Universal Codes for the Gaussian MAC Via Spatial Coupling

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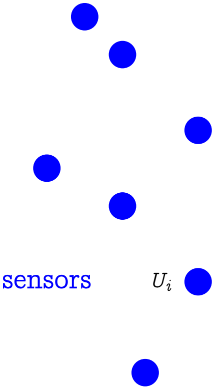
(Joint work with Phong Nguyen, Henry Pfister & Krishna Narayanan)

Texas A&M University

Allerton 2011
Montecello, IL

The Sensor Reachback Problem



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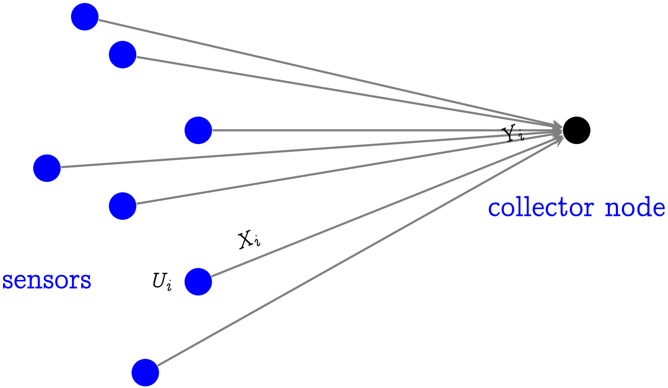

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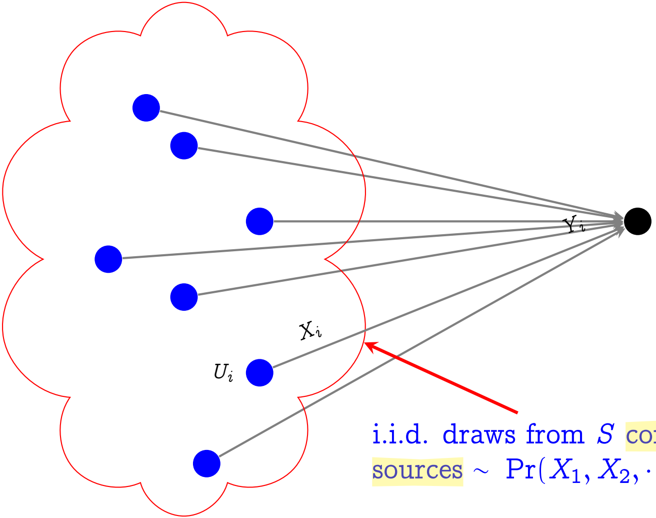

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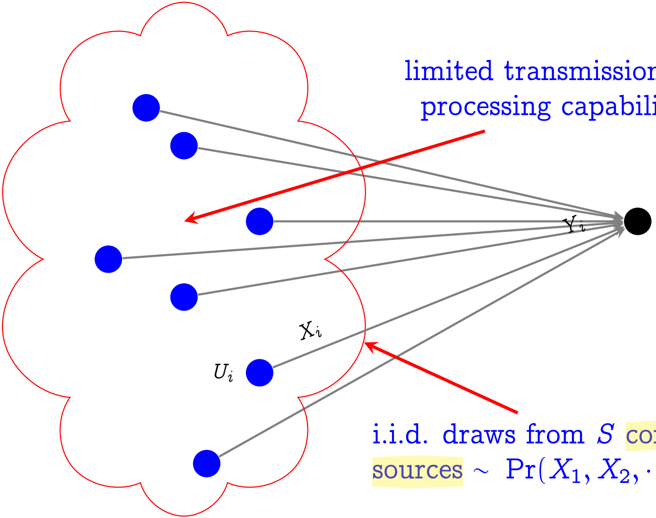

limited transmission and processing capabilities

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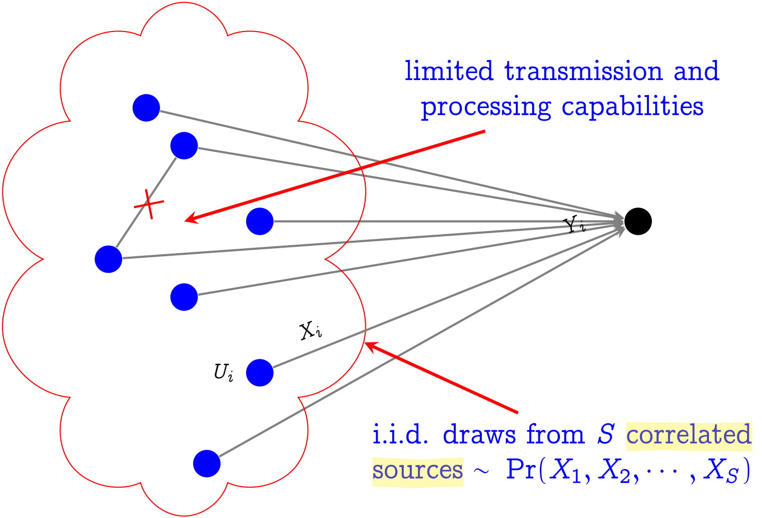

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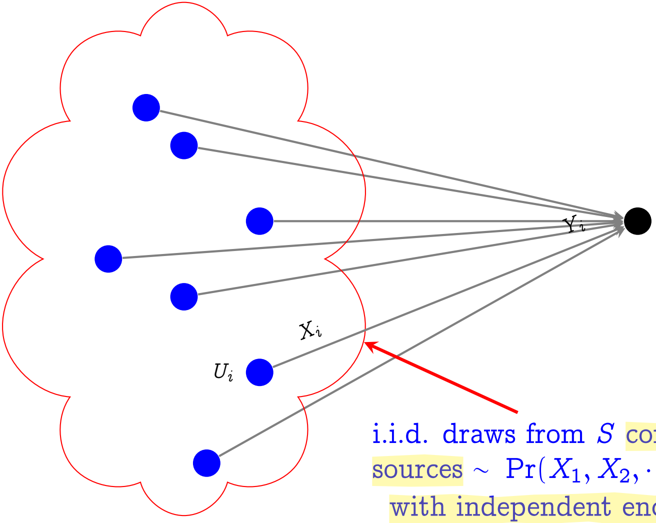

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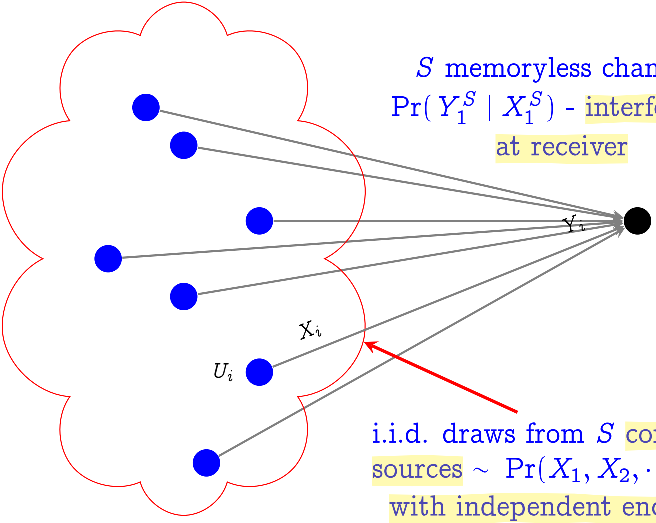

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$\Pr(Y_1^S \mid X_1^S)$ - interference at receiver

