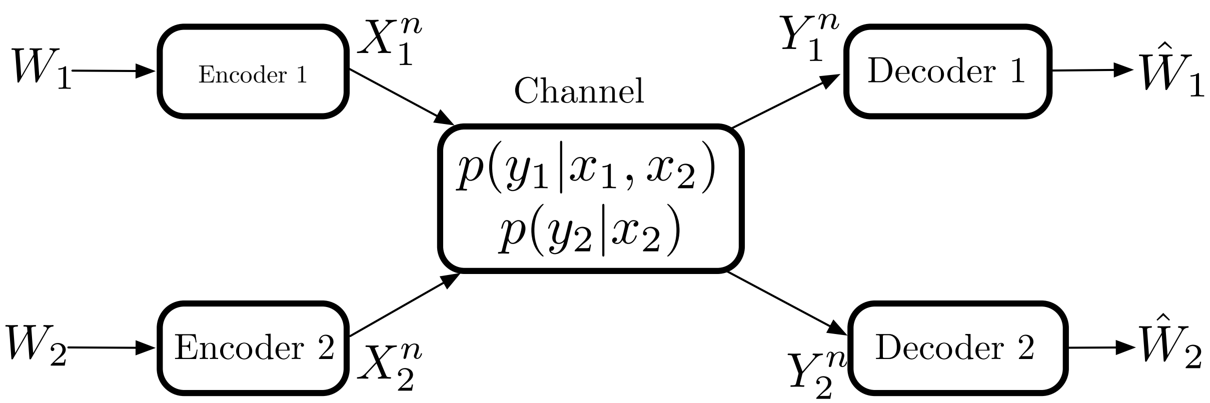
Superposition coding (without rate-splitting)

