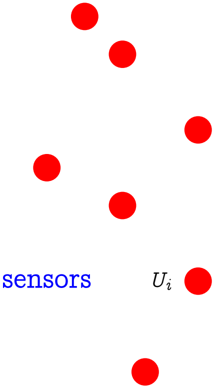
Universality for the Noisy Slepian-Wolf Problem via Spatial Coupling

Henry D. Pfister
(Joint work with Arvind Yedla & Krishna Narayanan)

Texas A&M University

ISIT 2011
St. Petersburg, Russia

The Sensor Reachback Problem



[BS03]

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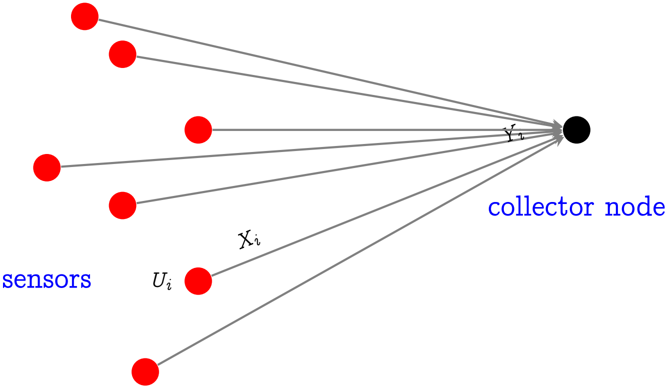

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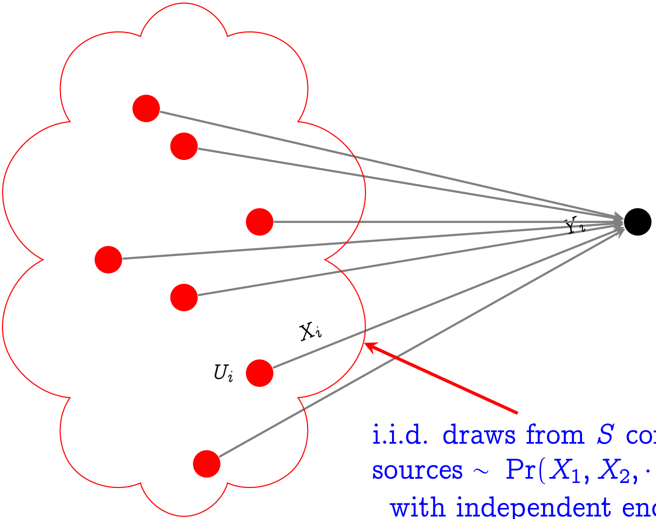

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i.i.d. draws from S correlated sources $\sim \Pr(X_1, X_2, \cdots, X_S)$ with independent encoding

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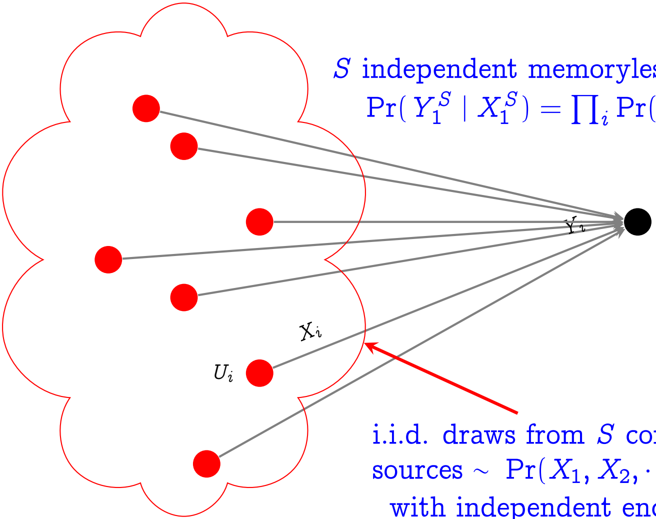

[BS03]

S independent memoryless channels
$\Pr(Y_1^S \mid X_1^S) = \prod_i \Pr(Y_i \mid X_i)$

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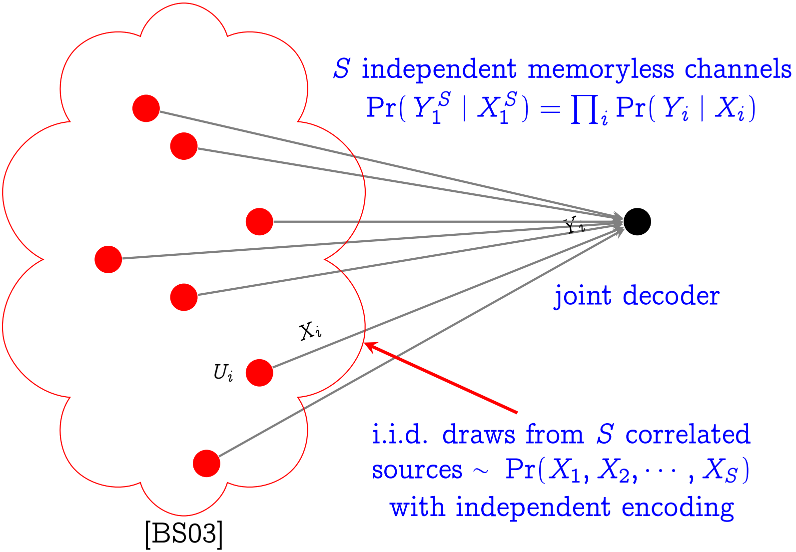

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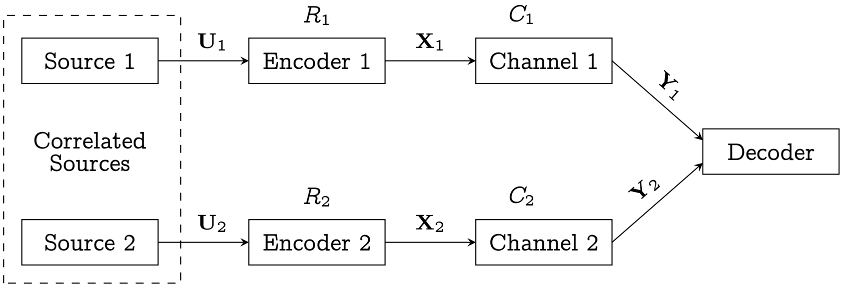

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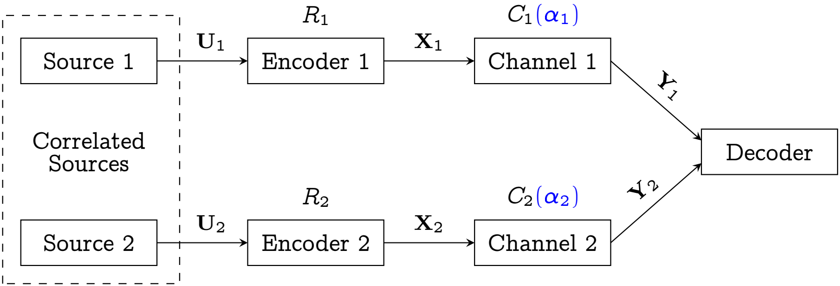