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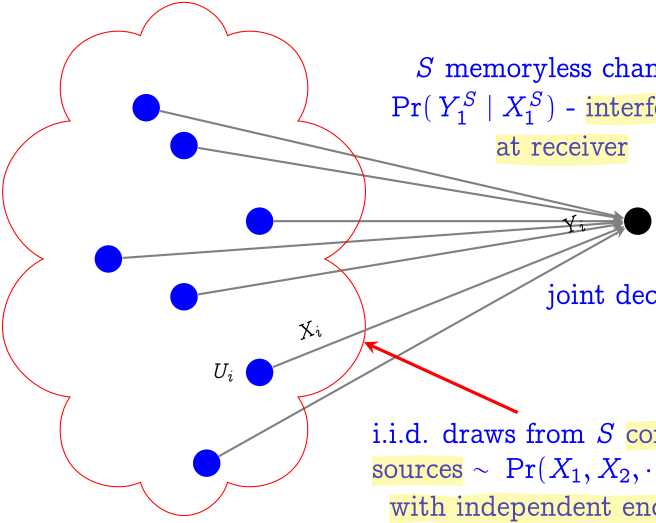

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joint decoding

i.i.d. draws from S correlated sources $\sim \Pr(X_1, X_2, \cdots, X_S)$ with independent encoding

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- recent prior work
  - optimized NS-LDPC codes good but not universal [YPN10]
  - spatially coupled codes are essentially universal [YPN11]

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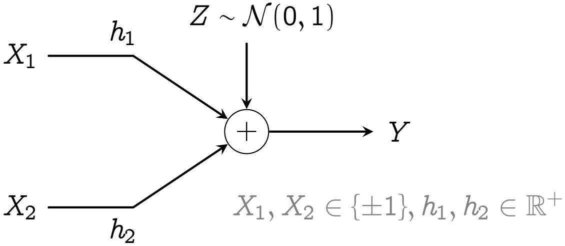

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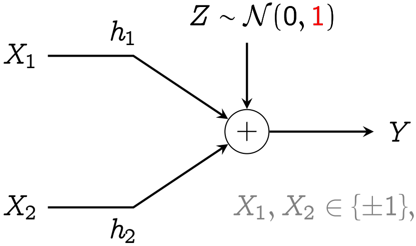

$X_1, X_2 \in \{\pm 1\}, h_1, h_2 \in \mathbb{R}^+$

- fixed noise variance

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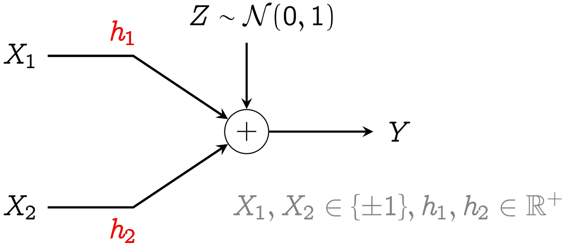

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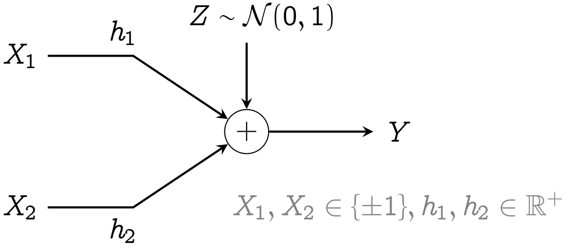

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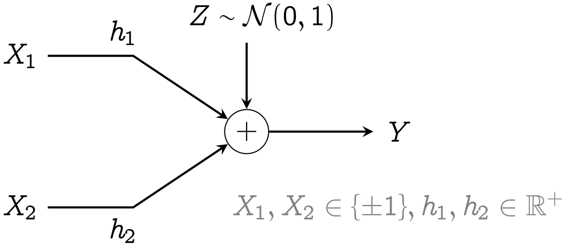

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- receiver knows h_1 and h_2
- each code has rate R

Universality & Multi-terminal Problems

point-to-point communication

bigger $\alpha \Rightarrow$ better

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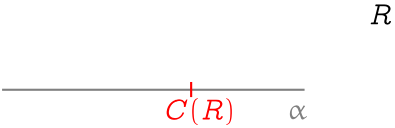

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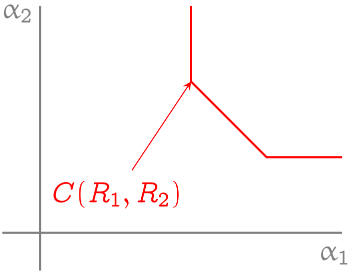

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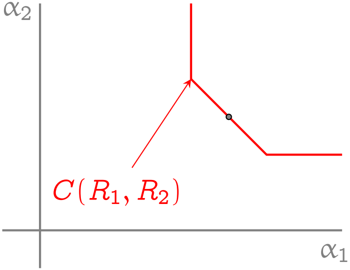

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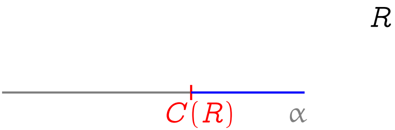
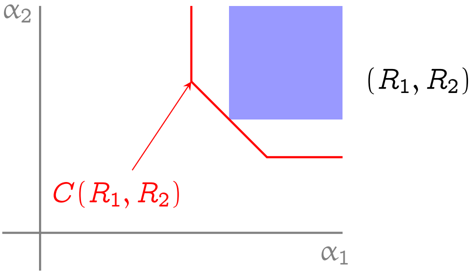


multi-terminal communication

channel degradation

$\Rightarrow$ partial region



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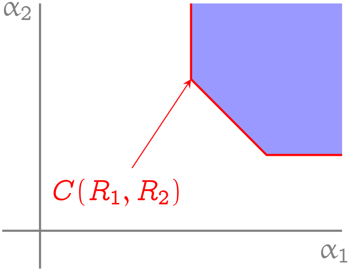

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universal codes

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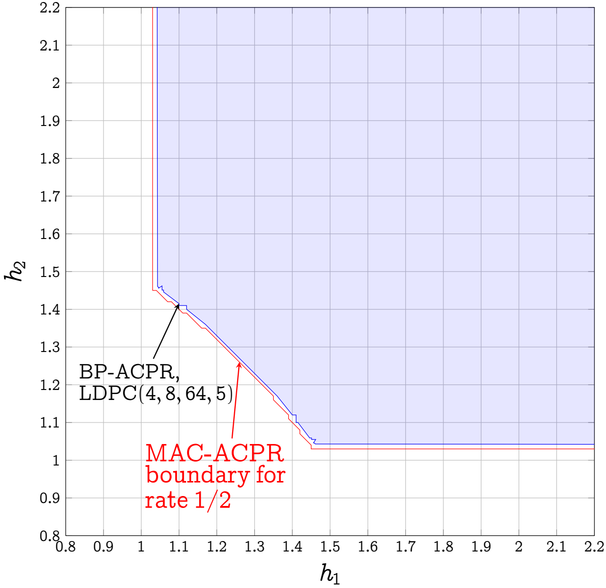
- performance degrades for different parameters
- need robustness against changes in channel conditions

SC codes are nearly universal



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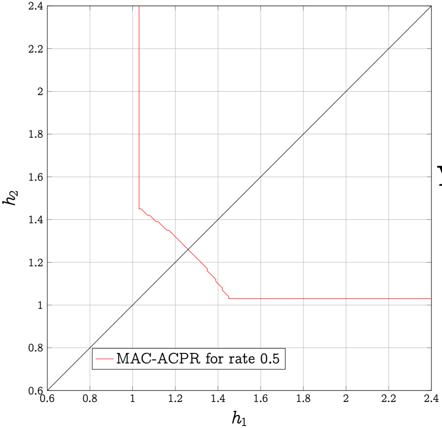