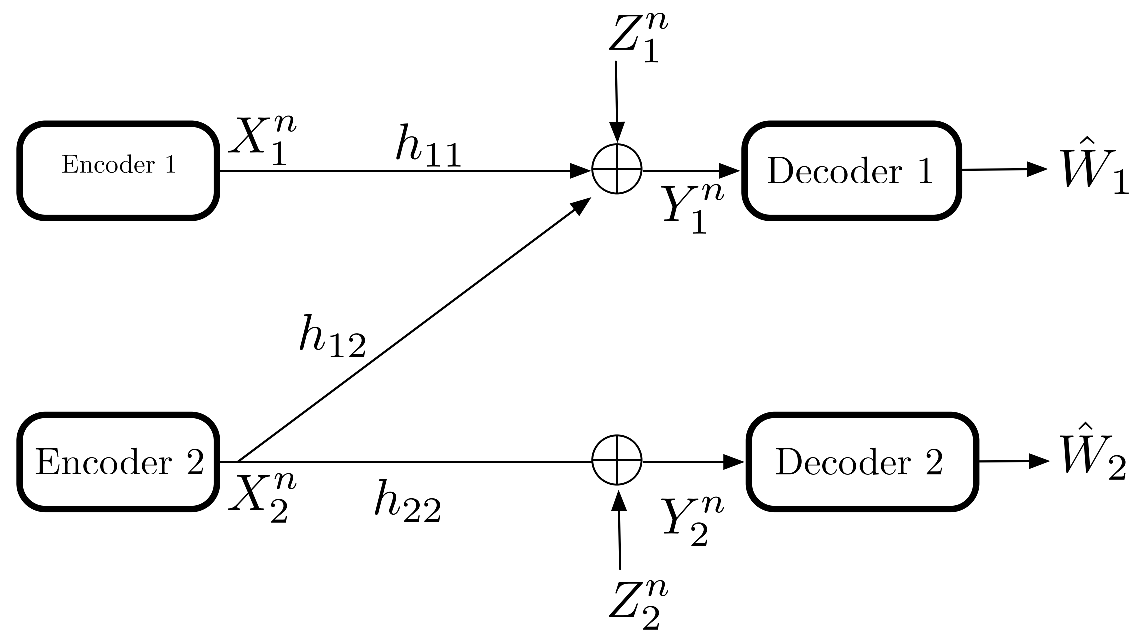
Staggered transmission

Sliding window and successive cancellation decoding

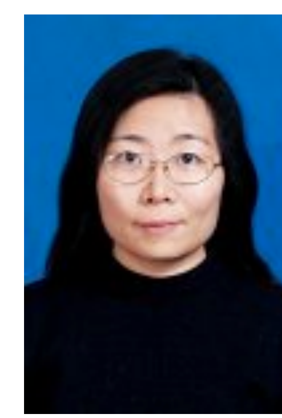
Variations

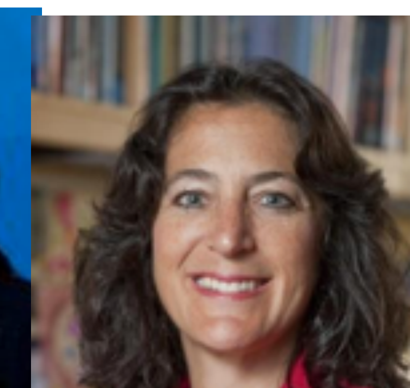
Z interference channel





[N. Liu and A. Goldsmith, "Capacity regions and bounds for a class of Z interference channels", IEEE Trans. on Info Theory, Vol. 55, No. 11, pp. 4986- 4994, Nov. 2009.]





Capacity unknown in general, except:

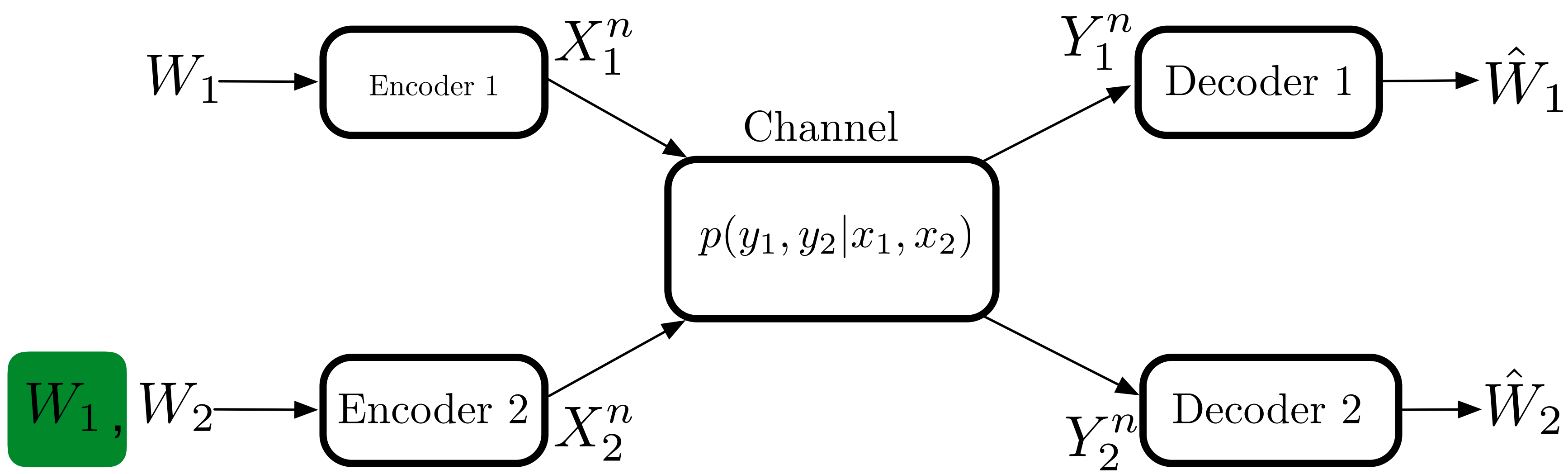
- sum-rate known for Gaussian Z-IC

[I. Sason, "On achievable rate regions for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1345–1356, Jun. 2004.]