- capacities characterized by parameters α_1, α_2

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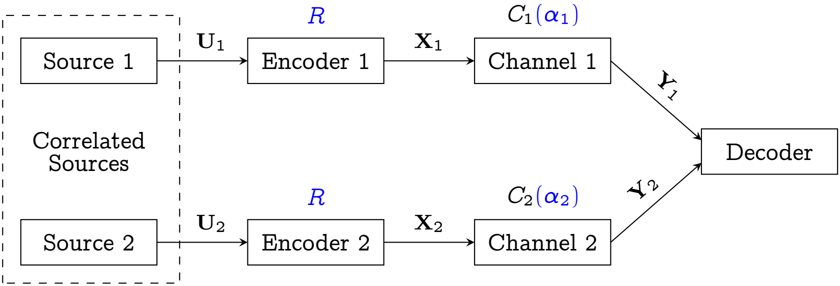

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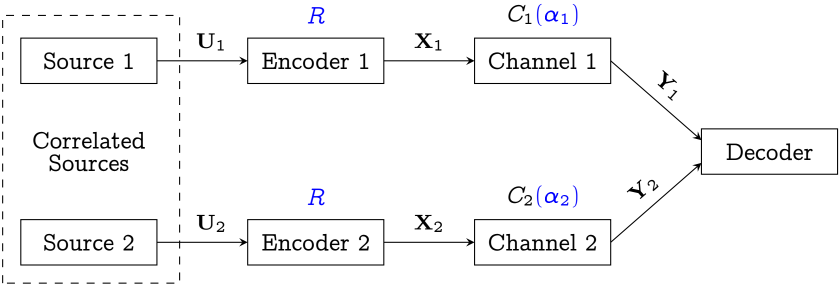

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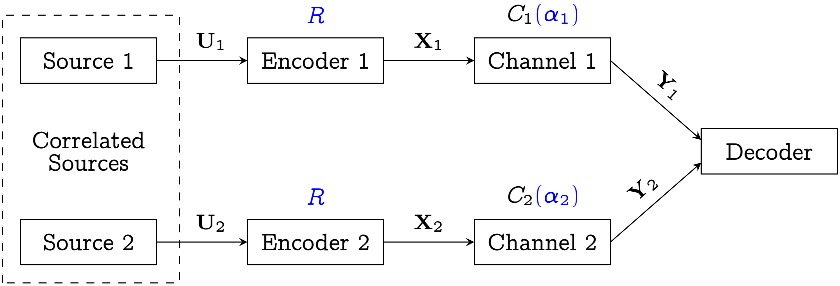

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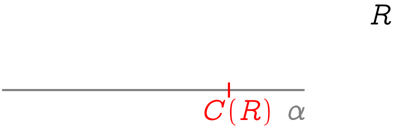
- BSC correlation over AWGN channels:
  $U_1 \sim \mathrm{Ber}(1/2),\ Z \sim \mathrm{Ber}(p),\ U_2 = U_1 + Z$

point-to-point communication

Universality & Multi-terminal Problems

point-to-point communication



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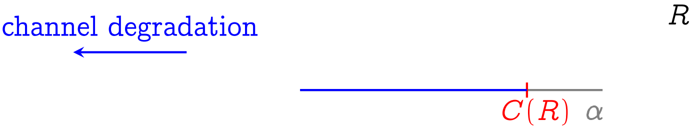

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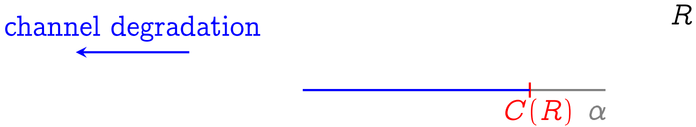

multi-terminal communication

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point-to-point communication

channel degradation

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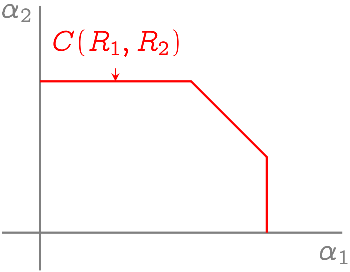

(R_1, R_2)

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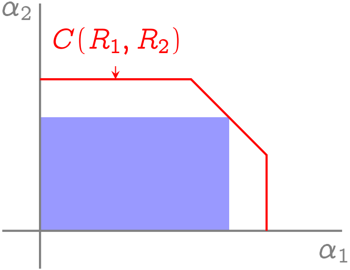
$\Rightarrow$ partial region



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Universality & Multi-terminal Problems

point-to-point communication

channel degradation ←

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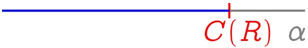

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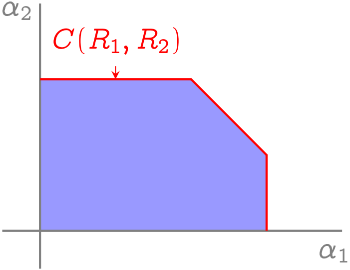
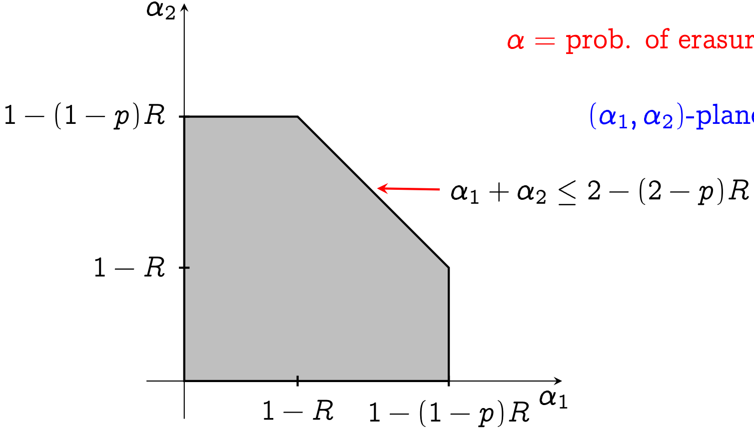
universal codes



(R_1, R_2)

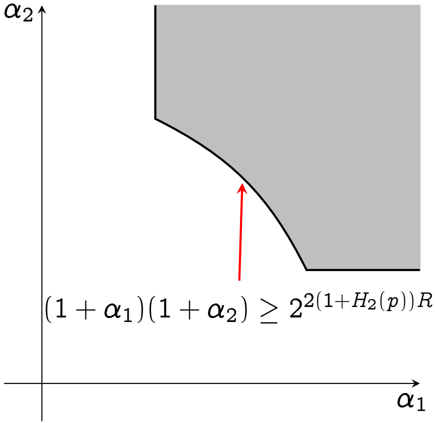
α = prob. of erasure

(α_1, α_2)-plane



erasure correlation, equal rates, and BEC noise

Slepian-Wolf Conditions



α = SNR

(α_1, α_2)-plane

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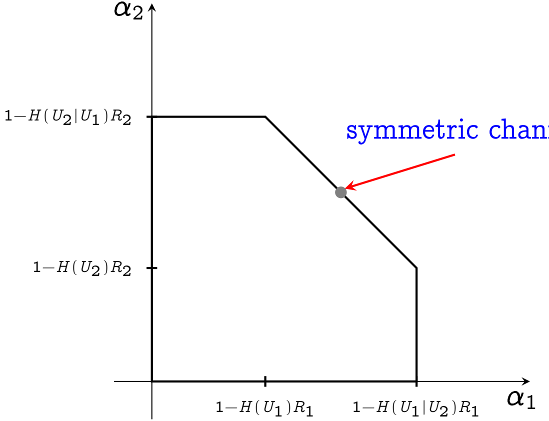