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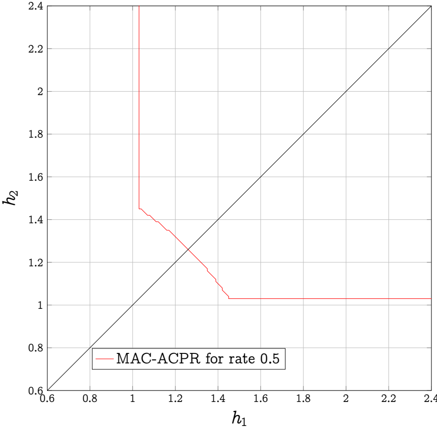

$$Y = h(X_1 + X_2) + Z$$
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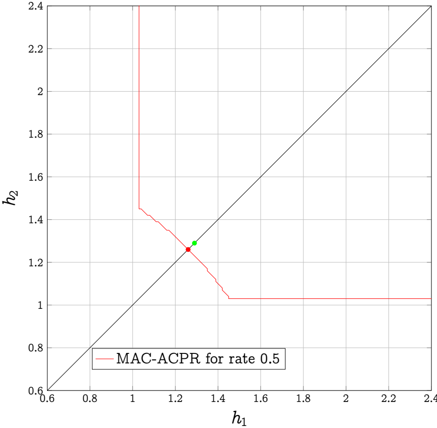

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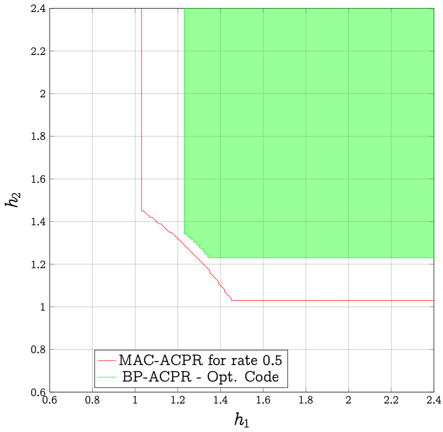

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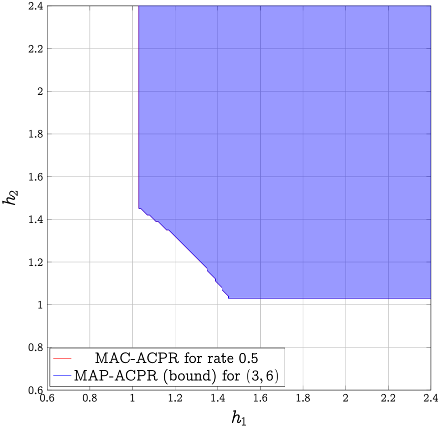
- problem that iterative decoding has not yet solved
- standard code optimization unable to achieve universality
- MAP decoding of simple codes appears to be universal



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- new paradigm for constructing capacity approaching codes under iterative decoding
- proven for the binary erasure channel
- this observation implies SC benefits many applications:
  - ISIT: BEC wiretap (Rathi et al.), erasure MAC / ISI (Kudekar & Kasai), CDMA (Takeuchi et al. / Schlegel & Truhachev), quantum (Hagiwara et al.), SW (Yedla et al.)
  - arXiv: ISI (Nguyen et al.)

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residual entropy: $H(\mathbf{X}_{\mathcal{C}} | \mathbf{Y}_{\mathcal{C}}(\alpha))$

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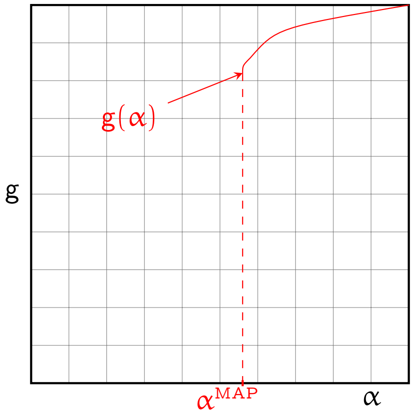

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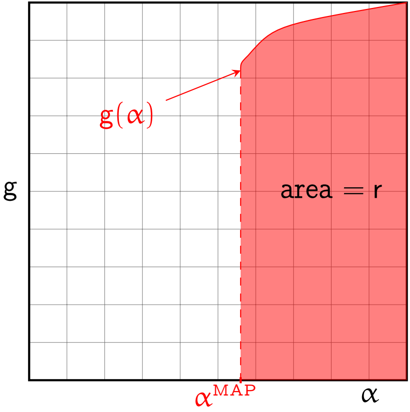

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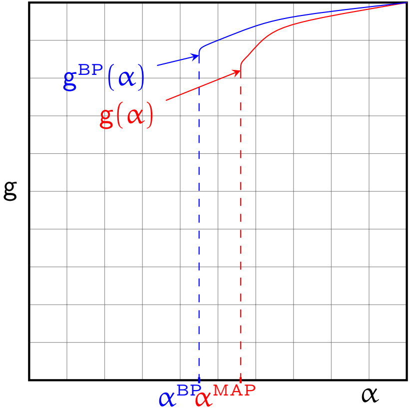

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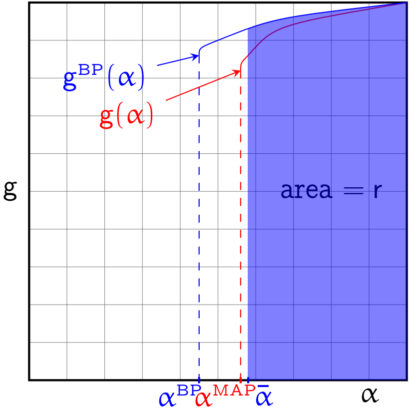

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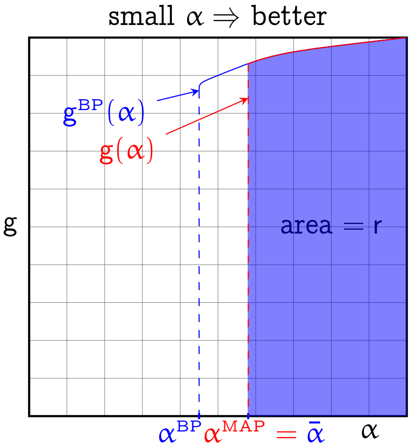

The BP GEXIT Curve of the Joint Decoder

- for $\mathbf{X} = [\mathbf{X}_1 \ \mathbf{X}_2]$ and $\mathbf{Y}(\alpha)$, the MAP GEXIT surface is defined by the gradient

