LDPC Code Design for Transmission of Correlated Sources Across Noisy Channels Without CSIT

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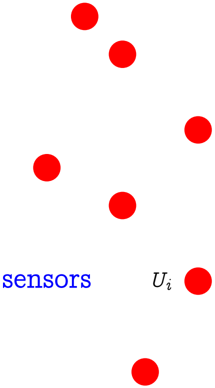
(Joint work with Arvind Yedla & Krishna Narayanan)

Texas A&M University

ISTC 2010
Brest, France

The Sensor Reachback Problem



[BS03]

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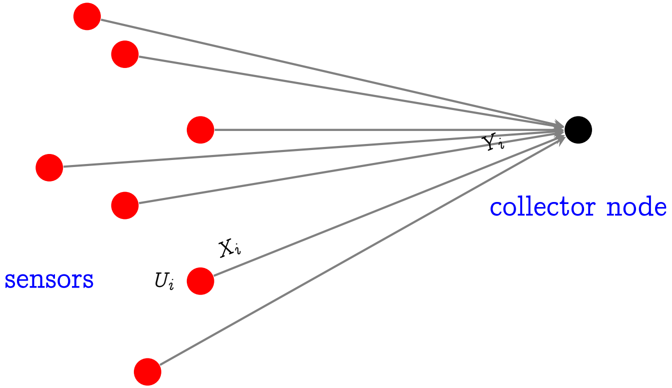

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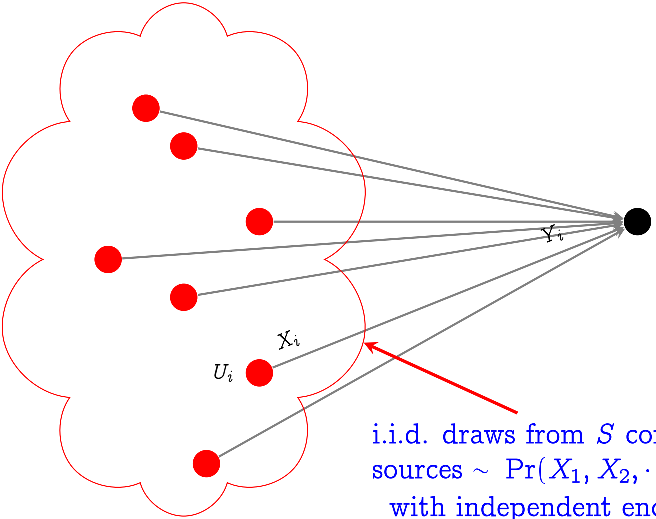

i.i.d. draws from S correlated sources $\sim \Pr(X_1, X_2, \cdots, X_S)$ with independent encoding

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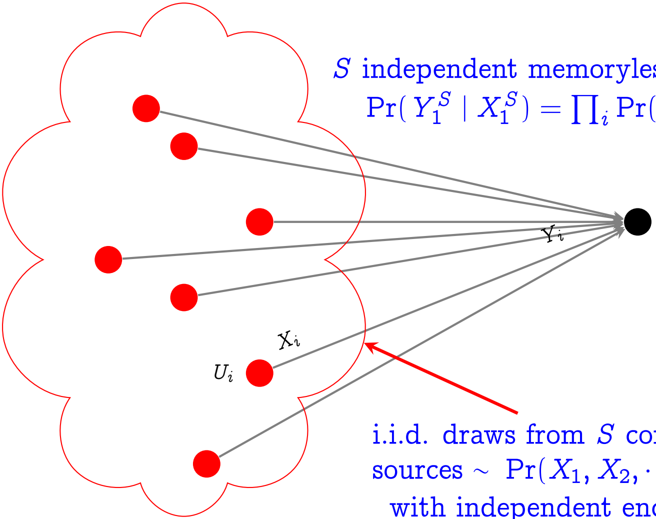

