Code Rate, Queueing Behavior and the Correlated Erasure Channel

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***Abstract*—This paper considers the relationship between code-rate selection and queueing performance for communication systems with time-varying parameters. While error-correcting codes offer protection against channel unreliability, there is a tradeoff between the enhanced protection of low-rate codes and the increased information transfer of high-rate codes. Hence, there exists a natural compromise between packet-level error protection and information rate. In the limiting regime where codewords are asymptotically long, this tradeoff is well-understood and characterized by the Shannon capacity. However, for delay-sensitive communication systems and finite code-lengths, a complete characterization of this tradeoff is still not fully developed. This paper offers a new perspective on the queueing performance of communication systems with finite block-lengths operating over correlated erasure channels. A rigorous framework that links code rate to overall system performance for random codes is presented. Guidelines for code rate selection in delay-sensitive systems are identified.**

I. Introduction

The transmission of digital information over noisy channels has become commonplace in modern communication systems. The high reliability of contemporary data links is due, partly, to the many successes of information theory and error-control coding [1]. In particular, the reliable transmission of digital information is possible at rates approaching Shannon capacity using asymptotically long codewords [2]. Indeed, many notable communication systems employ long codewords to provide high throughput and low probabilities of error [3].

One context where the insights offered by classical information theory do not necessarily apply directly is the broad area of delay-constrained communications and networks [4]. Real-time traffic and live interactive sessions are very sensitive to latency. Long codewords are therefore not particularly well-suited for real-time communication because they entail lengthy encoding/decoding delays. Alternative engineering methods, including power control, automatic repeat-request, scheduling and feedback, can be leveraged to establish rapid end-to-end connections [5], [6]. Moreover, delay considerations often force a system to operate below its maximum throughput [7].

Several recent articles in information theory focus on the tradeoff between throughput and delay. Popular approaches include effective capacity [8], outage capacity [9], [10], average delay characterizations [11], fluid analysis [12] and heavy-traffic limits [13]. While these contributions provide valuable insights about the design of delay-sensitive systems, many such articles make idealized assumptions about the behavior of coded transmissions. In particular, various authors adopt the notion of instantaneous capacity: individual data blocks are assumed, implicitly or explicitly, to possess enough degrees of freedom to support sophisticated coding schemes and thereby approach Shannon capacity within every time-slot. While reasonable for long codewords, such assumptions become somewhat of a concern for short data blocks. This is especially problematic for channels with memory, where correlation over time promotes deviations from typical behavior.

For delay-constrained communication systems that utilize short codewords, two opposite considerations underlie the selection of an error-correcting code. A low-rate code will, in general, result in a small probability of decoding failure; whereas the same system with a high-rate code is more prone to errors. Still, the successful decoding of a codeword associated with a high-rate code leads to the transmission of a larger number of information bits. In the limit of asymptotically long codewords, it is clear that code rate should only be slightly below Shannon capacity. However, the optimal operating point for systems with short block lengths is not so obvious. It may depend on the physical resources available and the service constraints imposed on the system.

Previous work in this area has either taken a higher-layer perspective, using a simplistic model of the physical layer; or adopted a channel-coding perspective, intentionally neglecting queueing considerations. Herein, we attempt to bridge the gap between these extremes in order to address two important questions. What is the optimal code rate for a particular implementation? What arrival rates can a system support under specific service requirements? Our approach in obtaining partial answers to these questions differ from previous work in that we strive to provide exact solutions. To facilitate the type of queueing analysis we wish to carry, we make the following assumptions. The packet arrival process at the transmitter is Bernoulli, with packet length having a geometric distribution at the bit level. The communication channel is a bit erasure channel with memory. Random codes, with maximum likelihood decoding, are employed to protect the sent information against erasures. Collectively, these assumptions are sufficient to conduct a rigorous analysis of the probability of block decoding failure at the receiver as well as a complete characterization of the ensuing queueing behavior. Implicit to our system model is the ability to acknowledge the reception of packets through instantaneous feedback.

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II. System Model

We initiate our study of the system we wish to consider with a description of the underlying communication channel. Bits are sent from the transmitter to the destination over an erasure channel with memory. The channel can be in one of two states, a *good* state g in which bits arrive unaltered at the destination, and a *bad* state e under which information is lost. The transitions of the channel over time follow a discrete-time Markov model, where the probability of transitioning from e to g is represented by α and the transition probability in the reverse direction is denoted by β. Using lexicographical state ordering, we can express the probability transition matrix as

$$P_{\mathrm{t}} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}.$$

This bit erasure channel is illustrated in Fig. 1. Correlation over time is captured through the probability transition matrix of the channel. We denote the state of the erasure channel at time t by c_t. The evolution of the channel over time then becomes a discrete random process, which we write $\{c_t\}$.