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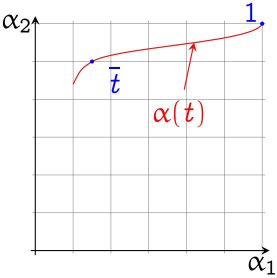
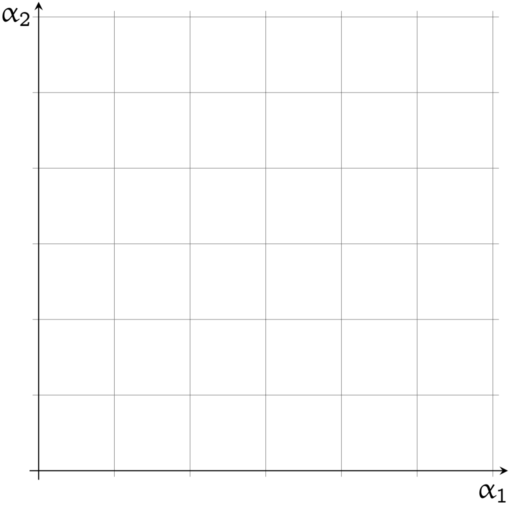
- the normalized area theorem is given by the line integral

$$\int_{t^{MAP}}^{1} \mathrm{g}^{MAP}(\alpha(t)) \cdot \alpha'(t) \mathrm{d}t = \mathrm{r} H(U_1, U_2),$$

where $\alpha(t)$ is a parametrized curve through channel space



MAP Boundary



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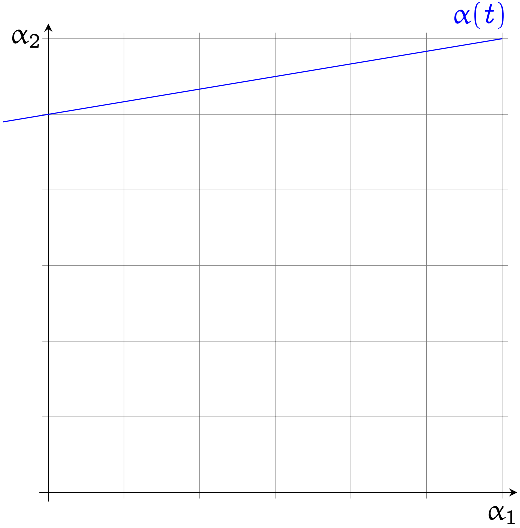

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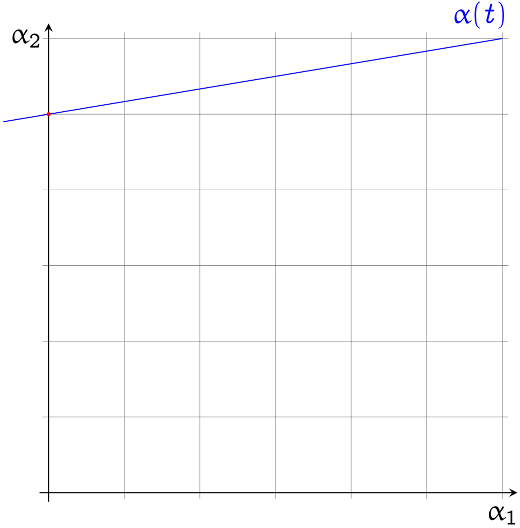

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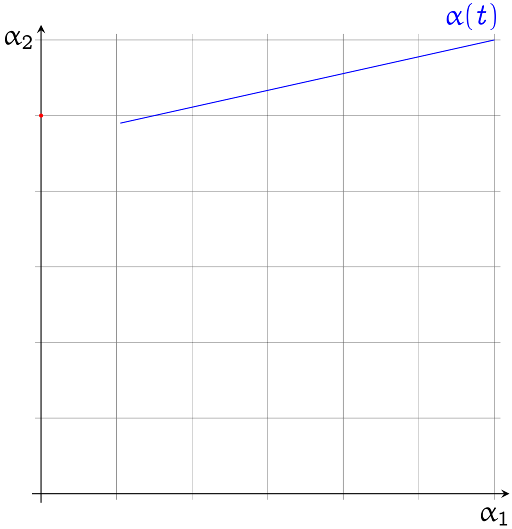

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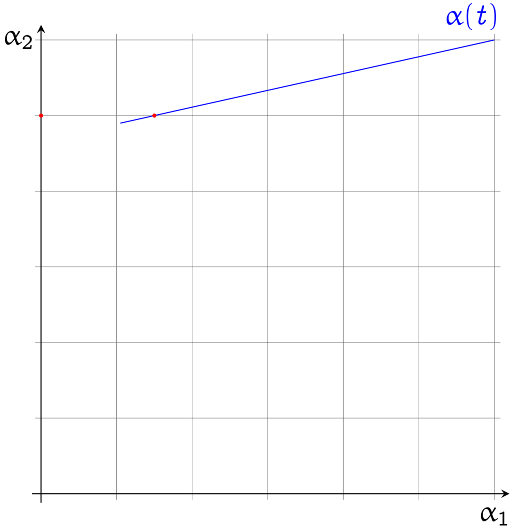

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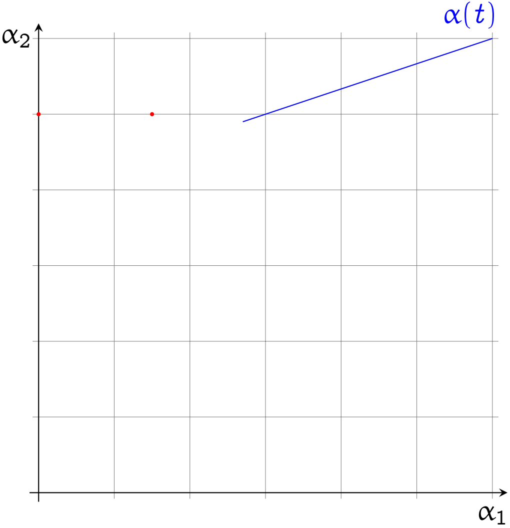

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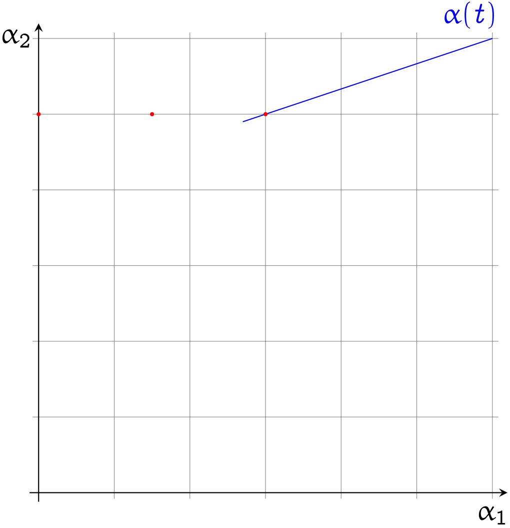

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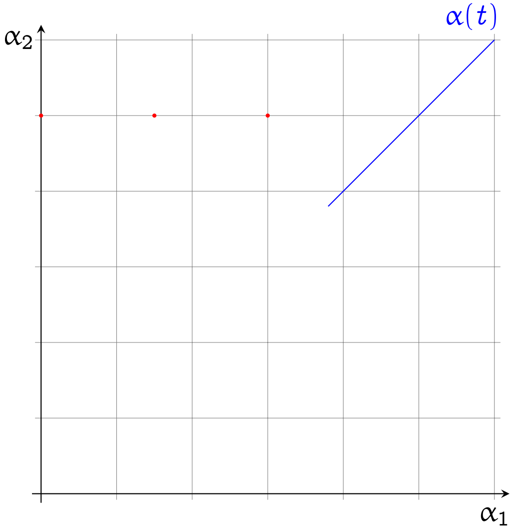

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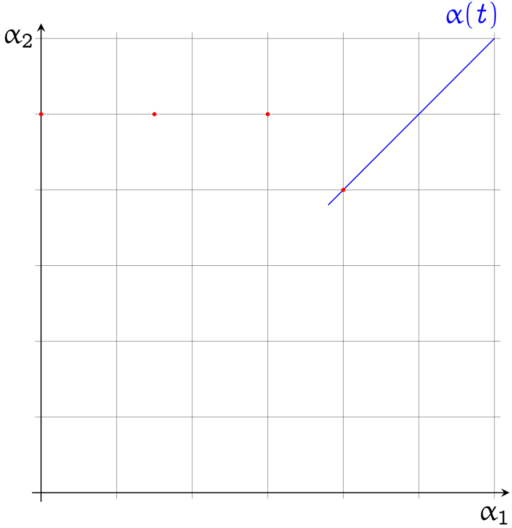