S independent memoryless channels

$\Pr(\,Y_1^S \mid X_1^S\,) = \prod_i \Pr(\,Y_i \mid X_i\,)$

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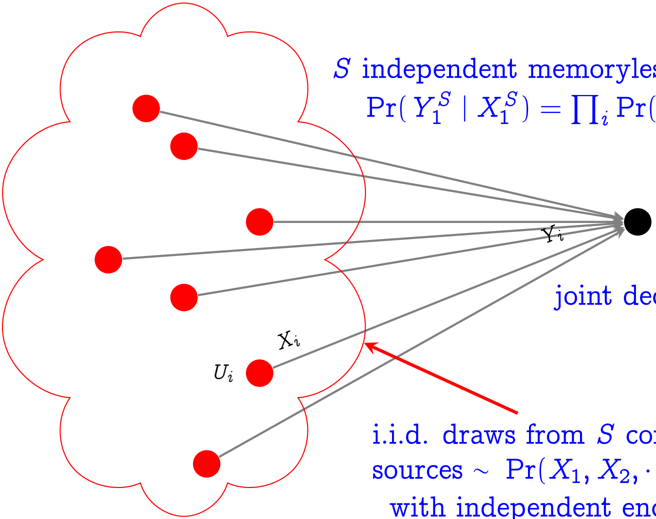
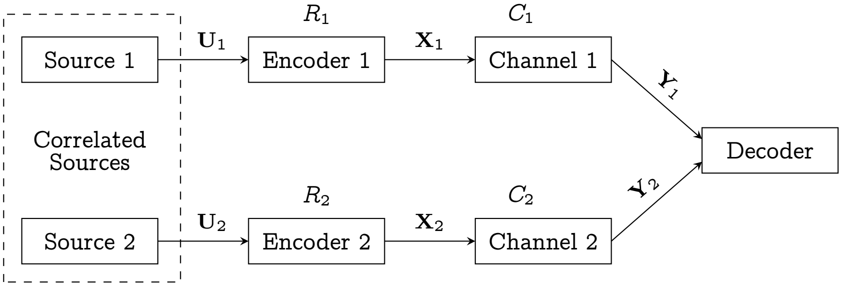


[BS03]

joint decoder

i.i.d. draws from S correlated sources $\sim \Pr(X_1, X_2, \cdots, X_S)$ with independent encoding

A Simplified Model



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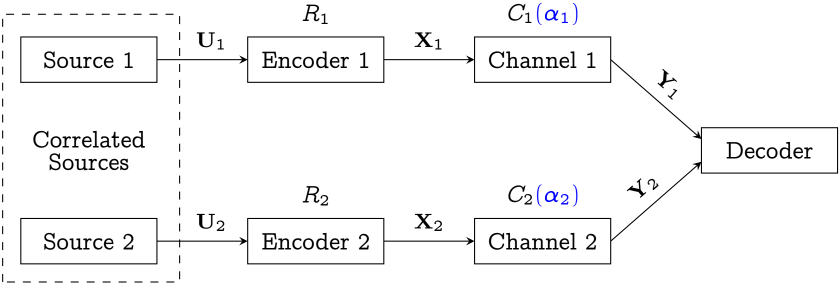

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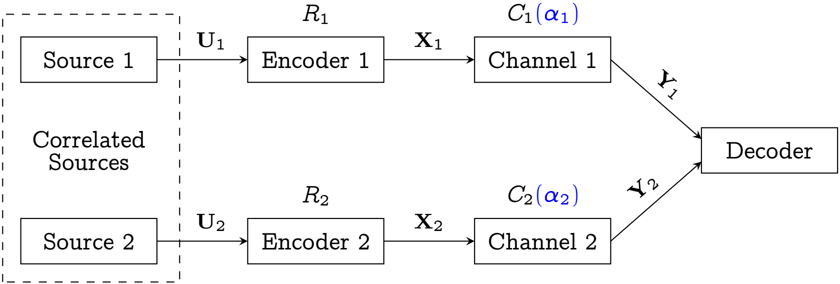