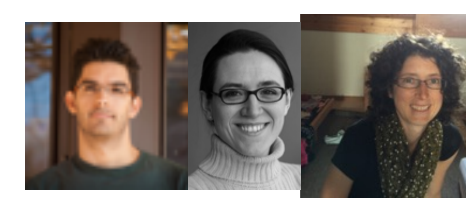
- sum-rate known when interference-free link is noise-free

[R. Ahlswede and N. Cai, "Codes with the identifiable parent property and the multiple-access channel," in *General Theory of Information Transfer and Combinatorics (Lecture Notes in Computer Science)*.]

Cognitive interference channel



Introduced: [N. Devroye, P. Mitran and V. Tarokh, "Achievable rates for cognitive radio channels," IEEE Trans. on Info. Theory, vol. 52, no. 5, pp. 1813-1827, May 2006.]



State of the art DM: [S. Rini, D. Tuninetti and N. Devroye, "New inner and outer bounds for the discrete memoryless cognitive interference channel and some capacity results ," IEEE Trans. on Info. Theory, vol. 57, no. 7, pp. 4087-4109, July 2011.]

State of the art Gaussian (capacity to within a constant gap): [S. Rini, D. Tuninetti and N. Devroye, "Inner and outer bounds for the Gaussian cognitive interference channel and some new capacity results," IEEE Trans. on Info. Theory, vol. 58, no. 2, pp. 820-848, Feb. 2012.]

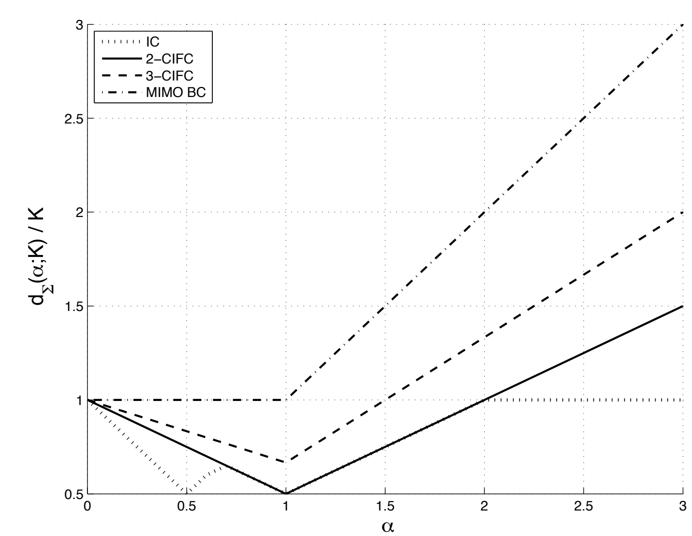
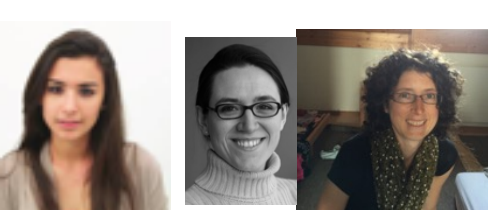


Fig. 3. $d_\Sigma(\alpha; K)/K$ for different channel models. The discontinuity at $\alpha =$