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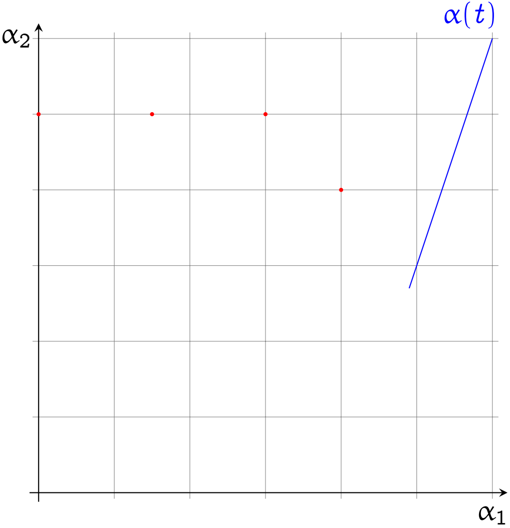

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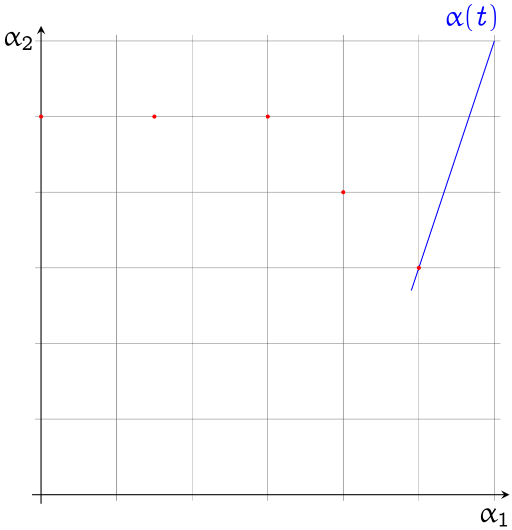

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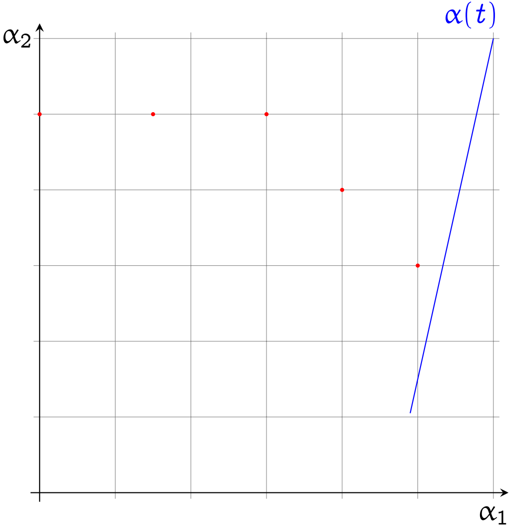

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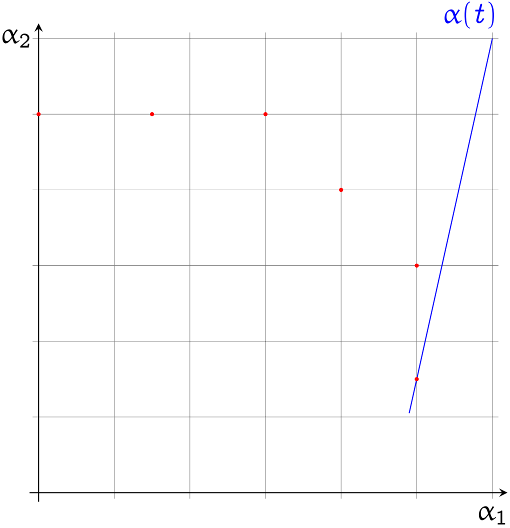

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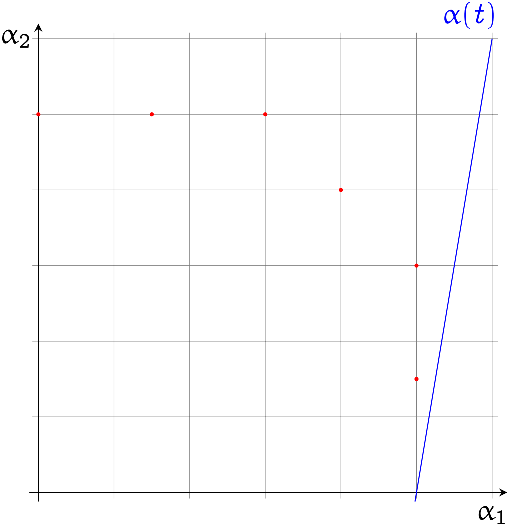

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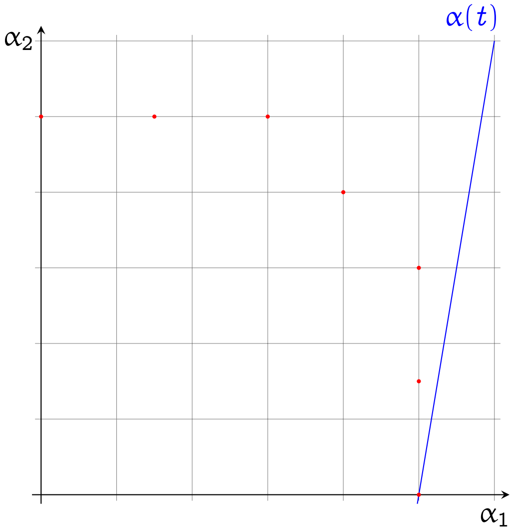

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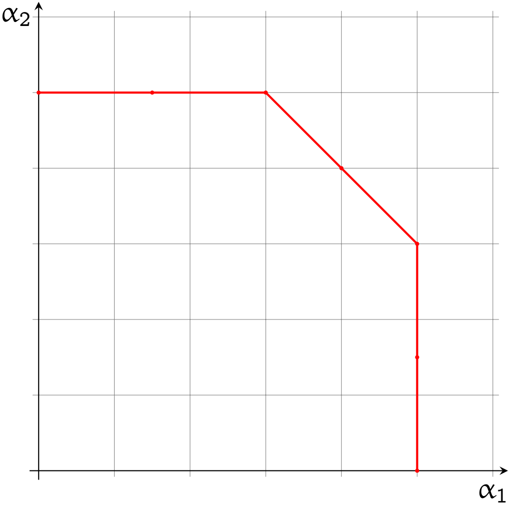

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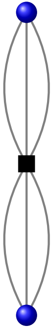

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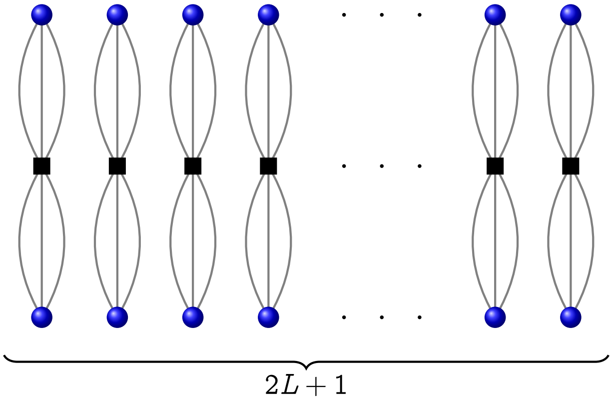