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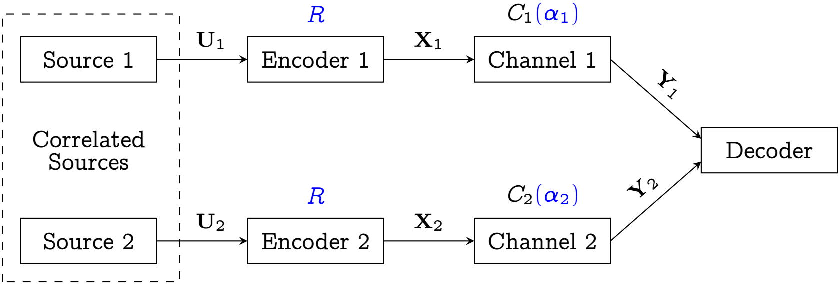
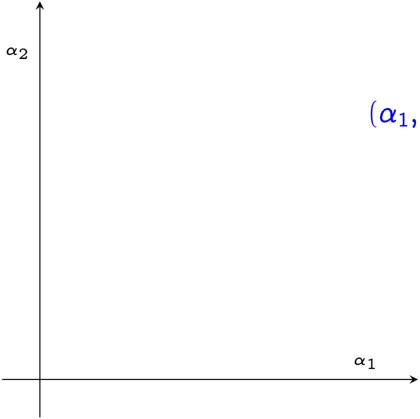


- each capacity characterized by a single parameter α
- α_1 and α_2 not known at transmitter
- each code has rate R

Slepian-Wolf Conditions



(α_1, α_2)-plane

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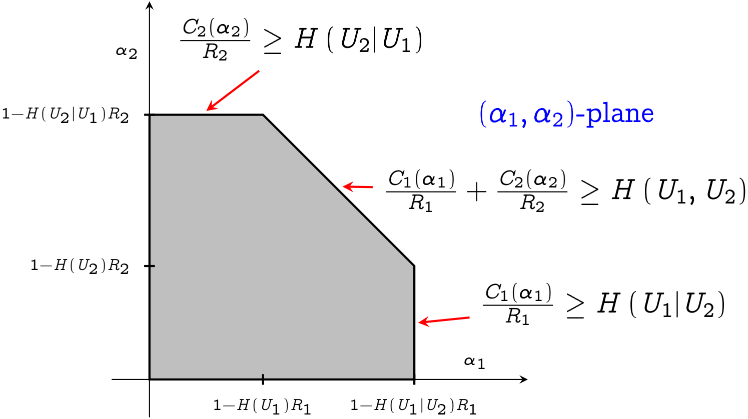
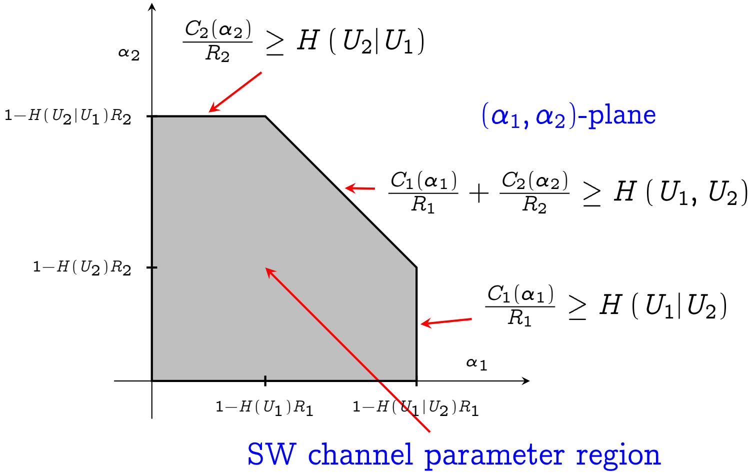


illustrated for the BEC case where $C(\alpha) = 1 - \alpha$

Slepian-Wolf Conditions



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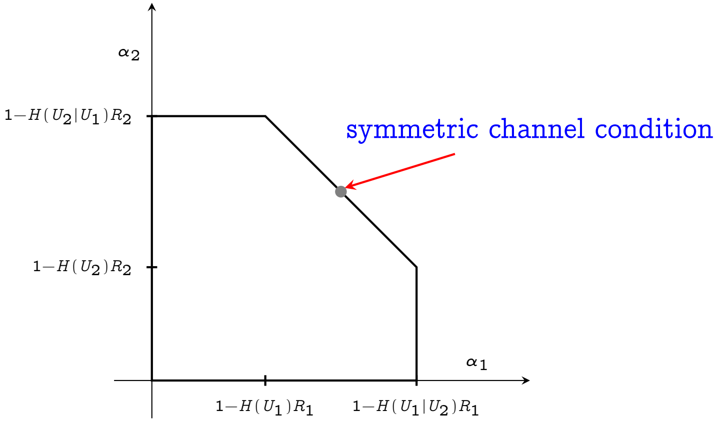

