IMP: A Message-Passing Algorithm for Matrix Completion

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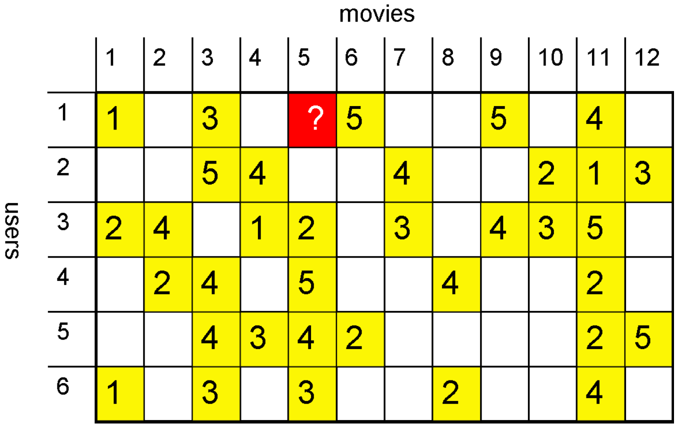
Outline

1. Matrix Completion via Iterative Information Processing
2. Factor Graph Model for Matrix Completion
3. IMP Algorithm and Performance Results
4. Conclusions and Open Problems

Matrix Completion Problem

> *"In its simplest form, the problem is to recover a matrix from a small sample of its entries,...*
>
> *Imagine now that we only observe a few entries of a data matrix. Then is it possible to accurately—or even exactly—guess the entries that we have not seen?..."*
>
> *- Candes and Plan '09*

Motivation: The Netflix Challenge ('06-'09)



	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		?	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

An overwhelming portion of the user-movie matrix (e.g., 99%) is unknown and the few observed movie ratings are noisy!

Recent Work on Matrix Completion

- Efficient Models and Practical Algorithms
  - Low rank matrix models and clustering models
  - Convex relaxation (SDP) and Bayesian approaches
- Exploration of the Fundamental Limits
  - Relationship between sparse observation and the recovery of missing entries
  - Cold-start problem in recommender systems

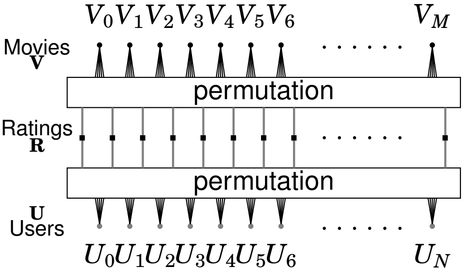
Main Differences

1. Focus on the matrix where the entries (drawn from a finite alphabet) are modeled by a factor graph
2. MP based algorithm to learn the missing information and eventually estimate missing entries

Our Goal: Matrix Completion via Iterative Processing

- Basic Approach
    - Establish a factor-graph model
    - Estimate parameters of model
    - Use for inference via message-passing (IMP)
- Benefits of our factor-graph model
    - Establishes a generative model for data matrices
    - Sparse observations reduce complexity
- Drawbacks of this approach
    - Mixes clustering and message-passing
    - Difficult to analyze behavior