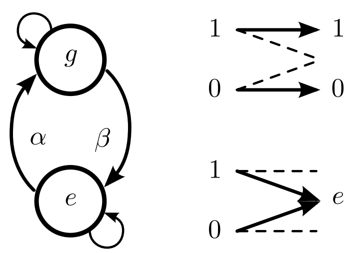

Fig. 1. A discrete-time Markov erasure channel is employed to model the operation of a communication link with memory.

In this study, we assume that the communication system employs an error correcting code with block length T. That is, every codeword is sent over T consecutive realizations of the erasure channel. Parameter T is determined by the requirements and specifications of the system. Ultimately, block length affects decoding delay and the amount of physical resources needed for feedback. In our analysis, T remains fixed throughout. The code rate r, however, is a parameter that can be optimized. For a particular code rate, the corresponding number of information bits per codeword is given by rT.

Having specified the channel structure, we turn to the definition of the arrival process. Data packets enter the queue according to a discrete-time process synchronized with the codeword transmission cycle. During every codeword transmission event, a data packet arrives at the queue with probability γ, else the queue remains unchanged. Arrivals are taken to be independent over time, hence forming a standard Bernoulli process. The length of a data packet is also random, following a geometric distribution. The probability that a packet contains exactly ℓ bits is given by

$$\Pr(L = \ell) = (1-\rho)^{\ell-1}\rho \qquad \ell = 1, 2, \ldots$$

where $\rho \in (0, 1)$. The arrival process and the packet length distribution have been selected, partly, to facilitate the analysis we wish to carry below. In particular, the memoryless property of the geometric distribution and the independence over time of the Bernoulli process are crucial properties that make for a tractable characterization of queueing behavior.

Once a code rate is specified, the number of successfully decoded codewords needed to complete a packet transmission is equal to $M = \lceil L/rT \rceil$. We note that L being geometric with parameter ρ implies that M is also geometric with parameter

$$\rho_r = \sum_{\ell=1}^{rT} (1-\rho)^{\ell-1}\rho = 1 - (1-\rho)^{rT}.$$

Thus, the probability that a data packet requires the successful transmission of exactly m codewords is equal to

$$\Pr(M = m) = (1-\rho_r)^{m-1}\rho_r \qquad m = 1, 2, \ldots$$

Key to our mathematical analysis is that the number of coded blocks per data packet, M, retains the memoryless property.

When discussing the size of the queue at the transmitter, two distinct characterizations are possible. A first option is to keep track of the number of packets present in the queue. The second choice is to count the number of data blocks of length rT remaining in the queue. Although the latter alternatives provides a more accurate representation of actual queue length in bits, the former option is much simpler to analyze and is more closely related to the concept of packet delay. For these reasons, we elect to define the state of the queue as the number of data packets awaiting transmission, as is customary in the classical queueing literature [14], [15].

The final building block of our system model is a coding strategy employed to correct transmission errors due to channel uncertainty. As mentioned above, our system utilizes random binary codes to provide protection against erasures. Maximum likelihood decoding is used at the destination to decode the received information. This completes the description of the communication system under study. We are now ready to conduct the performance analysis of this system, a task we initiate in the following section.

III. Probability of Decoding Failure

A first step in analyzing the performance of our system is to derive expressions for the probabilities of decoding failure at the receiver. To compute these probabilities, we need to obtain the distribution of the number of erasures within a codeword. Let $c_t \in \{e, g\}$ be the state of the erasure channel at time t, and define an aggregate state for block s by

$$B_s = \left(c_{sT+1}, c_{(s+1)T}\right)$$

where c_{sT+1} is the state of the channel at the beginning of block s and $c_{(s+1)T}$ is the channel state at the very end of the same block. We use N_s to represent the number of erasures during block B_s. Note that, because of the Markov property, random variable N_s is conditionally independent of $B_{\tilde{s}}$, given B_s, whenever $\tilde{s} \neq s$. Finding the joint distribution of $(N_s, c_{(s+1)T})$ conditioned on the value of c_{sT+1} is a straightforward problem of combinatorics. For $2 \leq n < T$,

$$\Pr\left(N_s = n, c_{(s+1)T} = e | c_{sT+1} = e\right) = \sum_{j=1}^{j_{\max}} \binom{n-1}{j}\binom{T-n-1}{j-1}\alpha^j(1-\alpha)^{n-j-1}\beta^j(1-\beta)^{T-n-j}$$

where $j_{\max} = \min\{n-1, T-n\}$ and j denotes the number of transitions from e to g in block s; for $0 < n \leq T-2$,

$$\Pr\left(N_s = n, c_{(s+1)T} = g | c_{sT+1} = g\right) = \sum_{j=1}^{j_{\max}} \binom{n-1}{j-1}\binom{T-n-1}{j}\alpha^j(1-\alpha)^{n-j}\beta^j(1-\beta)^{T-n-j-1}$$

where $j_{\max} = \min\{n, T-n-1\}$ and j represents the number of transitions from g to e in block s; for $0 < n < T$,

$$\Pr\left(N_s = n, c_{(s+1)T} = g | c_{sT+1} = e\right) = \sum_{j=1}^{j_{\max}} \binom{n-1}{j-1}\binom{T-n-1}{j-1}\alpha^j(1-\alpha)^{n-j}\beta^{j-1}(1-\beta)^{T-n-j}$$

where $j_{\max} = \min\{n, T-n\}$ and j stands for the number of transitions from e to g in block s; finally, for $0 < n < T$,

$$\Pr\left(N_s = n, c_{(s+1)T} = e | c_{sT+1} = g\right) = \sum_{j=1}^{j_{\max}} \binom{n-1}{j-1}\binom{T-n-1}{j-1}\alpha^{j-1}(1-\alpha)^{n-j}\beta^j(1-\beta)^{T-n-j}$$

where $j_{\max} = \min\{n, T-n\}$ and j designates the number of transitions from g to e in block s. From these equations, we can easily compute the distribution of N_s conditioned on the value of the aggregate state B_s, $\Pr(N_s | B_s)$.

Suppose that a random binary code of length T provides forward error correction to each transmitted block. The code is chosen by generating a random parity-check matrix H of size $(T-k)\times T$, where each entry is independent and equiprobable $\{0,1\}$. Given N_s, it is possible to find the probability of decoding failure at the destination in closed form; decoding will succeed if and only if the submatrix of H formed by choosing the n erased columns has rank n [16].

The probability that a random $n \times p$ matrix over $\mathbb{F}_2$, where $p = T - k$ is the number of parity bits, has rank n is given by the product

$$\prod_{i=0}^{n-1}\left(1 - 2^{i-p}\right).$$

We can then conclude that, given n erasures, the probability of decoding failure becomes

$$P_{\mathrm{f}}(n) = 1 - \prod_{i=0}^{n-1}\left(1 - 2^{i-p}\right).$$

The average probability of decoding failure at the receiver is given by $\mathbb{E}\left[P_{\mathrm{f}}(N_s)\right]$, where the expectation is over possible channel realizations within a block. We emphasize that this quantity alone is not sufficient to describe the queueing behavior of the system, as memory in the erasure channel will induce time-dependencies from block to block. This is detailed below, where queueing aspects of the communication systems are investigated.

IV. Queueing Behavior

Recall that a packet of length L is divided into M data segments, each of which is encoded separately into a codeword of length T and subsequently sent over the channel. A feedback mechanism informs the transmitter of reception status. When successful, the corresponding data segment is marked as delivered and transmission of the next data segment begins. In case of a decoding failure, the original data segment remains in-line for immediate retransmission. Data packets are stored in the queue upon arrival at the transmitter, and they remain in the queue until the corresponding data segments are successfully decoded at the destination. In other words, a packet composed of L bits will require the successful transmission of M codewords before it is removed from the queue. We use q_s to symbolize the length of the queue at the onset of block s. We also define an aggregate state

$$Q_s = (c_{sT+1}, q_s),$$

composed of the first channel state and queue length. We stress that $Q_s \in \{e, g\} \times \mathbb{N}$, a countable space.

We begin our analysis of queueing behavior by noticing that the sequence $\{Q_s\}$ forms a Markov chain. Furthermore, we can obtain the invariant distribution of the queue by determining the equilibrium distribution of the augmented process $\{Q_s\}$. We can write the transition probability from Q_s to Q_{s+1} as follows,

$$\begin{aligned}\Pr(Q_{s+1}|Q_s) &= \Pr\left(c_{(s+1)T+1}, q_{s+1} | c_{sT+1}, q_s\right) \\ &= \sum_{c_{(s+1)T}\in\{e,g\}} \Pr\left(c_{(s+1)T+1}, c_{(s+1)T}, q_{s+1} | c_{sT+1}, q_s\right) \\ &= \sum_{c_{(s+1)T}\in\{e,g\}} \Pr\left(c_{(s+1)T}, q_{s+1} | c_{sT+1}, q_s\right) \\ &\qquad \times \Pr\left(c_{(s+1)T+1} | c_{(s+1)T}\right).\end{aligned}$$

Recall that $\Pr\left(c_{(s+1)T+1} | c_{(s+1)T}\right)$ can be obtained directly from the definition of the erasure channel. More specifically, finding this probability amounts to locating an entry in P_{t}. Similarly, $\Pr(c_{i+j}|c_i)$ is given by the corresponding entry in P_{t}^j, where the jth power of P_{t} can be computed as

$$P_{\mathrm{t}}^j = \begin{bmatrix} \frac{\beta+\alpha(1-\alpha-\beta)^j}{\alpha+\beta} & \frac{\alpha-\alpha(1-\alpha-\beta)^j}{\alpha+\beta} \\ \frac{\beta-\beta(1-\alpha-\beta)^j}{\alpha+\beta} & \frac{\alpha+\beta(1-\alpha-\beta)^j}{\alpha+\beta} \end{bmatrix}.$$

We note that channel memory is a function of $1-\alpha-\beta$. Finding an expression for $\Pr\left(c_{(s+1)T}, q_{s+1} | c_{sT+1}, q_s\right)$ remains.

First, we assume that $q_s > 0$; admissible values for q_{s+1} are thus contained in $\{q_s - 1, q_s, q_s + 1\}$. Recall that two factors can affect the length of the queue, the arrival of a new data packet and the completion of a packet transmission. The latter will only occur if a codeword is successfully decoded at the destination and the head packet has no additional data segment left to send. Keeping these factors in mind, we get

$$\begin{aligned}
&\Pr(q_{s+1} = q_s + 1, c_{(s+1)T} | c_{sT+1}, q_s) \\
&= \sum_{n=0}^{T} \gamma \big(P_{\mathrm{f}}(n) + (1 - P_{\mathrm{f}}(n))(1-\rho_r) \big) \\
&\qquad \times \Pr(N_s = n, c_{(s+1)T} | c_{sT+1}), \\
&\Pr(q_{s+1} = q_s, c_{(s+1)T} | c_{sT+1}, q_s) \\
&= \sum_{n=0}^{T} \big((1-\gamma) \big(P_{\mathrm{f}}(n) + (1 - P_{\mathrm{f}}(n))(1-\rho_r) \big) \\
&\qquad + \gamma \, (1 - P_{\mathrm{f}}(n)) \, \rho_r \big) \Pr(N_s = n, c_{(s+1)T} | c_{sT+1}), \\
&\Pr(q_{s+1} = q_s - 1, c_{(s+1)T} |, c_{sT+1}, q_s) \\
&= \sum_{n=0}^{T} (1-\gamma) \, (1 - P_{\mathrm{f}}(n)) \, \rho_r \Pr(N_s = n, c_{(s+1)T} | c_{sT+1}),
\end{aligned}$$

When the queue is empty, only two possibilities can occur,

$$\begin{aligned}
&\Pr(q_{s+1} = 1, c_{(s+1)T} | c_{sT+1}, q_s = 0) \\
&\quad = \gamma \Pr(c_{(s+1)T} | c_{sT+1}) \\
&\Pr(q_{s+1} = 0, c_{(s+1)T} | c_{sT+1}), q_s = 0) \\
&\quad = 1 - \gamma \Pr(c_{(s+1)T} | c_{sT+1}).
\end{aligned}$$

Collecting these results together, we get the probability transition matrix of the Markov process $\{Q_s\}$. A compact graphical representation of the state transitions appears in Fig. 2.

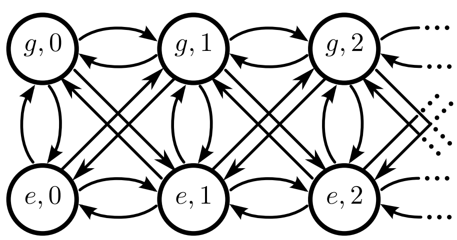

Fig. 2. State space and transition diagram for the aggregate queued process $\{Q_s\}$; self-transitions are intentionally omitted.

The transition probabilities being identified, we next analyze queueing behavior. For convenient, we define the following mathematical notation. For $k \in \mathbb{N}$ and $c, d \in \{e, g\}$, let

$$\begin{aligned}
\kappa_{cd} &= \Pr(Q_{s+1} = (d, k) | Q_s = (c, k)) \\
\lambda_{cd} &= \Pr(Q_{s+1} = (d, k+1) | Q_s = (c, k)) \\
\mu_{cd} &= \Pr(Q_{s+1} = (d, k-1) | Q_s = (c, k));
\end{aligned}$$

and, similarly, when the queue is empty, define

$$\begin{aligned}
\kappa_{cd}^0 &= \Pr(Q_{s+1} = (d, 0) | Q_s = (c, 0)) \\
\lambda_{cd}^0 &= \Pr(Q_{s+1} = (d, 1) | Q_s = (c, 0)).
\end{aligned}$$

Together, these equations define the 12 transition probabilities associated with a non-empty queue, and the 8 transition probabilities subject to the non-negativity constraint at zero.

For stable systems, performance is assessed using the equilibrium distribution of the queue, $\lim_{s \to \infty} \Pr(Q_s = (c, k))$. We represent the probabilities of the different states with a letter for the channel state and a number subscript for the queue occupancy. Accordingly, we can write the balance equations governing our Markov chain for $k \geq 2$ as

$$\begin{aligned}
&(\mu_{gg} + \mu_{ge} + \kappa_{ge} + \lambda_{ge} + \lambda_{gg}) g_k \\
&= \lambda_{gg} g_{k-1} + \lambda_{eg} e_{k-1} + \kappa_{eg} e_k + \mu_{eg} e_{k+1} + \mu_{gg} g_{k+1} \\
&(\mu_{ee} + \mu_{eg} + \kappa_{eg} + \lambda_{eg} + \lambda_{ee}) e_k \\
&= \lambda_{ee} e_{k-1} + \lambda_{ge} g_{k-1} + \kappa_{ge} g_k + \mu_{ge} g_{k+1} + \mu_{ee} e_{k+1}.
\end{aligned}$$

For $k = 1$, these equations become

$$\begin{aligned}
&(\mu_{gg} + \mu_{ge} + \kappa_{ge} + \lambda_{ge} + \lambda_{gg}) g_k \\
&= \lambda_{gg}^0 g_{k-1} + \lambda_{eg}^0 e_{k-1} + \kappa_{eg} e_k + \mu_{eg} e_{k+1} + \mu_{gg} g_{k+1} \\
&(\mu_{ee} + \mu_{eg} + \kappa_{eg} + \lambda_{eg} + \lambda_{ee}) e_k \\
&= \lambda_{ee}^0 e_{k-1} + \lambda_{ge}^0 g_{k-1} + \kappa_{ge} g_k + \mu_{ge} g_{k+1} + \mu_{ee} e_{k+1},
\end{aligned}$$

and, at zero, they reduce to

$$\begin{aligned}
(\kappa_{ge}^0 + \lambda_{ge}^0 + \lambda_{gg}^0) g_0 &= \kappa_{eg}^0 e_0 + \mu_{eg} e_1 + \mu_{gg} g_1 \\
(\kappa_{eg}^0 + \lambda_{eg}^0 + \lambda_{ee}^0) e_0 &= \kappa_{ge}^0 g_0 + \mu_{ge} g_1 + \mu_{ee} e_1.
\end{aligned}$$

The invariant distribution of the Markov chain can be derived from these recurrence relations using transform methods [15]. In particular, if we define the generating functions

$$E(z) = \sum_{z=0}^{\infty} e_k z^k \qquad G(z) = \sum_{z=0}^{\infty} g_k z^k,$$

then finding the stationary distribution of our Markov system is equivalent to solving a matrix equation of the form

$$A(z) \begin{bmatrix} E(z) \\ G(z) \end{bmatrix} = B(z) \begin{bmatrix} e_0 \\ g_0 \end{bmatrix},$$

where the entries in $A(z)$ and $B(z)$ are quadratic polynomials. The coefficients e_0, g_0 can be determined by requiring that the queue is stable (e.g., by canceling an unstable pole) and using the fact that

$$E(1) = \frac{\beta}{\alpha + \beta}, \qquad G(1) = \frac{\alpha}{\alpha + \beta}.$$

Although a closed-form solution exists and can be obtained using symbolic equation solvers, its form is too convoluted to be included in this publication. Still, we emphasize that this solution can be calculated efficiently using numerical methods. We include below an example to show how the methodology developed above can be applied to real-world systems.

V. Numerical Results

We present numerical results for a system with the following parameters. Codewords each span $T = 114$ code bits, and are transmitted every 4.615 ms. The probability that a new packet arrives at the source during a codeword transmission cycle is $\gamma = 0.25$. The expected size of a packet is 175 bits, which implies that $\rho = 1/175$. This leads to an average arrival rate of roughly 9.48 Kbps. Recall that, for every packet, the number of successfully decoded codewords needed to complete transmission is equal to $\lceil L/rT \rceil$, where rT is the number of information bits contained in a codeword. The erasure parameters are given by $\alpha = 0.08$ and $\beta = 0.02$. This implies that the expected bit-erasure probability is 0.2, and the memory of the channel decays exponentially at rate $(1 - \alpha - \beta) = 0.9$. A situation where these values offer a realistic assessment of system operation is the scenario where a pico-cell aggregator collects data from a sensor network and relays this information to a fusion center through a wireless GSM connection. Collectively, these parameters define the evolution of the Markov process governing the queue. System performance as a function of rT appears in Fig. 3. Each curve represents the tail probabilities in the

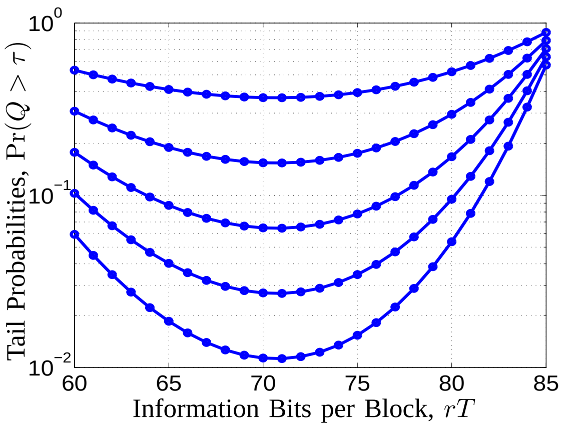

Fig. 3. This figure shows tail probabilities in the equilibrium packet distribution of the queue, $\Pr(Q > \tau)$, for threshold values $\tau \in \{5, 10, 15, 20, 25\}$. These probabilities are plotted as a function of the number of information bits per codeword, rT. The minimum on each curves represents an optimal operating point, and they are achieved uniformly at $rT = 71$.

equilibrium packet distribution of the queue, $\Pr(Q > \tau)$, for a specific threshold level, $\tau \in \{5, 10, \ldots, 25\}$. Low threshold values are selected to reflect delay-sensitive applications. In this figure, the optimal code rate ($r = 71/114$) appears robust to the choice of threshold value τ. This observation remains true for all the system parameters tested.

The optimal number of information bits per codeword, on the other hand, depends heavily on the channel memory factor. In our numerical study, we fixed the ratio $\alpha : \beta$ at one quarter, and varied the memory factor. When the channel is memoryless, the optimal rT is 80. For comparison, the capacity is 0.8 and gives an rT of roughly 91. As correlation increases, the optimal value of rT initially decreases, thereby improving protection against erasures. Still, at some point, the coherence time of the channel starts to approach the length of a codeword, $T = 114$. Under such condition, the code becomes ineffective at correcting long sequence of successive erasures. The optimal strategy progressively changes to including more information bits in every packet, and hoping that the channel remains in its good state. In the limit where the channel is very correlated, $(1 - \alpha - \beta) \to 1$, the optimal strategy is to transmit uncoded data bits, $rT = T$. Indeed, strong correlation is characterized by long strings of erasures followed by longer strings of reliable bits, and the best strategy is to send as many

TABLE I
OPTIMAL NUMBER OF INFORMATION BITS AND TAIL PROBABILITY $\Pr(Q > 5)$ AS A FUNCTION OF CHANNEL MEMORY FACTOR $1 - \alpha - \beta$.

$1 - \alpha - \beta$	Optimal rT	$\min \Pr(Q > 5)$
0	80	0.049
0.5	76	0.085
0.9	71	0.368
0.98	87	0.598
0.99	114	0.246

information bits as possible when the channel is good. At this point, the bit erasure channel essentially becomes a packet erasure channel. This is summarized in Table I.

VI. Discussion and Concluding Remarks

This article presents a new framework to analyze the relation between code rate and queueing behavior. The simplicity of the erasure channel and its closed-form characterization of error events are instrumental in conducting this analysis. For short block lengths and correlated channels, the optimal code rate appears to be linked to the relative size of a codeword compared to the coherence time. In some circumstances, it is better to adopt a rate beyond Shannon capacity.

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