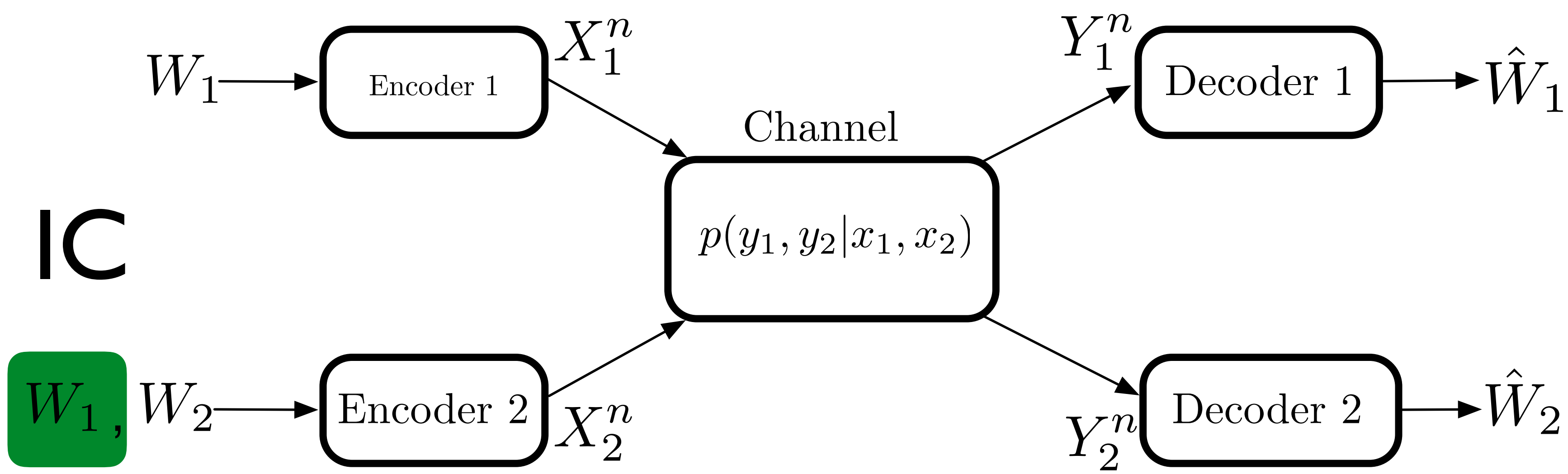
GDoF, cognitive with more users:



[D. Maamari, D. Tuninetti, and N. Devroye, ``Approximate Sum-Capacity of K-user Cognitive Interference Channels with Cumulative Message Sharing," IEEE Journal of Selected Areas in Communications -- Cognitive Radio Series, Vol. 32, No. 3, pp. 654-666, March 2014.]

Cognitive interference channel

BC and IC



New feature (like Broadcast Channel):

Encoder 2 can use

a) "dirty paper coding" to *eliminate* interference of W_2 at Rx 1, or

b) "cooperate" in sending W_1 to Rx 1

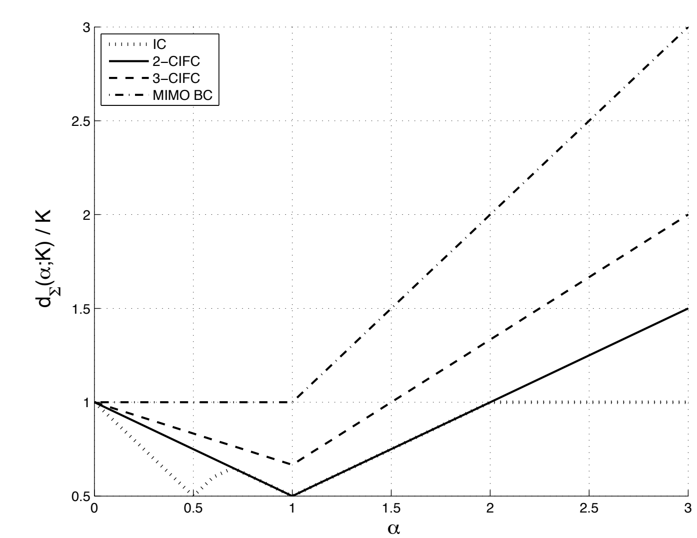
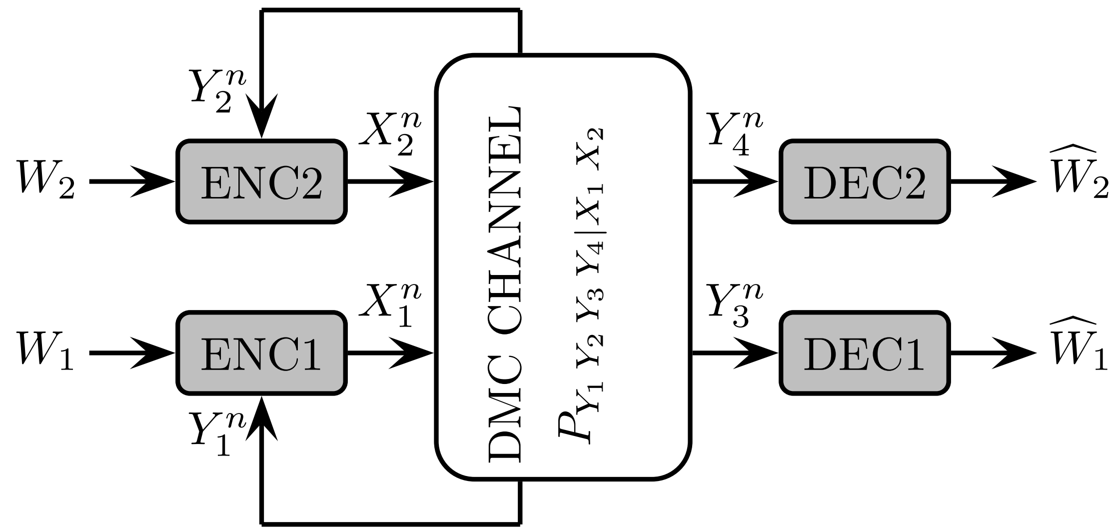