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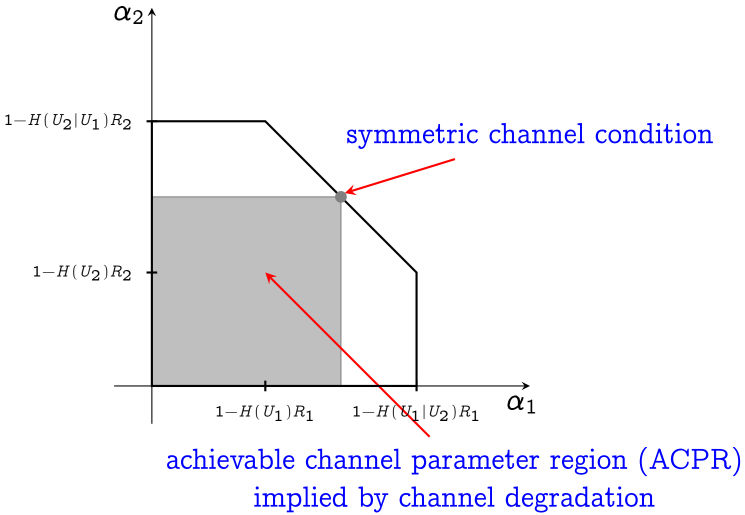

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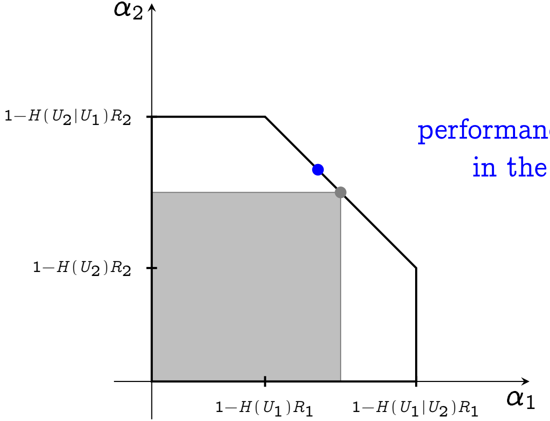
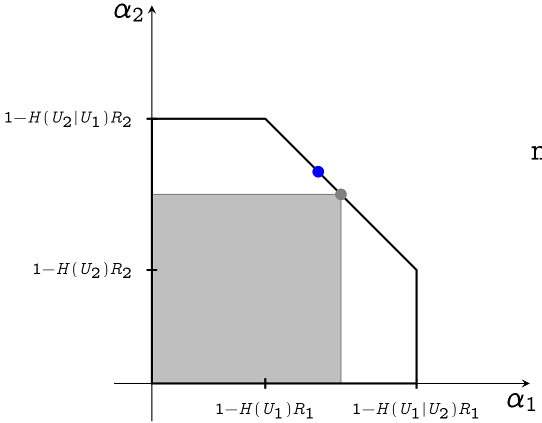


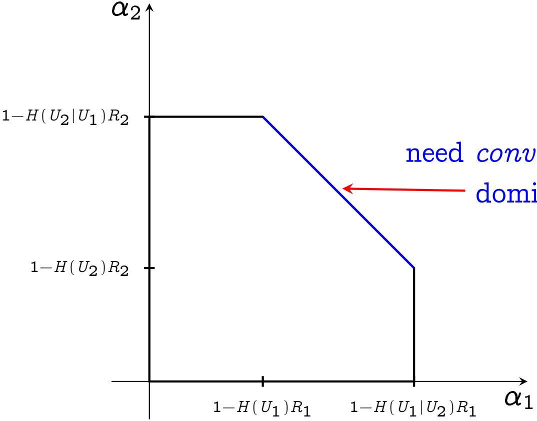
performance at another point
in the feasible region

Slepian-Wolf Conditions



may be bad

Slepian-Wolf Conditions



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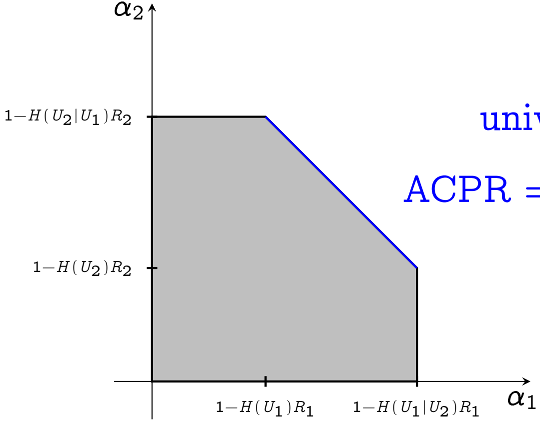

universal codes

ACPR = full SW region

Motivation and Prior Work

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- contribution of this work
  - spatially-coupled joint decoding is essentially universal

Review: BP vs MAP

$$X \xrightarrow{\text{BMS}(\alpha)} Y$$

[MMRU09]

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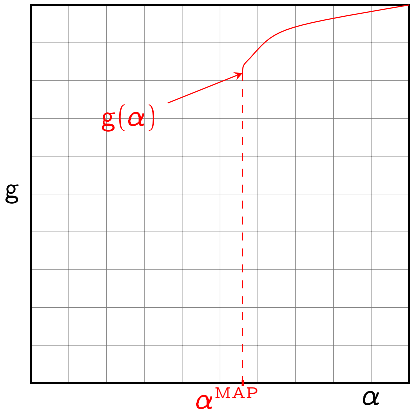

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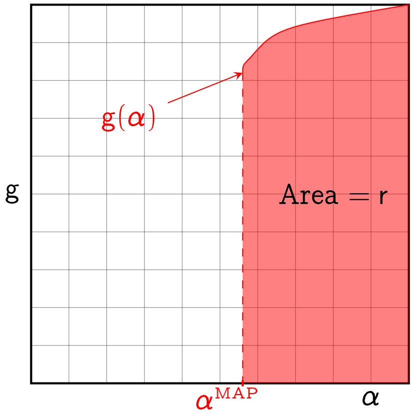

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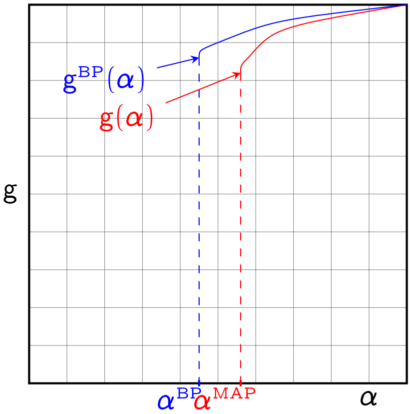

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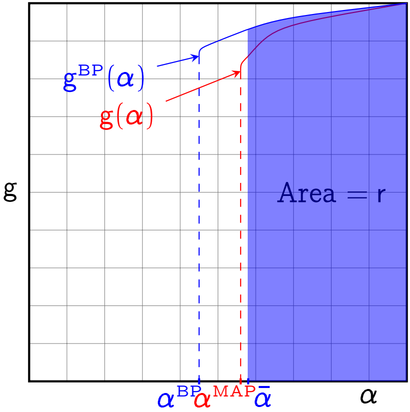

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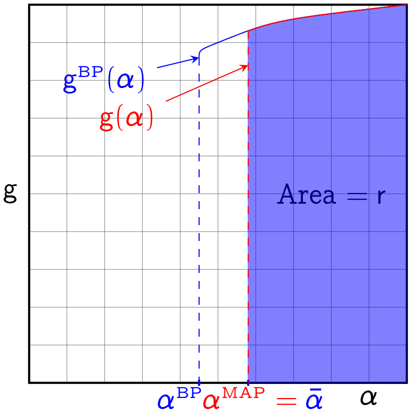

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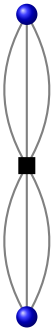
- recently, [KRU10] observed this as a larger phenomenon whereby the BP threshold of spatially-coupled LDPC codes saturates to the MAP threshold of their uncoupled cousins
- This observation implies SC benefits many applications:
  - ISIT: BEC Wiretap (Rathi et al.), MAC / ISI (Kudekar & Kasai), CDMA (Takeuchi et al. / Schlegel & Truhachev), Quantum (Hagiwara et al.)
  - arXiv: MAC (Yedla et al.), ISI (Nguyen et al.)