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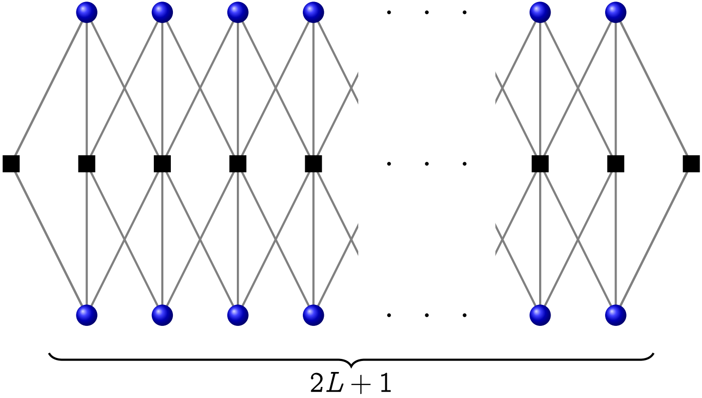
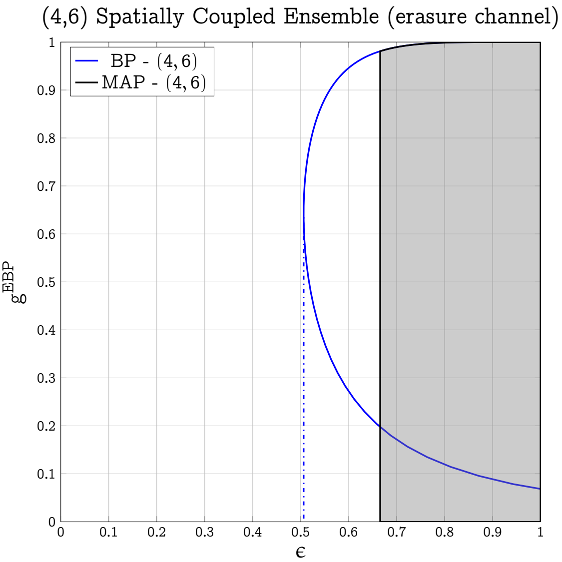


Review: The SC BP GEXIT Curve



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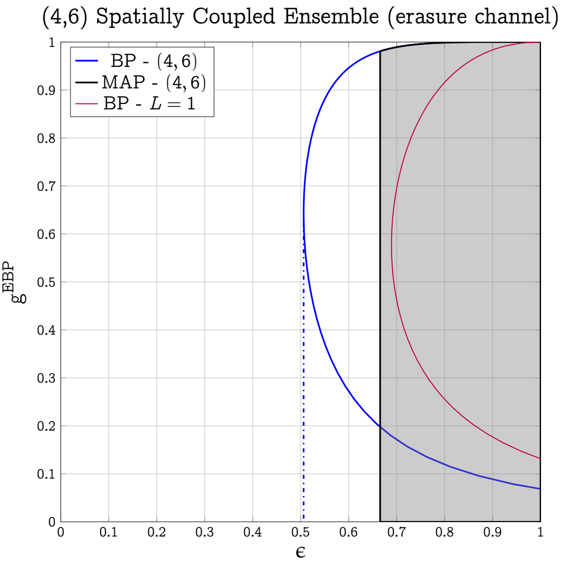

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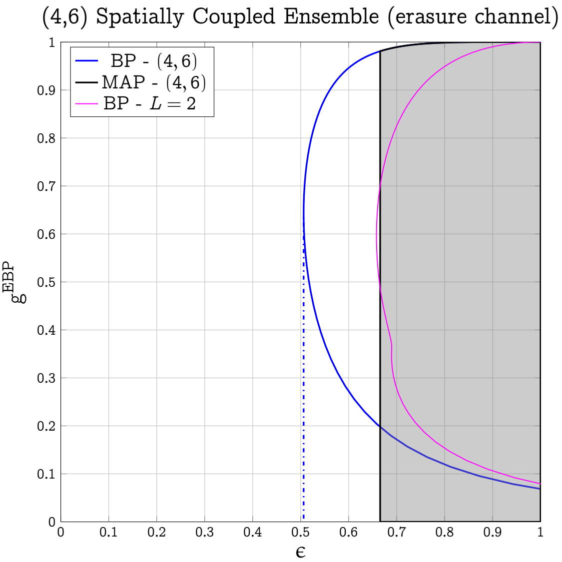

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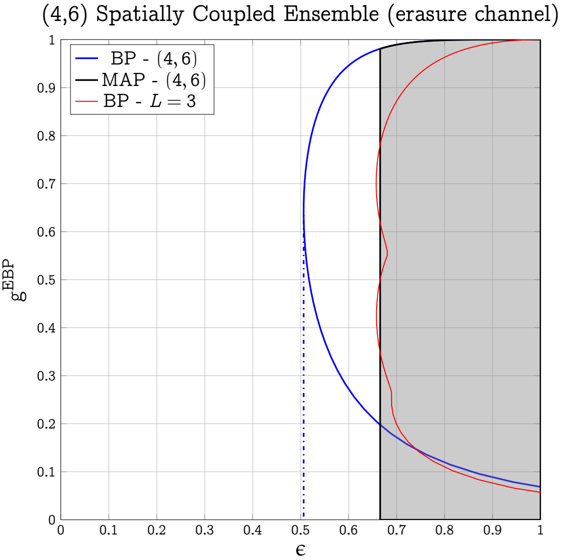

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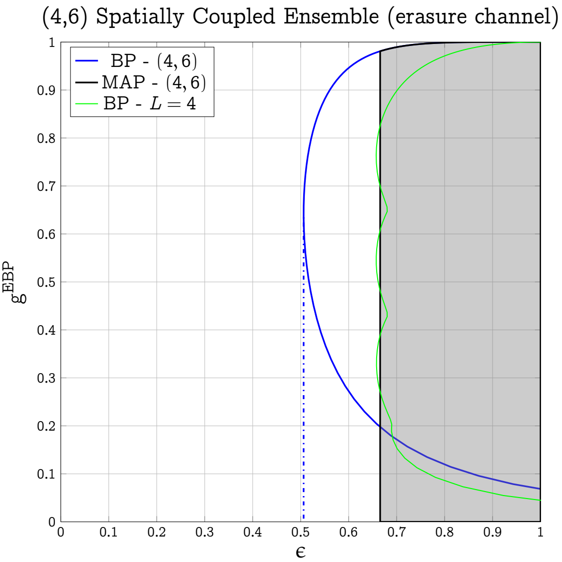

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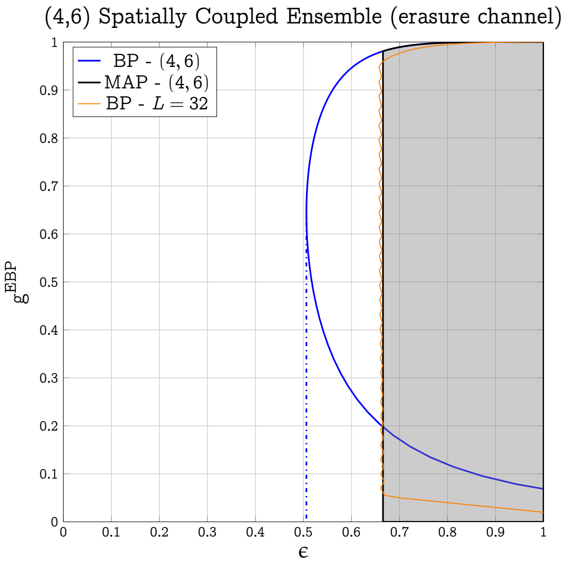

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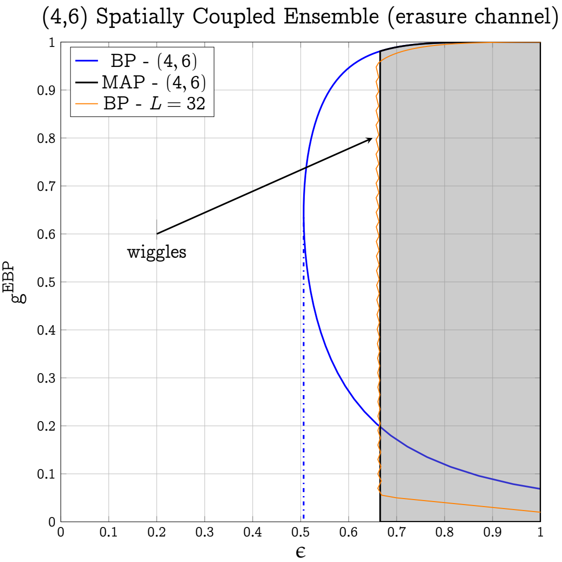

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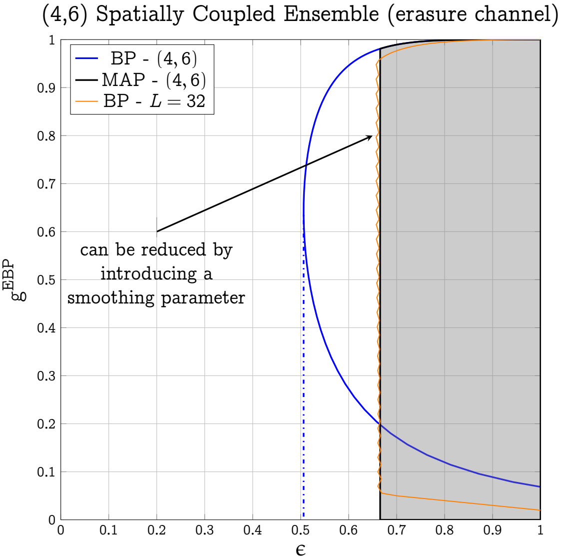
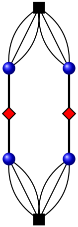


SC Protograph for Joint Decoder



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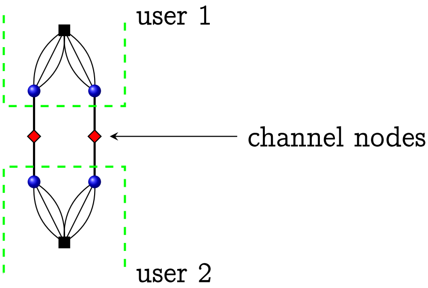

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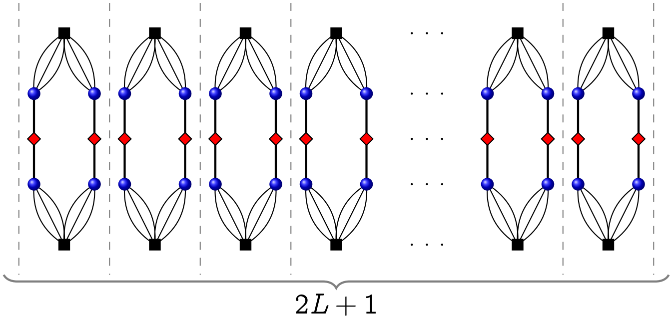

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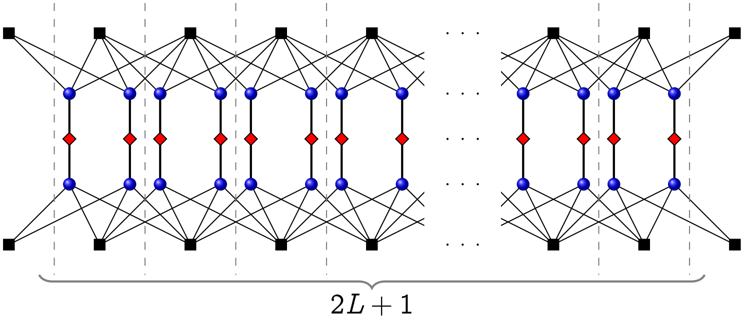

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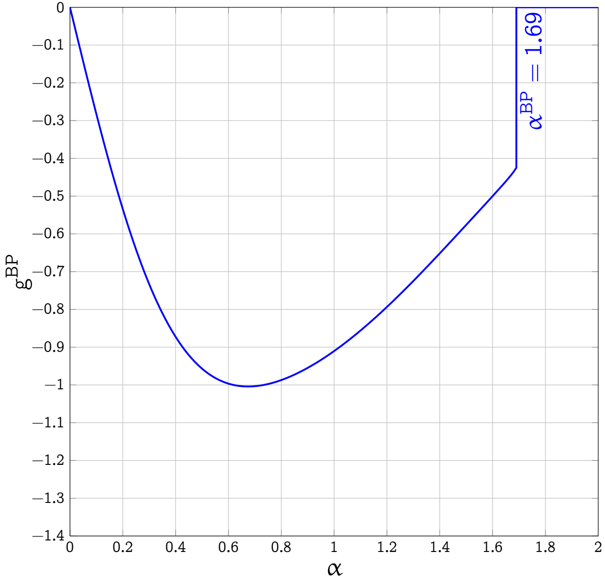

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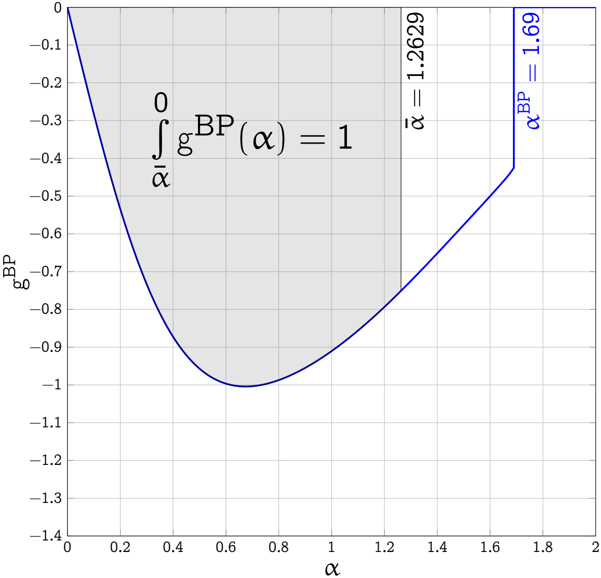

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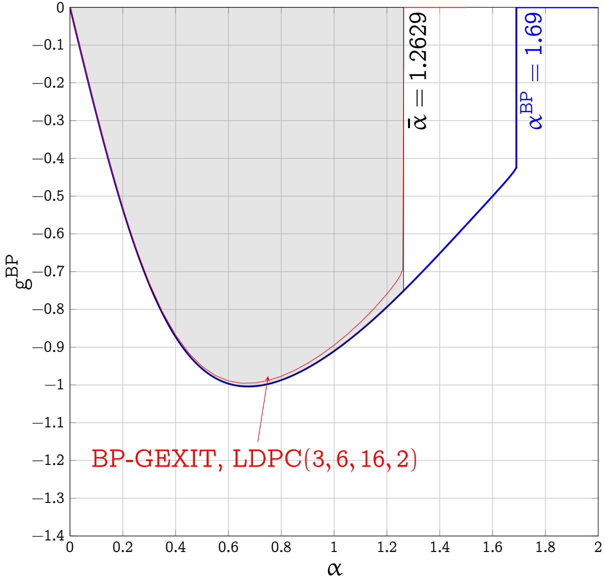

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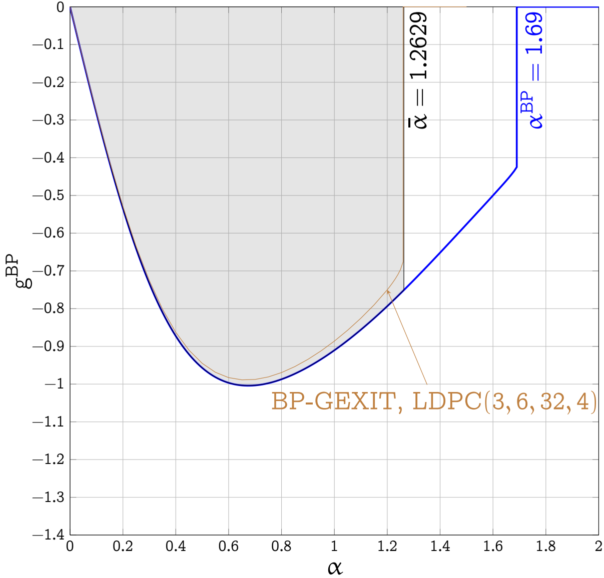
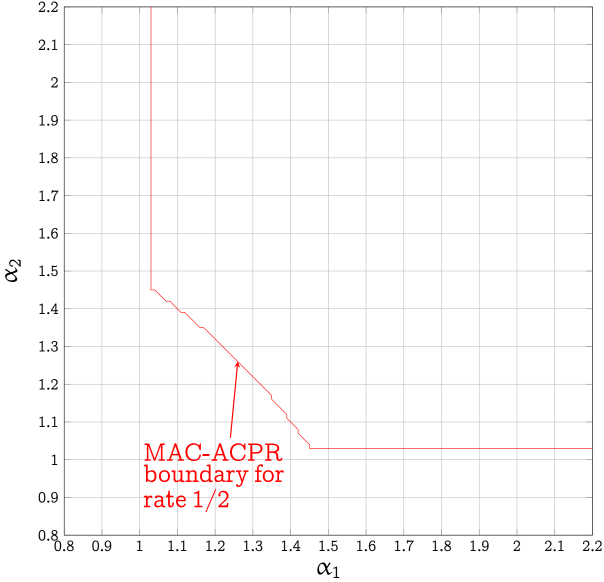


DE Performance of the Joint Decoder



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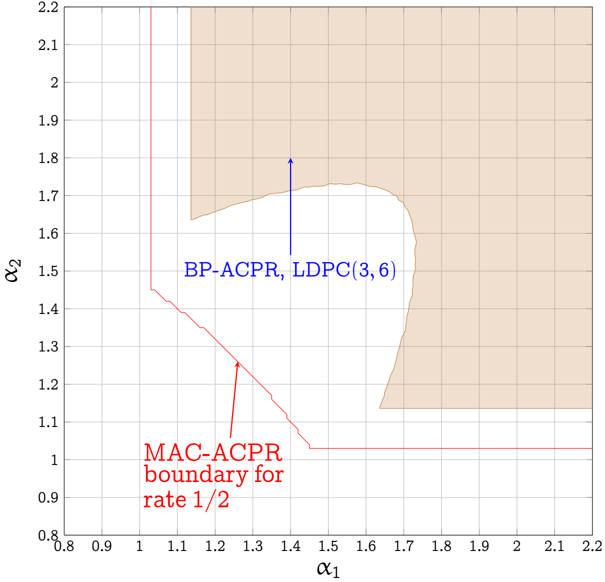

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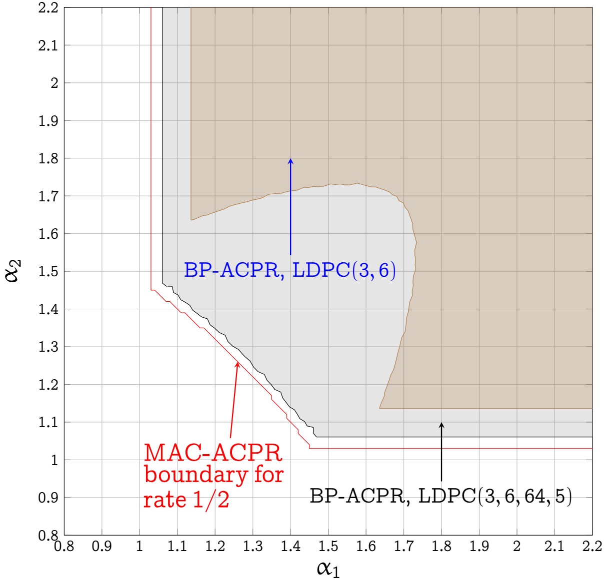

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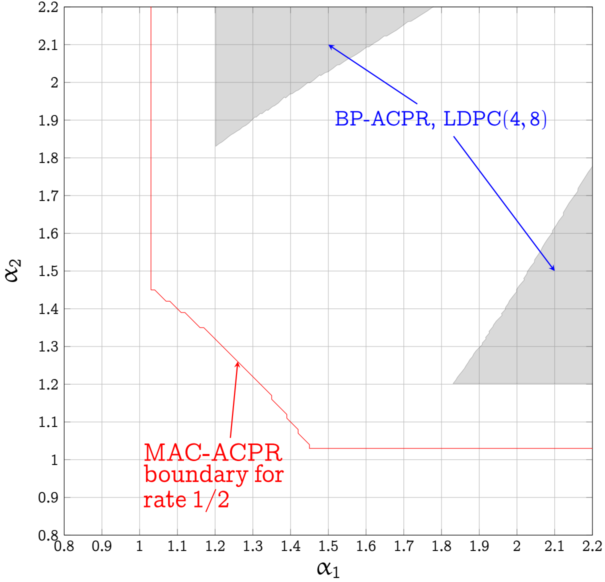

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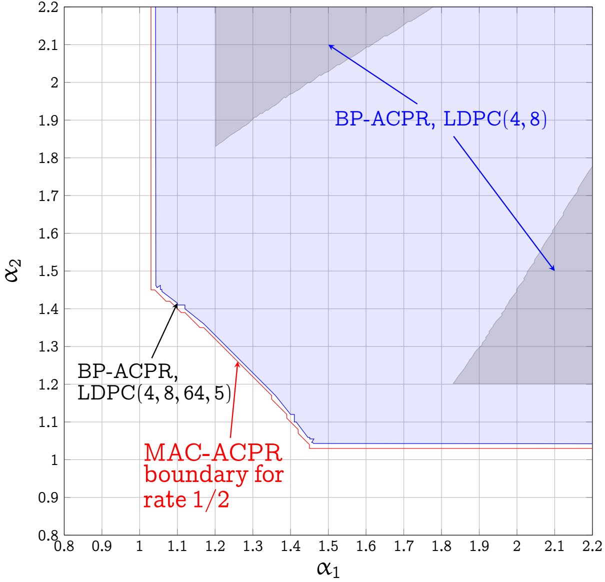

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Thank You!

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