Factor Graph Model (1): Conditional Independence



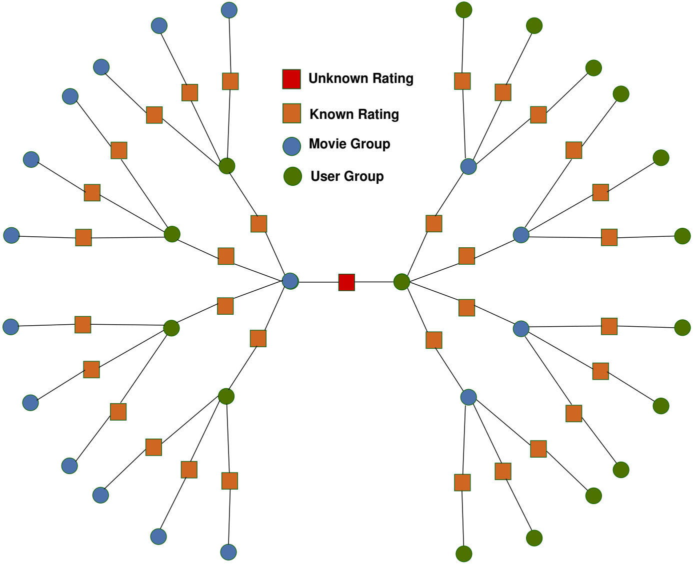
Assumption

The movie ratings R_{nm} (user n, movie m) are *conditionally independent* given the user group U_n and the movie group V_m

$$\Pr\left(\mathbf{R}_O | \mathbf{U}, \mathbf{V}\right) \triangleq \prod_{(n,m)\in O} w\left(R_{nm} | U_n, V_m\right)$$

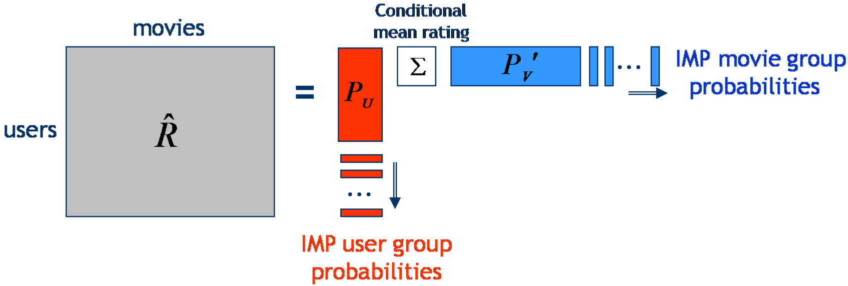
where $w(r|u,v) \triangleq \Pr(R_{nm} = r | U_n = u, V_m = v)$

Factor Graph Model (2): Computational Tree



Factor Graph Model (3): Matrix Decomposition

- MMSE estimates can be written as a matrix factorization



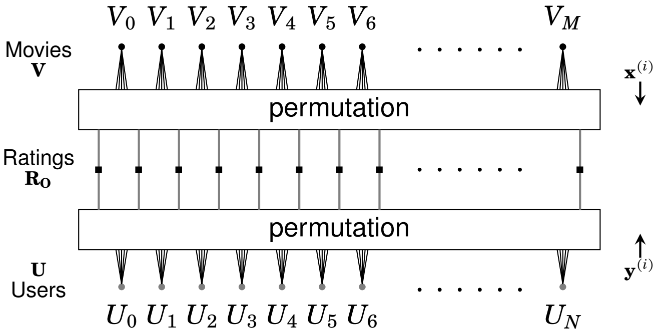
- In contrast to the standard low-rank matrix model, this adds non-negativity and normalization constraints

Inference by Message-Passing (IMP) Algorithm (1)

Step I. Initialize Group Rating Probabilities $w(r|u, v)$

- Cluster users (and movies) using variable-dimension vector quantization (VDVQ), which is a variant of VQ where the codebook vectors are full, but training vectors are sparse.
- Training is based on based on the generalized Lloyd algorithm (GLA) and the distance computed only on overlapping entries.
- For user clustering, the K codebook vectors can be thought of as K critics which have rated every movie.
- After clustering users/movies each into user/movie groups, estimate $w(r|u, v)$ from the frequencies of each user-group / movie-group / rating triple.

IMP Algorithm (2)



Notation

Let $\mathbf{T}^{(i)}_{m\to n}$ be $m \to n$ computation tree after i iterations, then message from movie m to user n is $\mathbf{x}^{(i)}_{m\to n} \triangleq \Pr(V_m = v|\mathbf{T}^{(i)}_{m\to n})$ and message from user n to movie m is $\mathbf{y}^{(i)}_{n\to m} \triangleq \Pr(U_n = u|\mathbf{T}^{(i)}_{n\to m})$

IMP Algorithm (3)

Step II. Message-Passing Update of Group Vectors

1. Initialization of user/movie to prior group probabilities

$$\mathbf{x}_m^{(0)}(v) = p_V(v),\ \mathbf{y}_n^{(0)}(u) = p_U(u)$$

2. Recursive update for user/movie group probabilities

$$\mathbf{x}_{m\to n}^{(i+1)}(v) \propto \mathbf{x}_m^{(0)}(v) \prod_{k\in\mathcal{U}_m\backslash n} \sum_u w\left(r|u,v\right) \mathbf{y}_{k\to m}^{(i)}(u)$$

$$\mathbf{y}_{n\to m}^{(i+1)}(u) \propto \mathbf{y}_n^{(0)}(u) \prod_{k\in\mathcal{V}_n\backslash m} \sum_v w\left(r|u,v\right) \mathbf{x}_{k\to n}^{(i)}(v)$$

$\mathcal{U}_m$: all users who rated movie m
$\mathcal{V}_n$: all movies whose rating by user n was observed

3. Update $w(r|u,v)$ for group ratings

IMP Algorithm (4)

Step III. Approximate Matrix Completion

After i iterations of message-passing update,

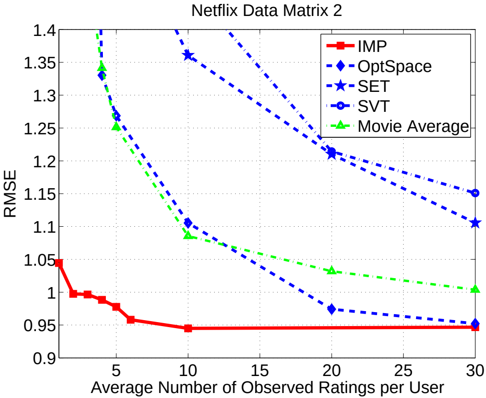
- Output probability of rating R_{nm} given observed ratings

$$\hat{p}^{(i+1)}_{R_{nm}|\mathbf{R}_O}(r) \propto \sum_{u,v} \mathbf{x}^{(i+1)}_{m\to n}(v) w\left(r|u,v\right) \mathbf{y}^{(i+1)}_{n\to m}(u)$$

- Minimize the mean-squared prediction error with

$$\hat{r}^{(i+1)}_{n,m} = \sum_{r\in\mathcal{R}} r\,\hat{p}^{(i+1)}_{R_{nm}|\mathbf{R}_O}(r)$$

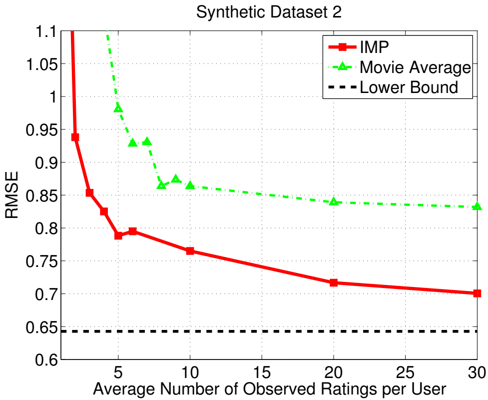
IMP Improves the Cold-Start Problem



Dataset Description

- Netflix submatrix of 5,035 movies and 5,017 users by avoiding movies and users with less than 3 ratings
- 16% of the users and 41% of the movies have less than 10 ratings

Generation of Synthetic Datasets



Dataset Description

- Synthetic Dataset generated after learning Netflix Data Matrix with 16 movie/user groups and randomly subsampled

Conclusion

Main Contributions

1. Factor Graph Model: Probabilistic low-rank matrix model
2. IMP Algorithm: Combines clustering with message-passing

Avenues for Future Research

1. Combine with clustering via message-passing to get a fully iterative approach
2. Obtain performance analysis of IMP algorithm via density evolution

Thank you