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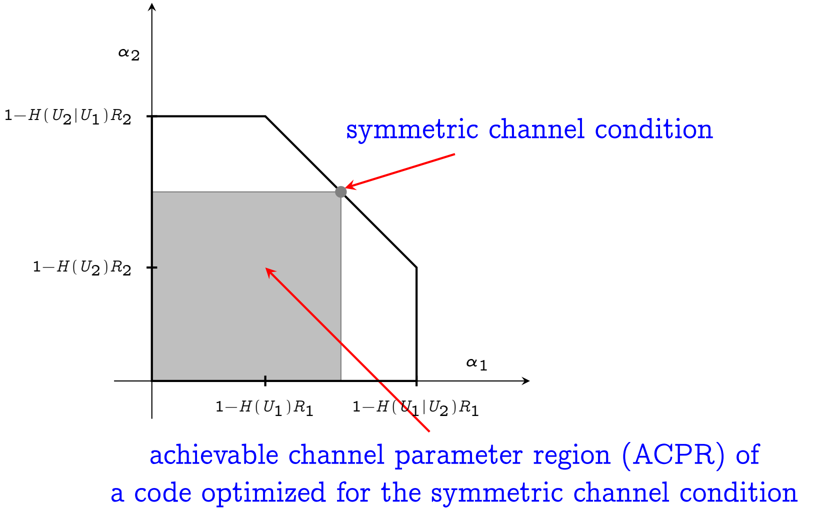

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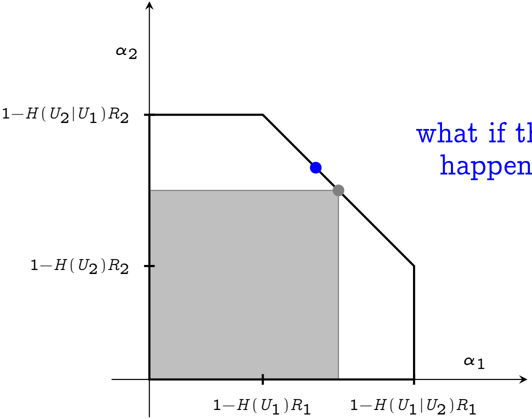
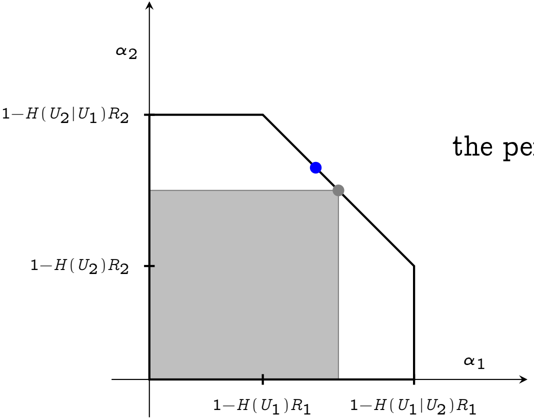


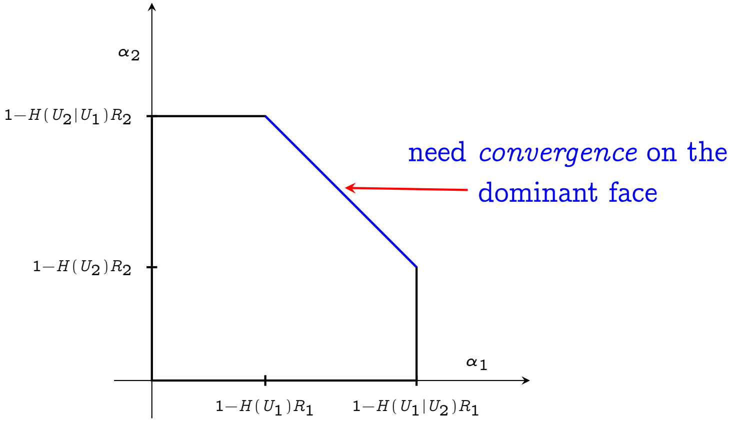
what if the channel condition happened to be different?

Slepian-Wolf Conditions



the performance is bad.