Review: Spatial Coupling contd.



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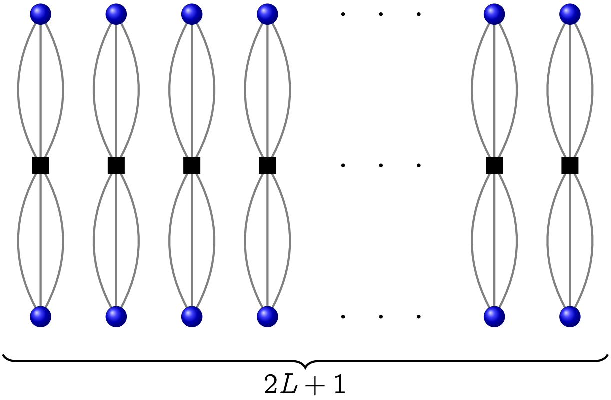

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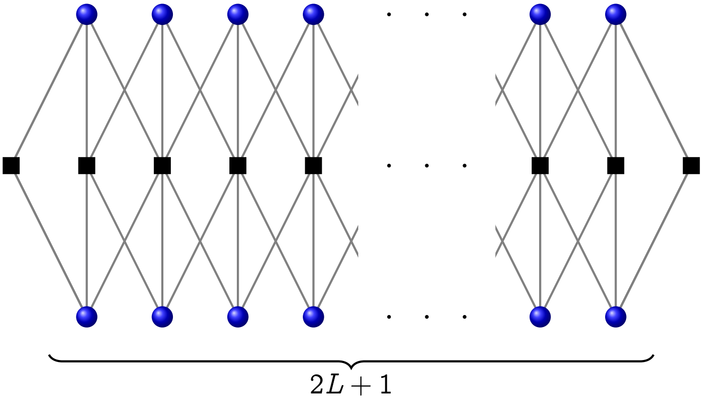
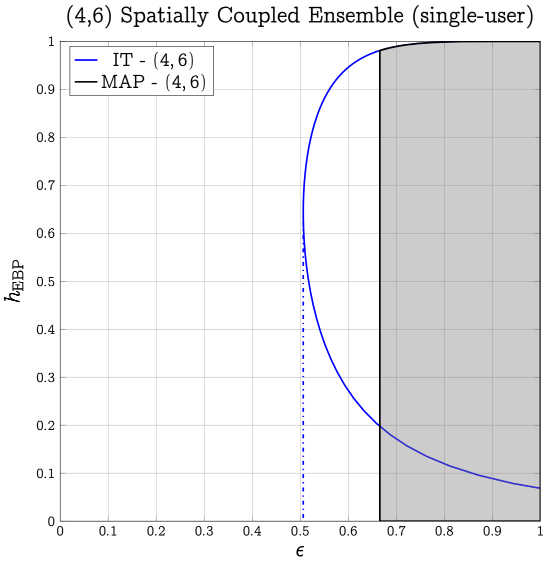


Review: The Extended BP GEXIT Curve



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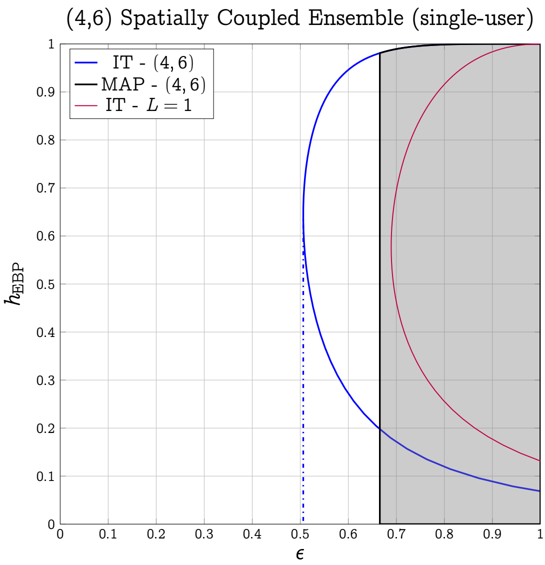

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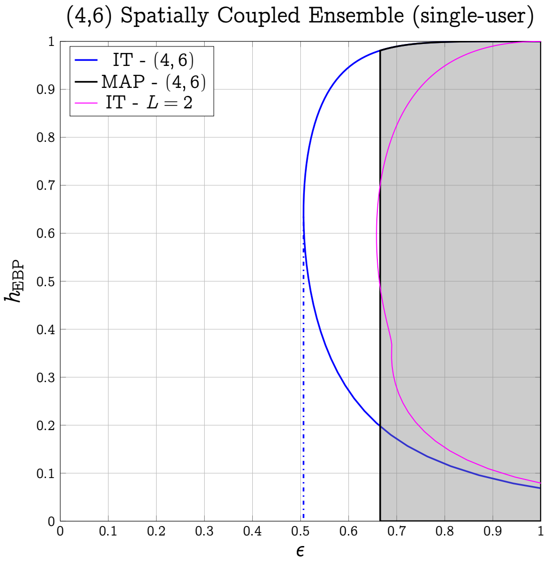

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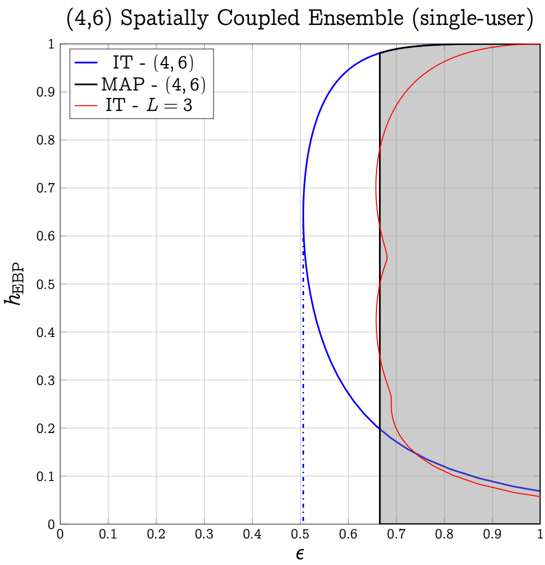

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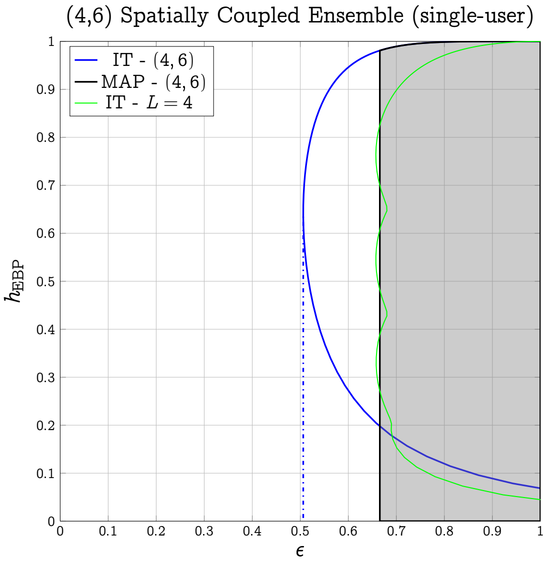

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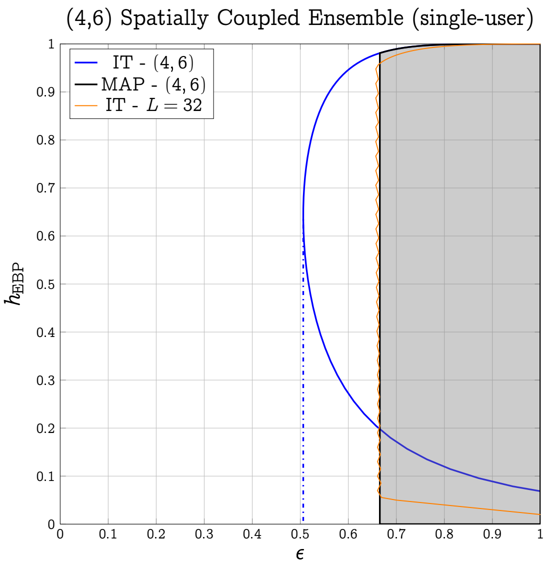

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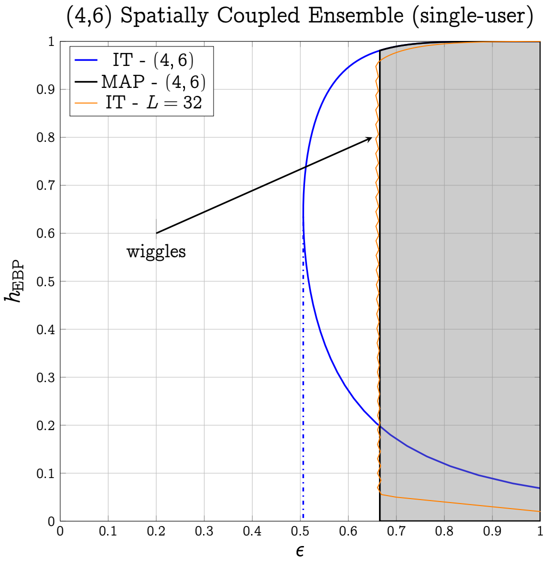

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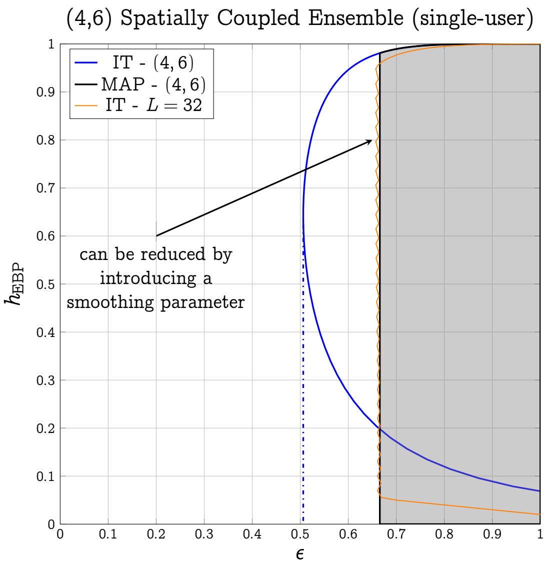

The BP GEXIT Curve of the Joint Decoder

- for $\mathbf{X} = [\mathbf{X}_1 \; \mathbf{X}_2]$ and $\mathbf{Y}(\alpha) = [\mathbf{Y}_1(\alpha) \; \mathbf{Y}_2(\alpha)]$, the MAP GEXIT surface is defined by the gradient

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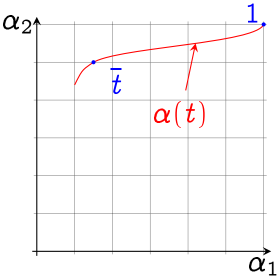
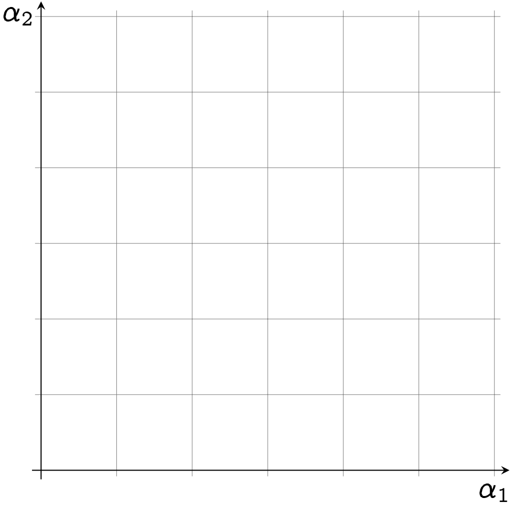
the normalized area theorem is given by the line integral



- $$\int_{t^{MAP}}^{1} \mathrm{g}^{MAP}(\alpha(t)) \cdot \alpha'(t)\, dt = \mathrm{r} H(U_1, U_2),$$

where $\alpha(t)$ is a parametrized curve through channel space

MAP Boundary



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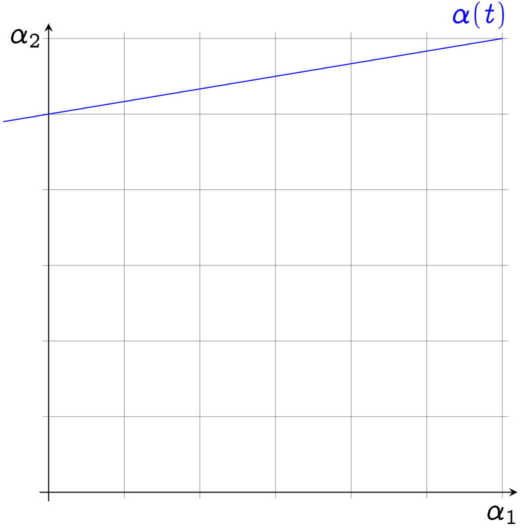

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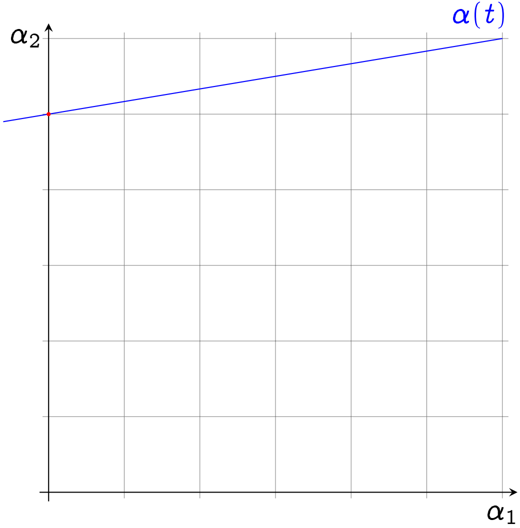

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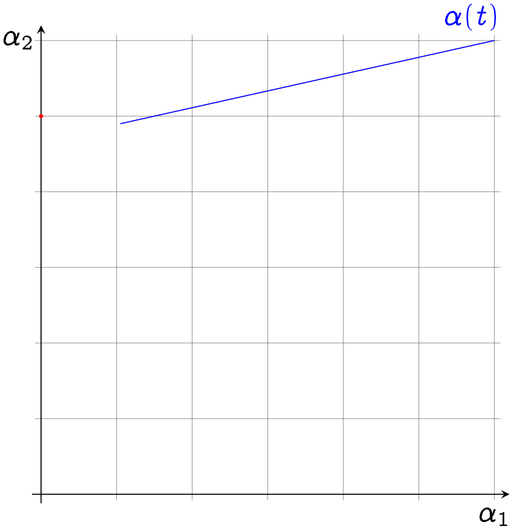

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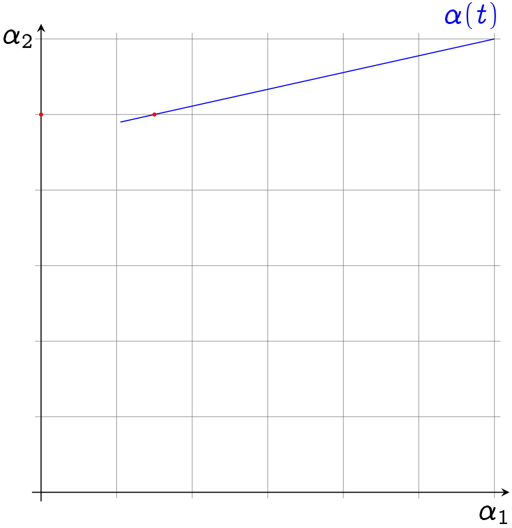

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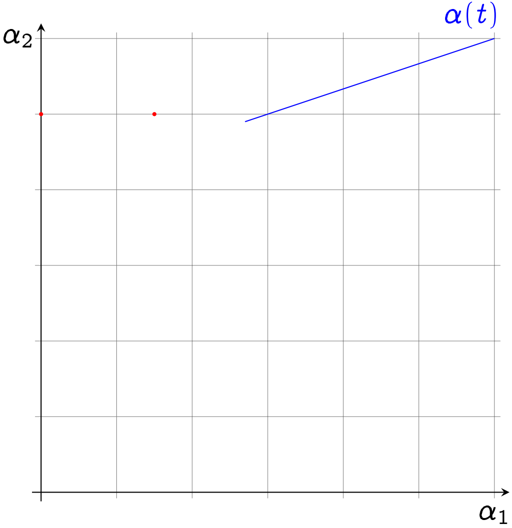

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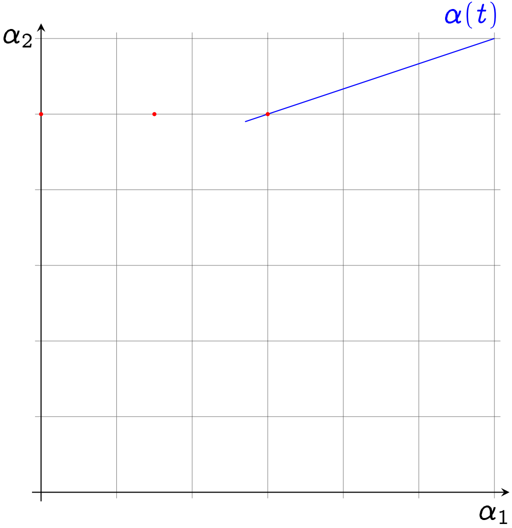

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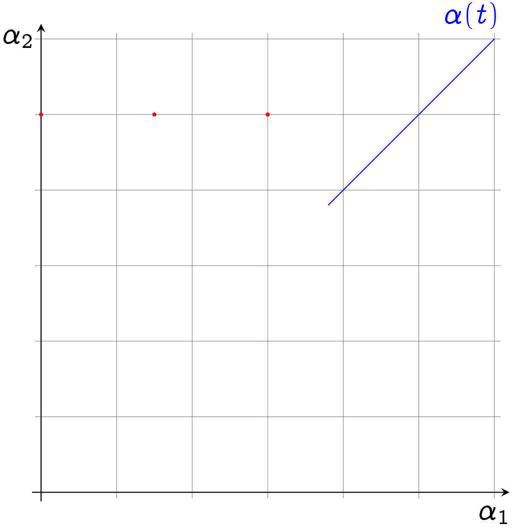

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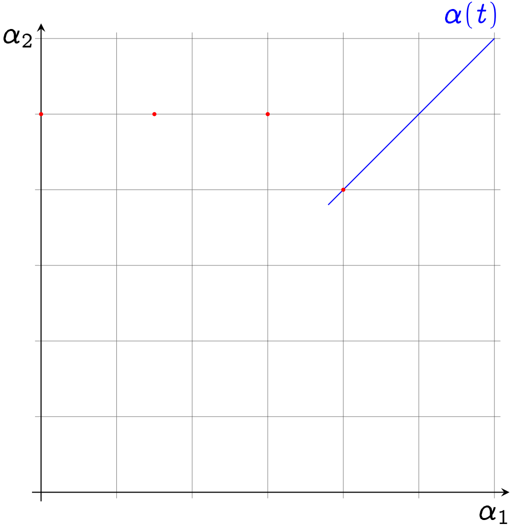

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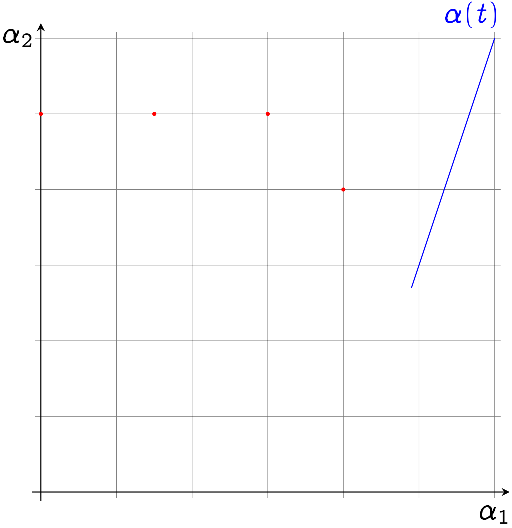

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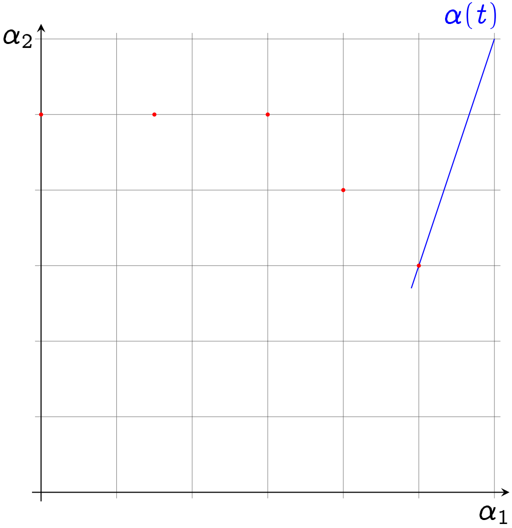

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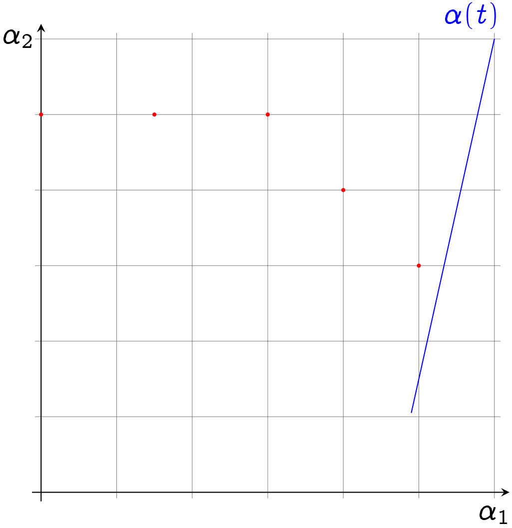

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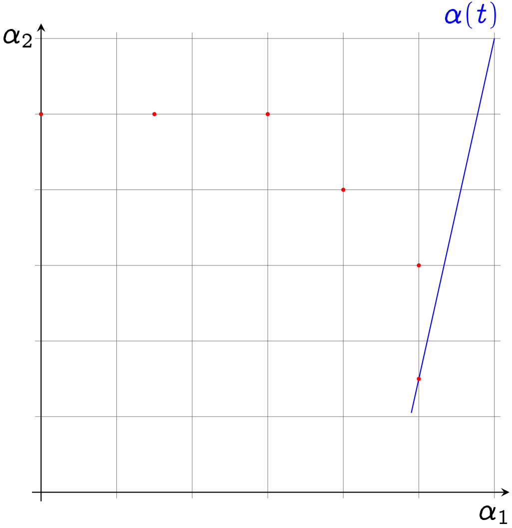

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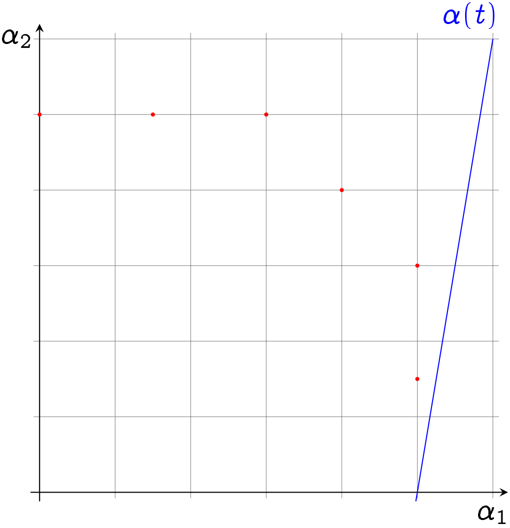

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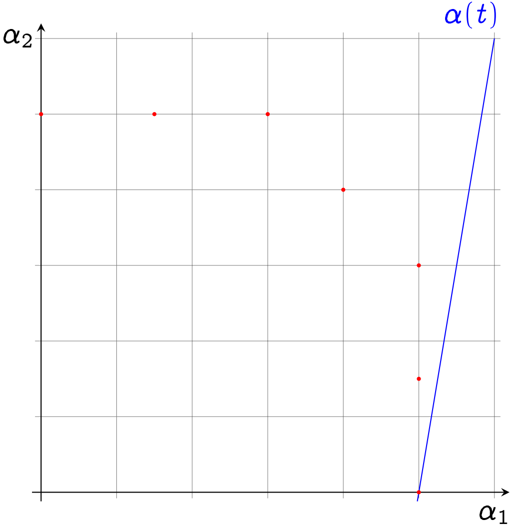

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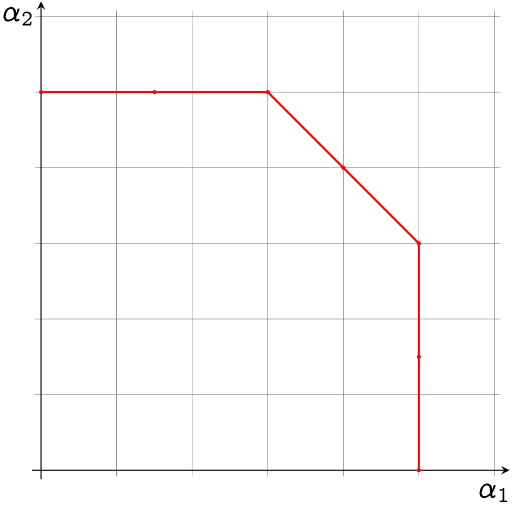
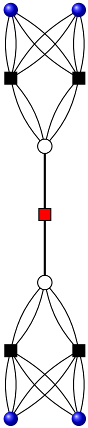


Spatially-Coupled Protograph for Joint Decoder



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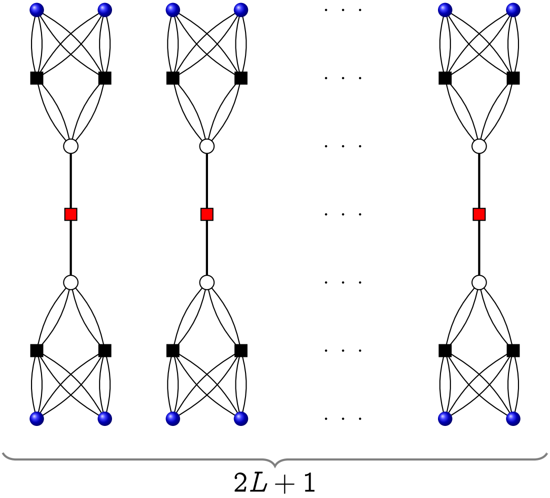

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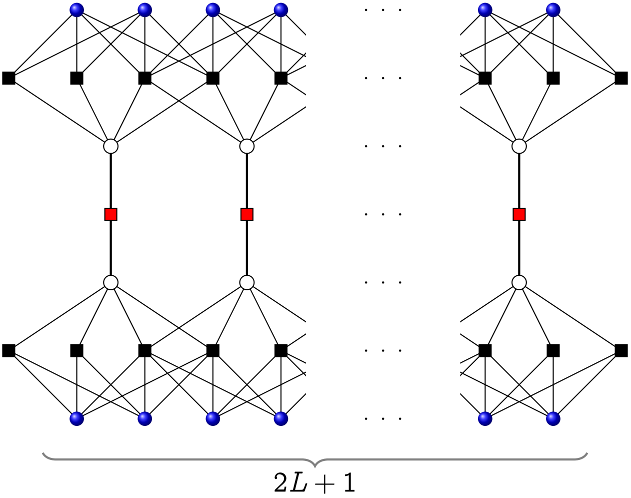
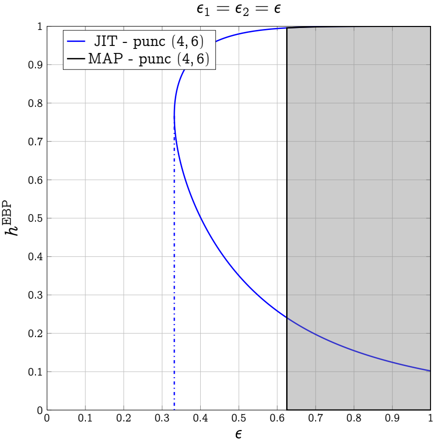
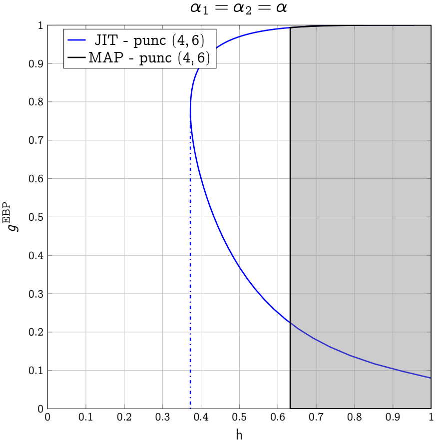


The Extended BP-GEXIT Curve of the Joint Decoder

BEC correlation



BSC correlation



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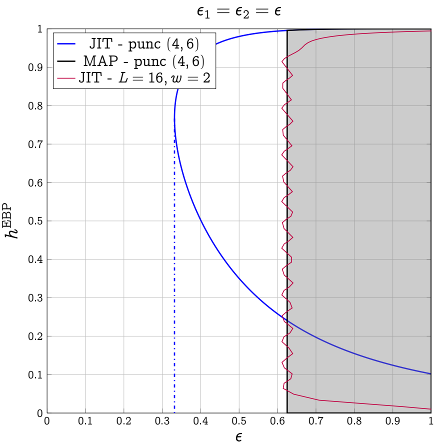
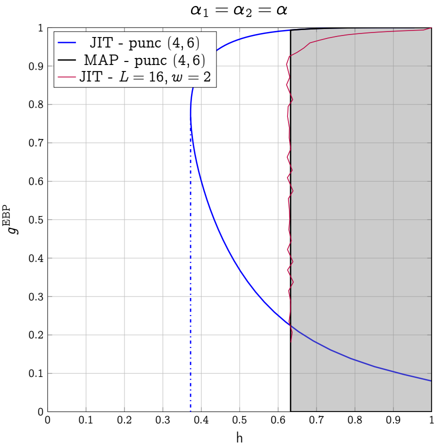


BSC correlation



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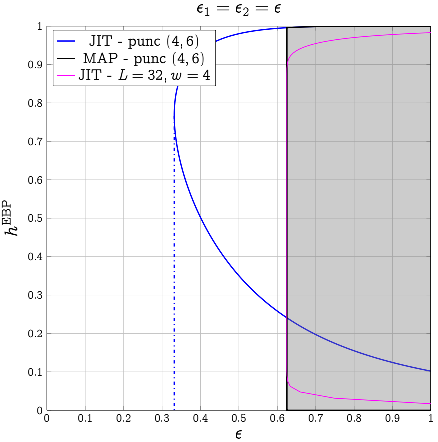

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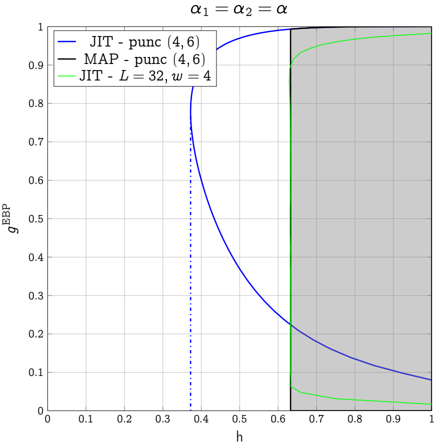
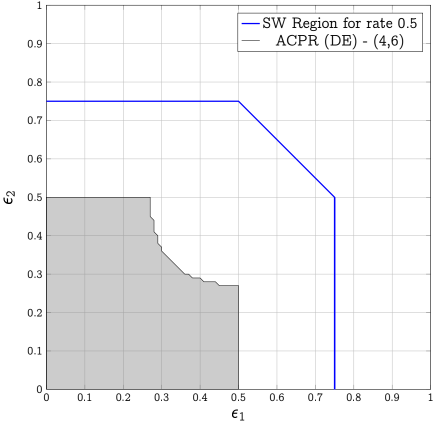
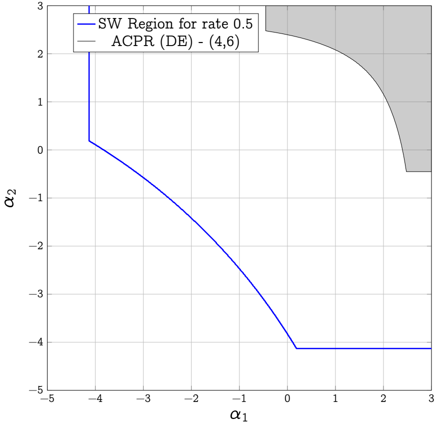


DE Performance of the Joint Decoder

BEC correlation



BSC correlation



DE Performance of the Joint Decoder

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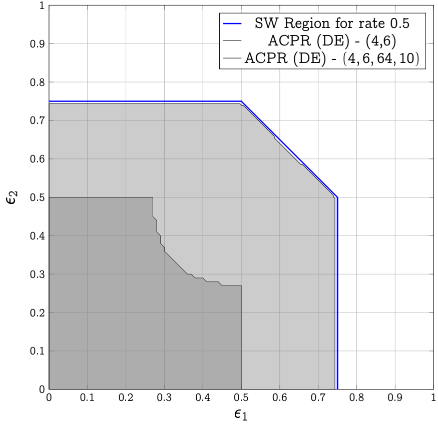
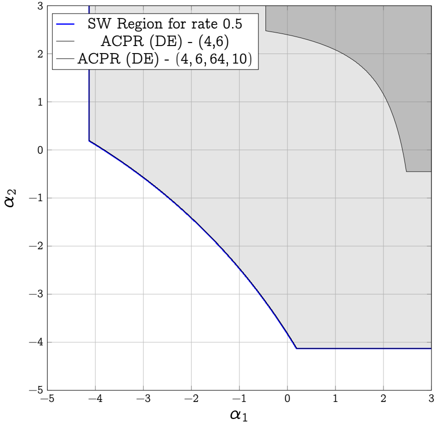


BSC correlation



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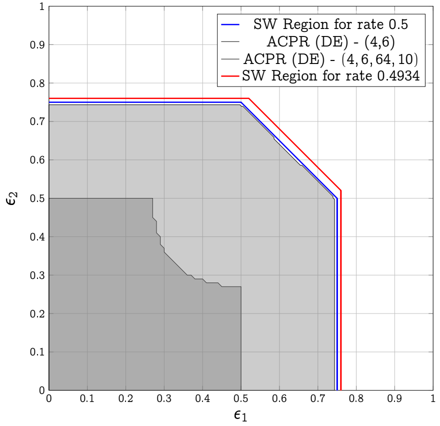
BEC correlation



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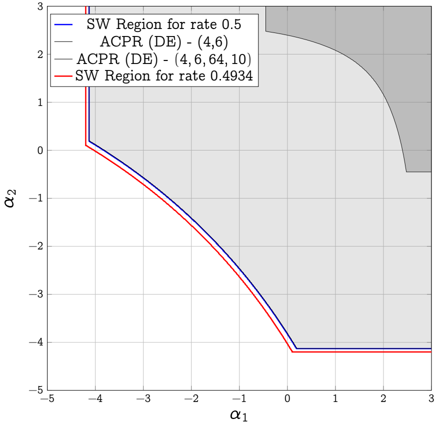

Closing Remarks

- conclusions
  - saturation to the MAP threshold for the SW problem
  - provides universality under iterative decoding

Closing Remarks

- conclusions
  - saturation to the MAP threshold for the SW problem
  - provides universality under iterative decoding
- future work
  - extend threshold saturation proof to this problem
  - investigate the phenomenon of threshold saturation for other multi-terminal problems

Thank You!

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