Fig. 3. $d_{\Sigma}(\alpha; K)/K$ for different channel models. The discontinuity at $\alpha = 1$ is not shown where the value is $\frac{1}{K}$.

Interference channel with generalized feedback

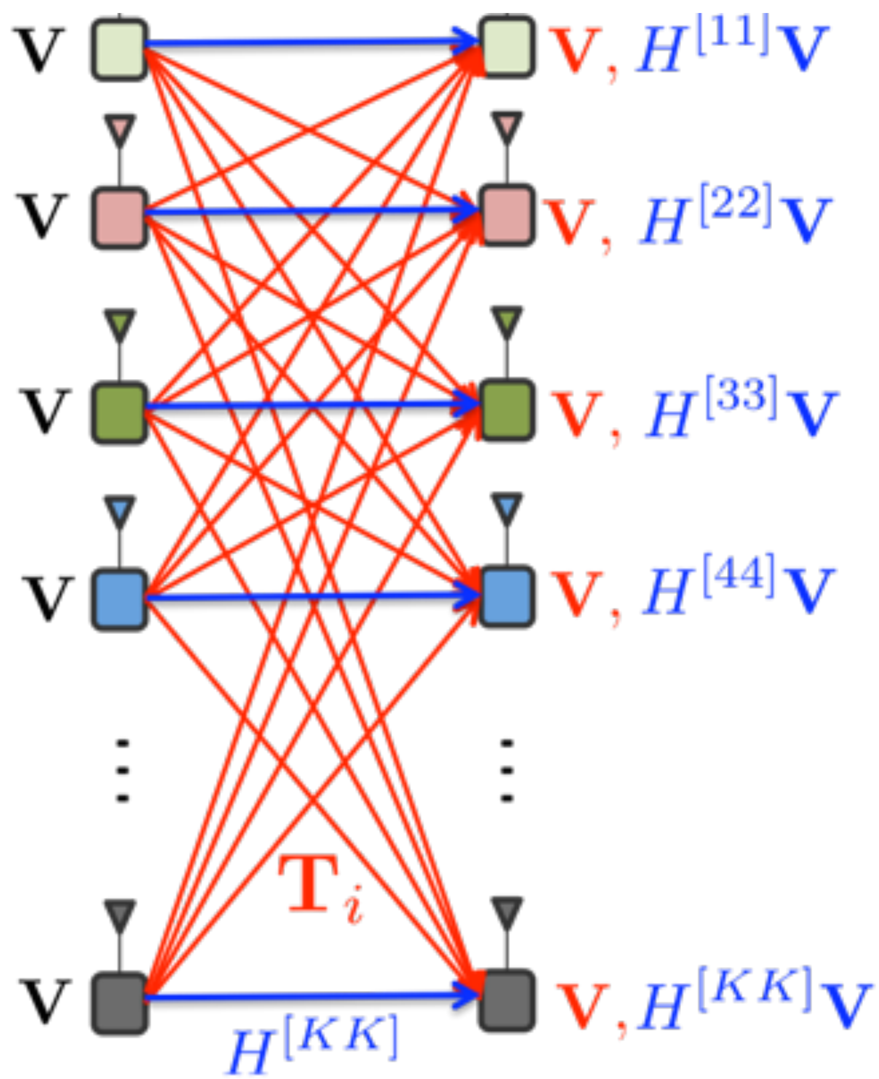


[S. Yang and D. Tuninetti, "Interference channel with generalized feedback (aka source cooperation) Part 1: achievable regions" IEEE Trans. on Info. Theory, Vo. 57, No. 5, pp. 2686-2710, May 2011.]