Slepian-Wolf Conditions



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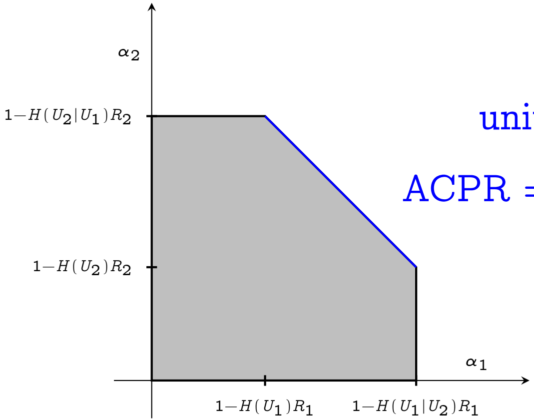

universal codes

ACPR = full SW region

random codes with ML decoding are universal.

what about LDPC codes with iterative decoding?

Prior Work and Summary

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  - Optimized LT codes are not universal [YPN09]

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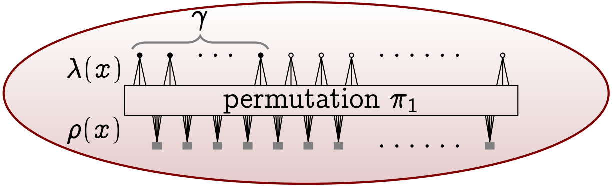
  - systematic codes $\Rightarrow$ correlated codes $\Rightarrow$ suboptimal
  - optimized non-systematic LDPC codes have large ACPRs

Tanner Graph & Density Evolution



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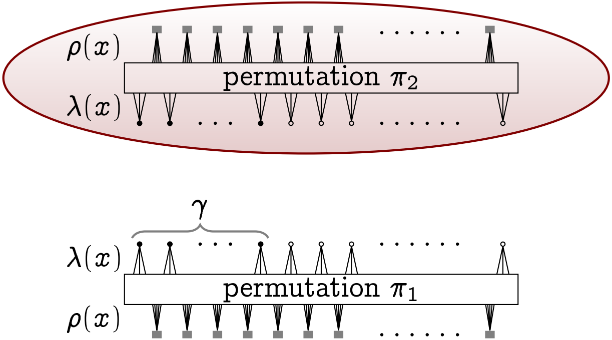

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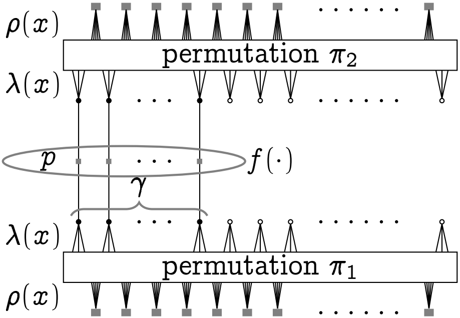

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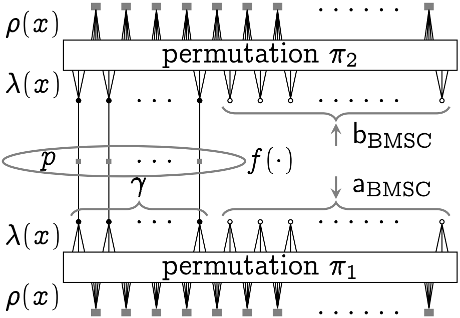

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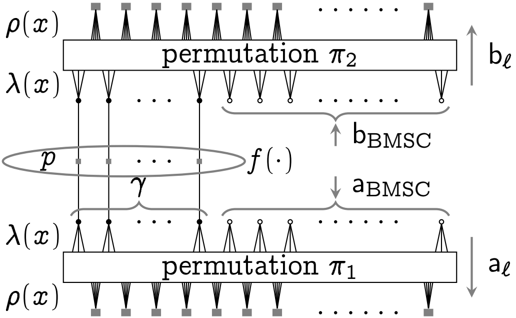

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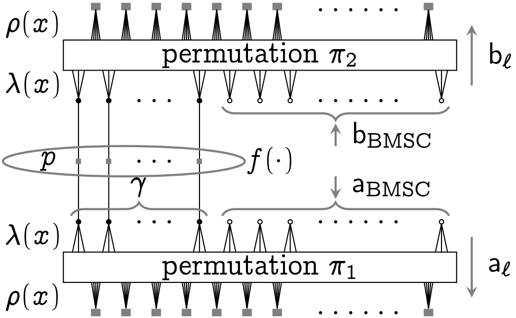

$$a_{\ell+1} = \left[\qquad\qquad\qquad \right] \circledast \lambda(\rho(a_\ell))$$

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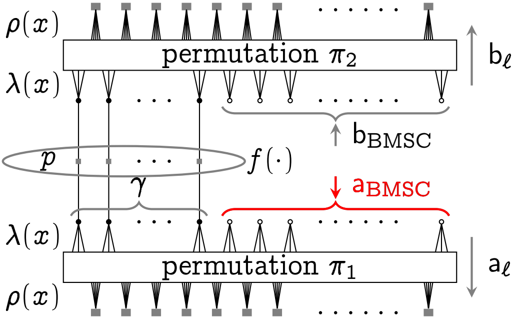

$$\mathsf{a}_{\ell+1} = \left[\qquad\qquad (1-\gamma)\mathsf{a}_{\mathrm{BMSC}} \right] \circledast \lambda(\rho(\mathsf{a}_\ell))$$

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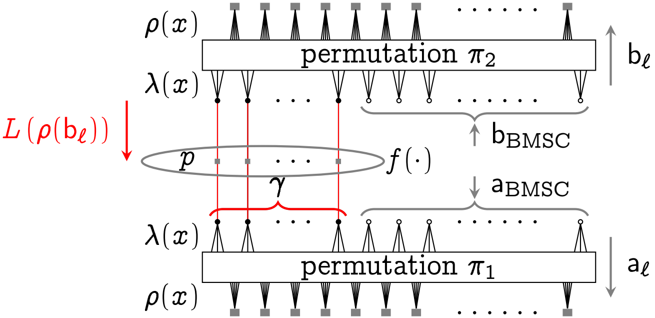

$$a_{\ell+1} = \left[\gamma f\left(L(\rho(b_\ell))\right) + (1-\gamma)a_{BMSC}\right] \circledast \lambda(\rho(a_\ell))$$

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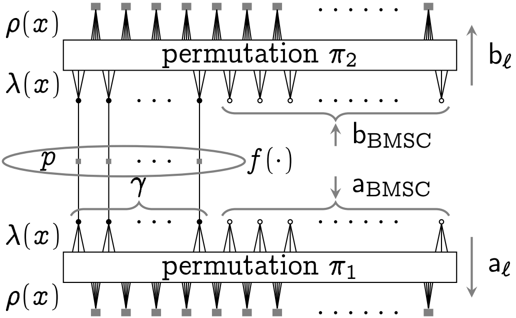

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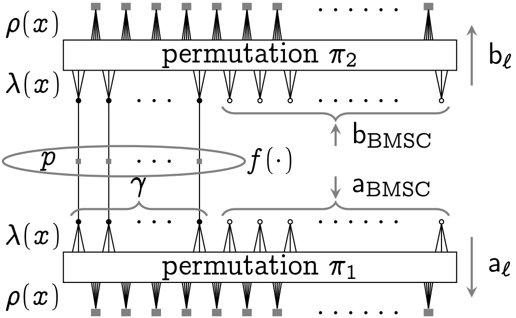
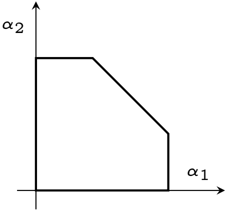


$\Gamma_{\lambda,\rho}(\alpha_1, \alpha_2)$ - residual error probability

Code Design

- fix the desired rate R_d



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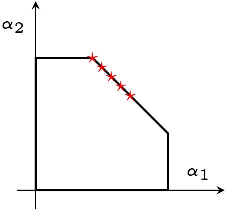

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- require *convergence* on $\mathcal{C}$

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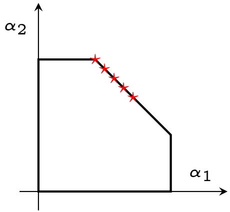

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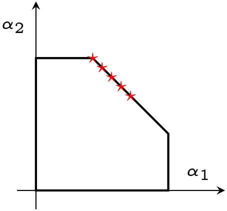
$$\mathcal{A}_x = \{(\alpha_1, \alpha_2) | \Gamma_x(\alpha_1, \alpha_2) \preceq (\tau, \tau)\}$$

$$\mathcal{F}(x) =$$

$$+$$



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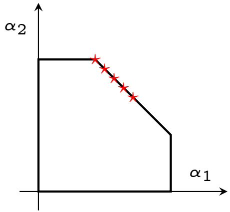

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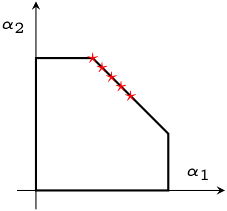

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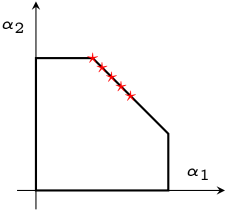

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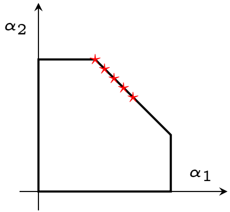

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- search using differential evolution

Code Design contd.,

- BSC correlated sources

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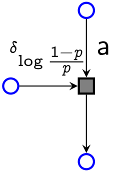
  - $\Pr(U_1 = U_2) = p$, $\Pr(U_1) = \Pr(U_2) = 1/2$



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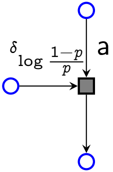

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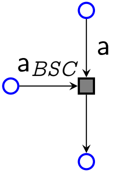

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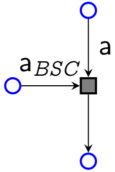

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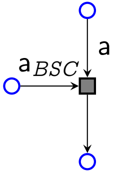
- erasure correlated sources



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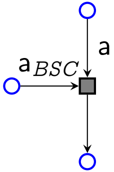
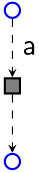


- erasure correlated sources
  - $Z \sim \text{Ber}(p)$

$$(U_1, U_2) = \begin{cases} \text{i.i.d. Bernoulli } \frac{1}{2} \text{ r.v.s, if } Z = 0 \\ \text{same Bernoulli } \frac{1}{2} \text{ r.v. } U \text{ , if } Z = 1 \end{cases}$$



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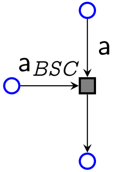

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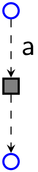
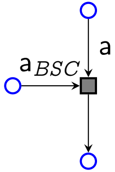


  - $f(\mathsf{a}) = (1-p) + p\mathsf{a}$

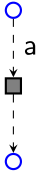
Code Design contd.,

- BSC correlated sources
  - $\Pr(U_1 = U_2) = p$, $\Pr(U_1) = \Pr(U_2) = 1/2$
  - all-zero codeword assumption not valid
  - $f(\mathsf{a}) = \mathsf{a}_{BSC(1-p)} \circledast \mathsf{a}$
  - transmission through AWGN channels



- erasure correlated sources
  - $Z \sim \text{Ber}(p)$

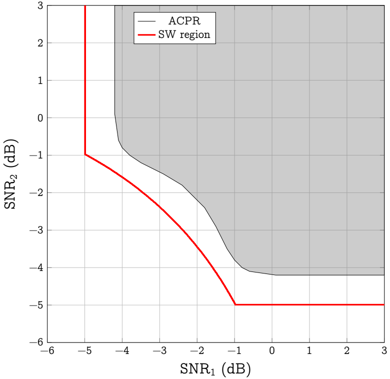
$$(U_1, U_2) = \begin{cases} \text{i.i.d. Bernoulli } \frac{1}{2} \text{ r.v.s, if } Z = 0 \\ \text{same Bernoulli } \frac{1}{2} \text{ r.v. } U \text{ , if } Z = 1 \end{cases}$$



  - $f(\mathsf{a}) = (1-p) + p\mathsf{a}$
  - transmission through erasure channels

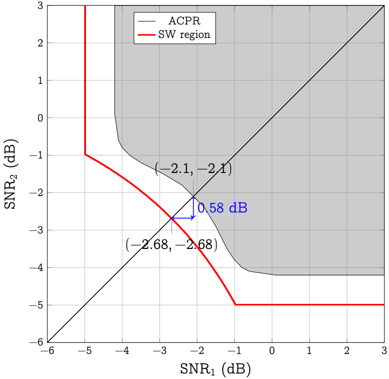
Results: AWGN ($p = 0.9$)

code rate $= 0.282$



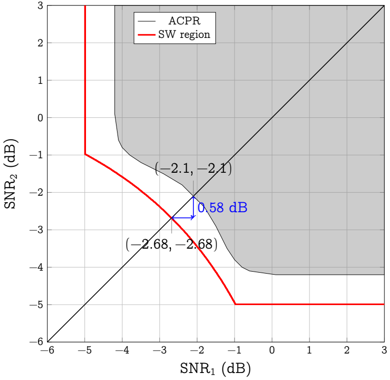
Results: AWGN ($p = 0.9$)

code rate $= 0.282$

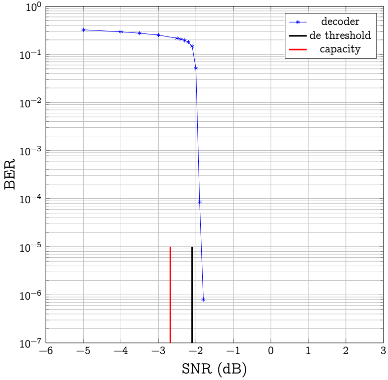


Results: AWGN ($p = 0.9$)

code rate $= 0.282$

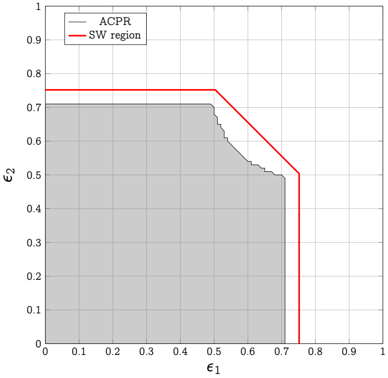


block length $= 10^5$



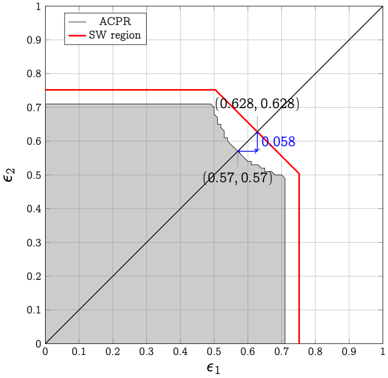
Results: BEC ($p = 0.5$)

code rate $= 0.330$



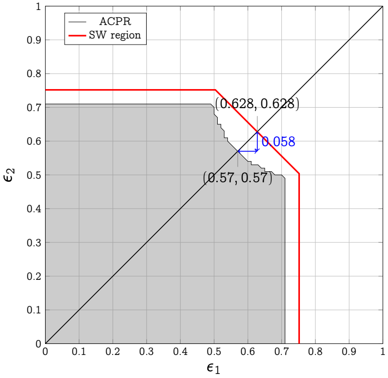
Results: BEC ($p = 0.5$)

code rate $= 0.330$

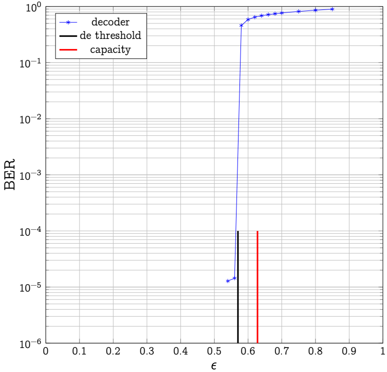


Results: BEC ($p = 0.5$)

code rate = 0.330



block length = 10^5



Thank You!

- J. Barros and S. Servetto, "The Sensor Reachback Problem," *submitted to IEEE Trans. Inform. Theory*, 2003.
- A. Yedla, H. D. Pfister, and K. R. Narayanan, "Can iterative decoding for erasure correlated sources be universal?" *47th Annual Allerton Conference on Communication, Control, and Computing.*, pp. 408–415, Sept. 2009.
- A. Abrardo, G. Ferrari, M. Martalo, M. Franceschini, and R. Raheli, "Optimizing Channel Coding for Orthogonal Multiple Access Schemes With Correlated Sources," in *Information Theory and Applications Workshop*, 2009, pp. 5–14.
- M. Martalo, G. Ferrari, A. Abrardo, M. Franceschini, and R. