Universality for the Noisy Slepian-Wolf Problem via Spatial Coupling

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***Abstract*—We consider a noisy Slepian-Wolf problem where two correlated sources are separately encoded and transmitted over two independent binary memoryless symmetric channels. Each channel capacity is assumed to be characterized by a single parameter which is not known at the transmitter. The receiver has knowledge of both the source correlation and the channel parameters. We call a system universal if it retains near-capacity performance without channel knowledge at the transmitter.**

Kudekar et al. recently showed that terminated low-density parity-check (LDPC) convolutional codes (a.k.a. spatially-coupled LDPC ensembles) can have belief-propagation thresholds that approach their maximum a-posteriori thresholds. This was proven for binary erasure channels and shown empirically for binary memoryless symmetric channels. They also conjectured that the principle of spatial coupling is very general and the phenomenon of threshold saturation applies to a very broad class of graphical models. In this work, we derive an area theorem for the joint decoder and empirically show that threshold saturation occurs for this problem. As a result, we demonstrate near-universal performance for this problem using the proposed spatially-coupled coding system. A similar result is also discussed briefly for the 2-user multiple-access channel.

***Index Terms*—LDPC codes, spatial coupling, EXIT functions, density evolution, correlated sources, non-systematic encoders, joint decoding, protograph, area theorem.**

I. Introduction

The phenomenon of threshold saturation via spatial coupling was introduced in [1], [2] to describe the excellent performance of convolutional LDPC codes over binary-input memoryless symmetric (BMS) channels [3]. Kudekar et al. prove that the belief propagation (BP) threshold of the spatially coupled ensemble is *essentially* equal to the maximum a-posteriori (MAP) threshold of the underlying ensemble when transmission takes place over a binary erasure channel (BEC) [1]. Empirical evidence of this phenomenon for BMS channels has been observed in [2], [3].

The underlying principle behind the impressive performance of spatially-coupled codes is very broad and Kudekar et al. conjecture that the same phenomenon occurs for more general channels. In this work, we consider a noisy Slepian-Wolf problem. The outputs of two discrete memoryless correlated sources, (U_1, U_2), are transmitted to a central receiver through two independent discrete memoryless channels with capacities C_1 and C_2, respectively. In [4], the authors consider the noisy Slepian-Wolf problem and observed that the MAP threshold of the punctured LDPC$(4, 6)$ ensemble is very close to the capacity region for transmission over erasure channels. Therefore, the phenomenon of threshold saturation motivates the use of spatial coupling as a potentially universal coding scheme for the noisy Slepian-Wolf problem. In this paper, this observation is extended to the 2-user Gaussian multiple access channel (MAC).

We will assume that the channels belong to the same channel family, and that each channel can be parameterized by a single parameter α (e.g., the erasure probability for erasure channels). We also assume that the channel parameters are not known at the transmitter. The system model is shown in Fig. 1. The two encoders are not allowed to communicate and hence they must use independent encoding functions. We also assume that both the encoders use identical rates $R = k/n$, i.e., they map k input symbols ($\mathbf{U}_1$ and $\mathbf{U}_2$) to n output symbols ($\mathbf{X}_1$ and $\mathbf{X}_2$), respectively. The decoder receives $(\mathbf{Y}_1, \mathbf{Y}_2)$ and computes an estimate of $(\mathbf{U}_1, \mathbf{U}_2)$.

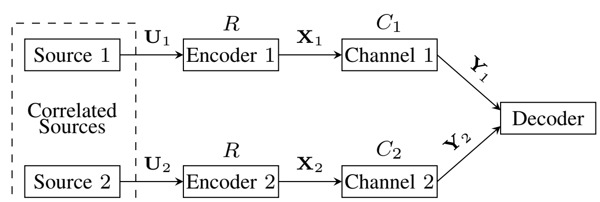

Figure 1. System Model

Reliable transmission over a channel pair (α_1, α_2) is possible as long as the pair satisfies the Slepian-Wolf conditions

$$\begin{aligned} \frac{C_1(\alpha_1)}{R} &\geq H(U_1|U_2)\,, \\ \frac{C_2(\alpha_2)}{R} &\geq H(U_2|U_1) \;\text{ and} \\ \frac{C_1(\alpha_1)}{R} + \frac{C_2(\alpha_2)}{R} &\geq H(U_1, U_2) \end{aligned} \tag{1}$$

are satisfied. For a given pair of rate-R encoding functions and a joint decoding algorithm, we say that a pair of channel parameters (α_1, α_2) is *achievable* if the encoder/decoder

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combination can achieve an arbitrarily low probability of error as the blocklength $n \to \infty$. As in [5], the achievable channel parameter region (ACPR) is defined as the set of all channel parameters which are achievable, and the Slepian-Wolf region (illustrated in Fig. 2 for erasure channels) is the set of all channel parameters (α_1, α_2) for which (1) is satisfied. Coding schemes for which the ACPR is equal to the Slepian-Wolf region are said to be *universal*. Such schemes are important because, in some practical situations, it is unreasonable to have knowledge of the channel parameters at the transmitter. Hence a single coding scheme needs to perform well over a large set of channel parameters.

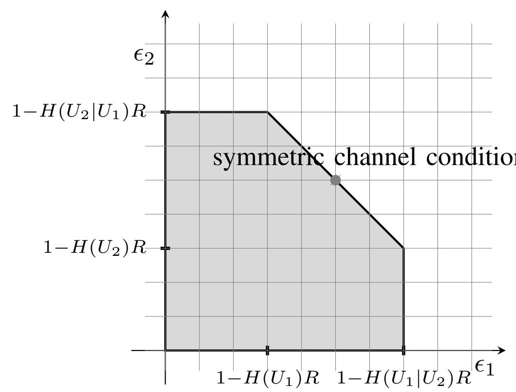

Figure 2. The Slepian-Wolf region for erasure channels for a rate pair (R, R).

In this paper, we consider the following scenarios:

1) The channels are binary erasure channels (BECs) and the source correlation is modeled through erasures. Let Z be a Bernoulli-p random variable. The sources U_1 and U_2 are defined by

$$(U_1, U_2) = \begin{cases} \text{i.i.d. Bernoulli } \frac{1}{2} \text{ r.v.s, if } Z = 0 \\ \\ \text{same Bernoulli } \frac{1}{2} \text{ r.v. } U \text{ , if } Z = 1 \end{cases}$$

This gives $H(U_1|U_2) = H(U_2|U_1) = 1 - p$ and $H(U_1, U_2) = 2 - p$. In this model, the decoder has access to the side information Z. While this model is not realistic, it is useful as a toy model that enables us to gain a better understanding of the problem.

2) A more realistic model is one where the channels are binary-input additive white Gaussian noise channels (BAWGNC) and the source correlation is modeled through a virtual correlation channel analogous to a binary symmetric channel (BSC). It is useful to visualize this correlation by the presence of an auxiliary binary symmetric channel (BSC) with parameter $1-p$ between the sources. In other words, U_2 is the output of a BSC with input U_1 (a Bernoulli-$1/2$ random variable) i.e., $U_2 = U_1 + Z$. Here Z is a Bernoulli-$(1-p)$ random variable and can be thought of as an *error*. Let $H_2(\cdot)$ denote the binary entropy function. Then, $H(U_1|U_2) = H(U_2|U_1) = H_2(p)$ and $H(U_1, U_2) = 1 + H_2(p)$. In this model, the side information Z is not available at the decoder.

Although separation between source and channel coding is known to be optimal for this problem [6], it can be beneficial to take a joint source-channel coding approach (via direct channel coding and joint decoding at the receiver) [7]. This problem is considered in [8], [9], where the authors choose a code that performs well at one point on the Slepian-Wolf region and evaluate its performance for different channel parameters. As a result, the performance of the code is far from the optimal performance for some channel parameters. Even the optimized degree profiles of LDPC codes for this problem are far from universal [5]. In this paper, we derive the area theorem for the joint decoder and compute (G)EXIT curves for transmission over symmetric channel conditions. This provides empirical evidence that the phenomenon of threshold saturation occurs for the noisy Slepian-Wolf problem. Moreover, density evolution (DE) results suggest that the spatially-coupled punctured-systematic LDPC$(4, 6)$ ensemble is near universal.

II. Density Evolution and (G)EXIT Curves

We assume that the sequences $\mathbf{U}_1$ and $\mathbf{U}_2$ are encoded using a punctured-systematic encoder for LDPC codes whose degree distribution functions are given by (λ, ρ). The advantages of using a punctured-systematic encoder are discussed in [5]. Based on standard notation [10], we let $\lambda(x) = \sum_i \lambda_i x^{i-1}$ be the degree distribution (from an edge perspective) corresponding to the variable nodes and $\rho(x) = \sum_i \rho_i x^{i-1}$ be the degree distribution (from an edge perspective) of the parity-check nodes in the decoding graph. The coefficient λ_i (resp. ρ_i) gives the fraction of edges that connect to the variable nodes (resp. parity-check nodes) of degree i. Likewise, L_i is the fraction of variable nodes with degree i. The fraction of punctured (i.e., systematic) bits is given by

$$\gamma \triangleq 1 - \frac{\int_0^1 \rho(x)\,\mathrm{d}x}{\int_0^1 \lambda(x)\,\mathrm{d}x}. \tag{2}$$

The remainder of this section makes heavy use of the terminology and notation from [10] for DE analysis and (G)EXIT curves. Let a_ℓ and b_ℓ denote the L-density[1] of the messages emanating from the variable nodes at iteration ℓ, corresponding to codes 1 and 2. The density evolution (DE) equations [5] can be written as follows

$$\begin{aligned} \mathsf{a}_{\ell+1} &= \Big[\gamma f\Big(L\left(\rho(\mathsf{b}_\ell)\right)\Big) + (1-\gamma)\mathsf{a}_{\mathsf{BMSC}}\Big] \circledast \lambda(\rho(\mathsf{a}_\ell)) \\ \mathsf{b}_{\ell+1} &= \Big[\gamma f\Big(L\left(\rho(\mathsf{a}_\ell)\right)\Big) + (1-\gamma)\mathsf{b}_{\mathsf{BMSC}}\Big] \circledast \lambda(\rho(\mathsf{b}_\ell)), \end{aligned} \tag{3}$$

where $\lambda(\mathsf{a}) = \sum_i \lambda_i \mathsf{a}^{\circledast(i-1)}$, $L(\mathsf{a}) = \sum_i L_i \mathsf{a}^{\circledast(i-1)}$, $\rho(\mathsf{a}) = \sum_i \rho_i \mathsf{a}^{\boxast(i-1)}$, $\mathsf{a}_{\mathsf{BMSC}}$ and $\mathsf{b}_{\mathsf{BMSC}}$ are the densities of the log-likelihood ratios received from the channel. The operators $\circledast$ and $\boxast$ are the standard density transformation operators at the variable and check nodes respectively [10]. The operator f at the correlation nodes depends on the equivalent channel

[1] Assuming that the transmission alphabet is $\{\pm 1\}$, the densities are conditioned on the transmission of a $+1$.

corresponding to the correlation model, as described in [11]. For the correlation models considered, one can derive a generalized symmetry condition that allows the function f to be chosen so that DE can be performed under the all-zero codeword assumption.

In Sections II-A and II-B, we describe the (G)EXIT curves for the joint decoder. For simplicity, we consider (G)EXIT curves for symmetric channel conditions throughout this work. In this case, the DE equations collapse into the recursion $\mathsf{a}_{\ell+1} = \mathsf{D}(\mathsf{a}_{\text{BMSC}}, \mathsf{a}_\ell)$. Similar to single user channels, the area theorem for the joint decoder can be used to compute an upper bound on the MAP threshold of the joint decoder. For example, this technique was applied to the joint decoding of a finite-state channel and an LDPC code in [12]. It has been observed that this upper bound is tight for regular LDPC ensembles transmitted over the BEC [13]. For asymmetric channel conditions, we can define 2-dimensional (G)EXIT surfaces analogously and the area theorem gives outer bounds on the MAP boundary. As described in [10], it is useful to assume that each bit of user 1 (user 2) has been transmitted through a channel with parameter $\alpha_i^{(1)}$ ($\alpha_i^{(2)}$), and suppose that these parameters are differentiable functions of a common parameter α. The area theorem follows trivially from the definition of the MAP (G)EXIT function and is given by

$$\int_{\alpha^{MAP}}^{1} h^{MAP}(\alpha)\mathrm{d}\alpha = \frac{\gamma H(U_1, U_2)}{2(1-\gamma)}.$$

A. Erasure Correlation

For erasure correlation with probability p, there is a parity-check at the correlation-node with probability p and with probability $1-p$ there is no parity-check, so $f(\mathsf{a}) = (1-p) + p\mathsf{a}$. For simplicity, we consider the EXIT curves for the case when the channel erasure probabilities for both users are equal. The extended belief propagation (EBP) EXIT curve for the joint decoder with symmetric channel conditions, is given in parametric form by

$$h^{\text{EBP}} = (\epsilon(x), L(1-\rho(1-x))),\ x \in [0,1],\ \text{where}$$

$$\epsilon(x) = \frac{1}{1-\gamma}\left[\frac{x}{\lambda(1-\rho(1-x))} - \gamma f\Big(L(1-\rho(1-x))\Big)\right].$$

The EBP EXIT curve and the MAP threshold for the punctured LDPC$(4,6)$ ensemble are shown in Fig. 4. The MAP threshold at symmetric channel conditions is $\epsilon^{\text{MAP}} \approx 0.6245$, while the Slepian-Wolf bound is $\epsilon = 0.625$.

B. BSC Correlation

Using the generalized symmetry condition for this model results in the correlation node function $f(\mathsf{a}) = \mathsf{a}_{\text{BSC}(p)} \boxast \mathsf{a}$. It turns out that the GEXIT kernel for the joint decoder is the same as that of the standard BAWGNC, given by

$$l^{\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}}(y) = \left(\int \frac{\mathrm{e}^{-\frac{(2-2/\sigma^2)^2\sigma^2}{8}}}{1+\mathrm{e}^{z+y}}\mathrm{d}z\right)\Big/\left(\int \frac{\mathrm{e}^{-\frac{(2-2/\sigma^2)^2\sigma^2}{8}}}{1+\mathrm{e}^{z}}\mathrm{d}z\right),$$

where σ is the unique positive number such that the BAWGNC with noise variance σ^2 has entropy h. Let

$$\mathsf{G}(\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}, \mathsf{a}) = \int \mathsf{a}(y) l^{\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}}(y)\mathrm{d}y$$

be the GEXIT functional applied to the density a. For each fixed point (not necessarily stable) of density evolution $(\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}, \mathsf{a})$, satisfying $\mathsf{a} = \mathsf{D}(\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}, \mathsf{a})$, the point $(\mathsf{h}, \mathsf{G}(\mathsf{a}_{\text{BAWGNC}(\mathsf{h})}, \mathsf{a}))$ lies on the EBP GEXIT curve. The EBP GEXIT curve is the set of all these points and can be computed numerically as outlined in [14]. The EBP GEXIT curve and the MAP threshold for the punctured LDPC$(4,6)$ ensemble are shown in Fig. 5. The MAP threshold for symmetric channel conditions is $\mathsf{h}^{\text{MAP}} \approx 0.6324$ and the Slepian-Wolf bound is $\mathsf{h} \approx 0.6328$.

III. Spatial Coupling

Spatial coupling is best described by the (l, r, L) ensemble through a protograph [1], [15]. We briefly review the protograph structure at the joint decoder here. Consider the protograph of a standard LDPC$(4,6)$ ensemble. There are two check nodes and three variable nodes. For each user, take a collection of $(2L+1)$ protographs at positions $[-L, L] \triangleq \{-L, \cdots, L\}$ and couple them as described in [1]. One variable node at each position $i \in [-L, L]$ from the first user is punctured and connected to a punctured variable node at the same position of the second user. The resulting protograph, shown in Fig. 3, is then expanded M times to form the parity-check matrix of the joint system. This structure is fundamental to the phenomenon of threshold saturation observed at the joint decoder. It is simply not sufficient to use spatially-coupled codes with random connections between the information nodes. Such a coupling will only result in pushing the threshold of the component codes to the MAP threshold, but may have little effect on the BP threshold of the joint system.

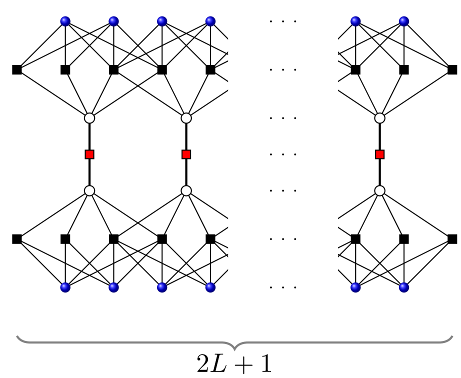

Figure 3. Protograph of the joint decoder

Although this ensemble is very instructive in understanding the universality of spatially-coupled codes, the EBP curves for this ensemble exhibit wiggles around the MAP threshold (similar to the single user channels as discussed in [1]) , for the case of transmission over erasure channels. The magnitude of these wiggles appears to remain constant with increasing L and their presence implies that the BP threshold is smaller than the MAP threshold of the underlying ensemble. Therefore, the $(4, 6, L)$ ensemble cannot be universal. To overcome this, we use the (l, r, L, w) ensemble introduced in [1] for the remainder of this work.

A. The (l, r, L, w) ensemble

The (l, r, L, w) spatially-coupled ensemble can be described as follows: Place M variable nodes at each position in $[-L, L]$. The check nodes are placed at positions $[-L, L+w-1]$, with $\frac{l}{r}M$ check nodes at each position. The connections are made as described in [1]. This procedure generates a Tanner graph for the (l, r, L, w) ensemble.

For this work we consider codes of rate $1/3$, punctured to a rate $1/2$. Two such graphs (generated by the above procedure) are taken and $2M/3$ variable nodes ($M/3$ from each graph) at each position are connected by a random (uniform) permutation of size $M/3$ via correlation nodes. This procedure ensures that all the variable node positions are symmetric (as opposed to Fig. 3) with respect to puncturing and correlation, enabling us to write down the density evolution (DE) equations as described in the following section.

B. Density Evolution of the (l, r, L, w) Ensemble

Let $\mathsf{a}_i^{(\ell)}$ and $\mathsf{b}_i^{(\ell)}$ denote the average density emitted by the variable node at position i, at iteration ℓ, for codes 1 and 2 respectively. Let $\Delta_{+\infty}$ denote the delta function at $+\infty$ and set $\mathsf{a}_i^{(\ell)} = \mathsf{b}_i^{(\ell)} = \Delta_{+\infty}$ for $i \notin [-L, L]$. The channel densities for codes 1 and 2 are denoted by a_{BMSC} and b_{BMSC} respectively. All the above densities are L-densities conditioned on the transmission of the all-zero codeword (see Section II). We consider the parallel schedule for each user (as described in [1]) and update the correlation nodes before proceeding to the next iteration. Let us define

$$g(\mathsf{x}_{i-w+1}, \cdots, \mathsf{x}_{i+w-1}) \triangleq \left(\frac{1}{w}\sum_{j=0}^{w-1}\left(\frac{1}{w}\sum_{k=0}^{w-1}\mathsf{x}_{i+j-k}\right)^{\boxast(r-1)}\right)^{\circledast(l-1)},$$

$$\Gamma(\mathsf{x}_{i-w+1}, \cdots, \mathsf{x}_{i+w-1}) \triangleq \left(\frac{1}{w}\sum_{j=0}^{w-1}\left(\frac{1}{w}\sum_{k=0}^{w-1}\mathsf{x}_{i+j-k}\right)^{\boxast(r-1)}\right)^{\circledast l}.$$

The DE equations for the joint spatially-coupled system can be written as

$$\begin{aligned}
\mathsf{a}_i^{(\ell+1)} &= \left[\gamma f\left(\Gamma(\mathsf{b}_{i-w+1}^{(\ell)}, \cdots, \mathsf{b}_{i+w-1}^{(\ell)})\right) + (1-\gamma)\mathsf{a}_{\text{BMSC}}\right] \circledast \\
&\qquad g(\mathsf{a}_{i-w+1}^{(\ell)}, \cdots, \mathsf{a}_{i+w-1}^{(\ell)}), \\
\mathsf{b}_i^{(\ell+1)} &= \left[\gamma f\left(\Gamma(\mathsf{a}_{i-w+1}^{(\ell)}, \cdots, \mathsf{a}_{i+w-1}^{(\ell)})\right) + (1-\gamma)\mathsf{b}_{\text{BMSC}}\right] \circledast \\
&\qquad g(\mathsf{b}_{i-w+1}^{(\ell)}, \cdots, \mathsf{b}_{i+w-1}^{(\ell)}),
\end{aligned}$$

for $i \in [-L, L]$. For a further discussion of the DE equations for the (l, r, L, w) spatially-coupled ensembles on BMS channels, see [2]. Let $\underline{\mathsf{a}} = (\mathsf{a}_{-L}, \cdots, \mathsf{a}_L)$. The fixed points of DE for symmetric channel conditions are given by $(\mathsf{a}_{\text{BMSC}(\mathsf{h})}, \underline{\mathsf{a}})$, which satisfy

$$\begin{aligned}
\mathsf{a}_i &= \left[\gamma f\left(\Gamma(\mathsf{a}_{i-w+1}, \cdots, \mathsf{a}_{i+w-1})\right) + (1-\gamma)\mathsf{a}_{\text{BMSC}(\mathsf{h})}\right] \circledast \\
&\qquad g(\mathsf{a}_{i-w+1}, \cdots, \mathsf{a}_{i+w-1}). \qquad (4)
\end{aligned}$$

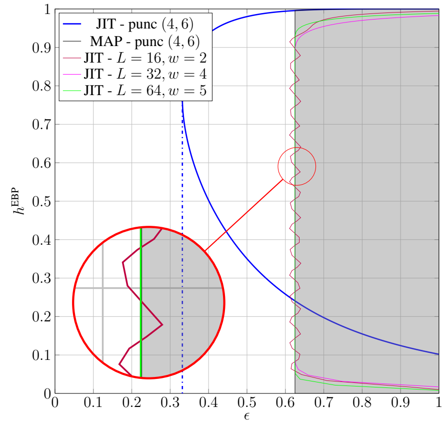

Figure 4. EBP EXIT curves of the $(4, 6, L, w)$ and $(4, 6)$ ensembles for transmission over erasure channels with erasure correlated sources.

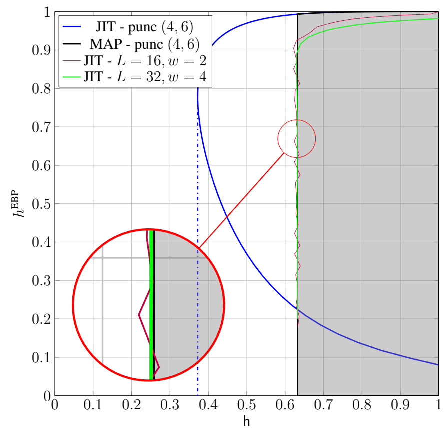

Figure 5. EBP GEXIT curves of the $(4, 6, L, w)$ and $(4, 6)$ ensembles for transmission over AWGN channels which BSC correlation between the sources.

We can use the procedure outlined in [14] to compute both the stable and unstable fixed points which satisfy (4). Define

$$\mathsf{G}(\mathsf{a}_{\text{BMSC}(\mathsf{h})}, \underline{\mathsf{a}}) = \frac{1}{2L+1}\sum_{i=-L}^{L}\mathsf{G}(\mathsf{a}_{\text{BMSC}(\mathsf{h})}, \mathsf{a}).$$

The EBP GEXIT curve is the set of points $(\mathsf{h}, \mathsf{G}(\mathsf{a}_{\text{BMSC}(\mathsf{h})}, \underline{\mathsf{a}}))$. The resulting curves for the erasure channel with erasure correlated sources are shown in Fig. 4 and those for the AWGN channel with BSC correlated sources are shown in Fig. 5. These curves are very similar to the single user case and demonstrate the phenomenon of threshold saturation at the joint decoder, for symmetric channel conditions. For channel parameters not on the symmetric line, these plots imply threshold saturation towards the MAP boundary.

IV. The 2-user Gaussian MAC

Consider a 2-user additive Gaussian multiple access channel (MAC), given by

$$Y = h_1 X_1 + h_2 X_2 + Z,$$

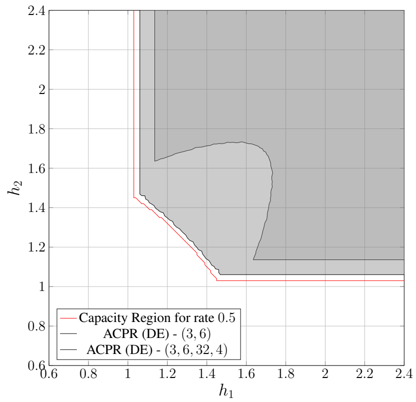

Figure 6. DE ACPR of a spatially coupled $(3, 6, 32, 4)$ LDPC code for the two user Gaussian MAC. The DE results for the regular LDPC$(3, 6)$ ensemble are shown for comparison.

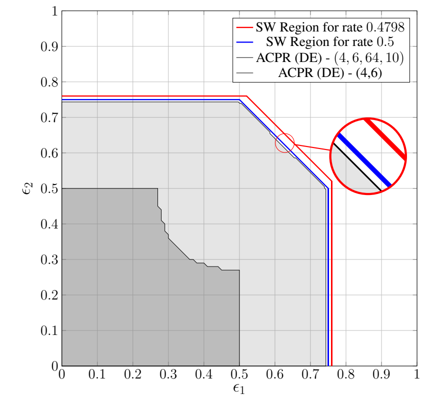

Figure 7. DE ACPR of the spatially coupled punctured $(4, 6, 64, 10)$ LDPC and the regular punctured LDPC$(4, 6)$ ensembles for transmission over erasure channels with erasure correlated sources.

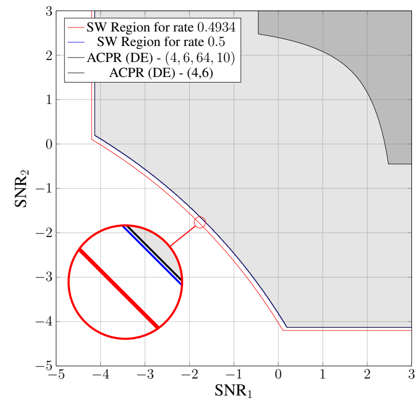

Figure 8. DE ACPR of the spatially coupled punctured $(4, 6, 64, 10)$ LDPC and the regular punctured LDPC$(4, 6)$ ensembles for transmission over AWGN channels with BSC correlated sources.

with $Z \sim \mathcal{N}(0, 1)$, $X_1, X_2 \in \{\pm 1\}$ and $h_1, h_2 \in \mathbb{R}$. We assume that the fading coefficients are not known at the transmitter and we consider the notion of universality with respect to fading coefficients. The factor graph of the joint decoder consists of two single user Tanner graphs, whose variable nodes are connected through a function node [10, p. 308]. Using the notation described in Section II, the DE equation for the joint decoder with symmetric fading coefficients is given by

$$\mathsf{a}_{\ell+1} = f\Big(L\left(\rho(\mathsf{a}_\ell)\right), \mathsf{a}_{\text{BAWGNC}}\Big) \circledast \lambda(\rho(\mathsf{a}_\ell)).$$

Here, f denotes the operation at the function node for transformation of densities, and is chosen under the assumption of transmission of a random coset of the LDPC code. Preliminary DE results show that spatial coupling allows for near universal performance on the Gaussian MAC (see Fig. 6). In our future work, we will derive the area theorem (with an appropriate GEXIT kernel) for this problem to formalize this result.

V. Results and Concluding Remarks

It was shown in [1], that for transmission over erasure channels, the BP threshold of spatially-coupled ensembles is essentially equal to the MAP threshold of the underlying ensemble. This was observed numerically for general BMS channels in [2], [3]. In this work, we numerically show that the phenomenon of threshold saturation is very general and can provide universality for multi-user scenarios. In particular, we considered the noisy Slepian-Wolf problem and showed that spatial coupling boosts the BP threshold of the joint decoder to the MAP threshold of the underlying ensemble. The density evolution ACPRs for the two scenarios considered in this paper are shown in Fig. 7 and 8. These figures show that spatially coupled ensembles are near universal for this problem. The analytic proof of this result remains an open problem. Such a proof would essentially show that it is possible to achieve universality for the noisy Slepian-Wolf problem under iterative decoding.

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