[D. Tuninetti, "An outer bound for the memoryless two-user interference channel with general cooperation" Information Theory Workshop, pp. 217-221, 2012.]

general model that captures causal source cooperation, all forms of feedback

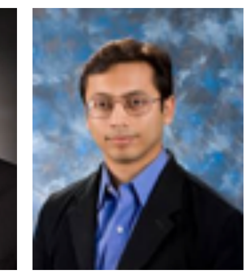
K-user interference channels



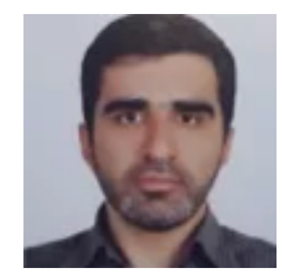
(a) K user interference channel

Interference alignment

[VR Cadambe and SA Jafar, “Interference alignment and the degrees of freedom of the K user interference channel,” IEEE Transactions on Information Theory, Vol. 54, No. 8, pp. 3425-3441, Aug. 2008.



[M. Maddah-Ali, A. Motahari, and A. Khandani, “Communication over X channel: Signalling and multiplexing gain,” in Technical Report. UW-ECE-2006-12, University of Waterloo, July 2006.]



Ask Helmut + Aylin!

Each user can send up to half of interference-free rate

DoF = K/2

image taken from https://sites.google.com/site/interferencealignment/home

highly recommended site for interference alignment

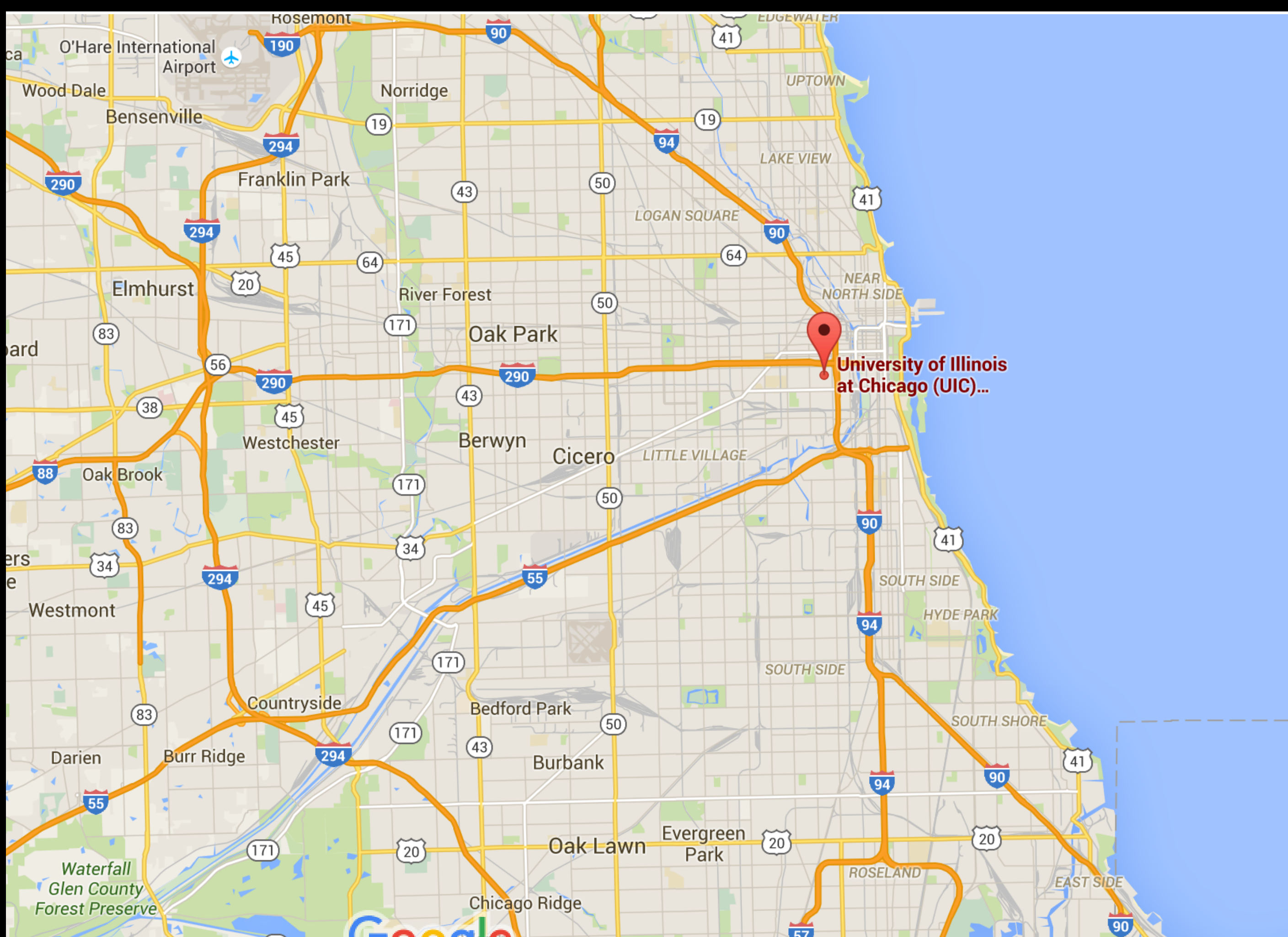
Other variations

- with state, known at….?
- secrecy….?
- with cognition and state….?
- ergodic capacity…..?
- game-theoretic issues….?
- Wyner model …?
- zero-error capacity….?
- MIMO interference channel….?
- IC with various flavors of feedback….?
- I-MMSE approach to understanding the IC ….?
- ICs with relays….?
- two-way ICs……?

unsolicited advice: chip away at fundamental problems!

Forced UIC advertisement

UIC is a public school in downtown Chicago



“Trump cancels Chicago rally (UIC Pavilion); protesters fill streets near UIC”

UIC is a great place to visit

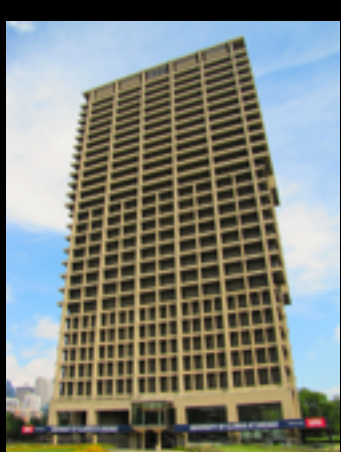
- home of the "NICEST" lab

Networks Information Communications and Engineering Systems Laboratory




Natasha Devroye | Hulya Seferoglu | Besma Smida | Daniela Tuninetti

- home of the best "Brutalist" architecture in the world

Key lessons learned

Simple channel model, what is it?

Many ways of treating interference, such as?

Is capacity known?

What is surprising?

Open problems

Are Gaussian inputs optimal for the Gaussian IC? depends what expression!

[S. Beigi, S. Liu, C. Nair, and M. Yazdanpanah, "Some results on the scalar Gaussian interference channel," ISIT, July 2016.]

Progress on evaluating the multi-letter capacity expression?

[S. Beigi, S. Liu, C. Nair, and M. Yazdanpanah, "Some results on the scalar Gaussian interference channel," ISIT, July 2016.]

Does the sum-capacity decrease or increase with the symmetric interference coefficient (btw 0 and 1)?

[I. Sason, "On achievable rate regions for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1345–1356, Jun. 2004.]

Questions + discussions now, later, email are always welcome

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I may be looking for a post-doc January 2017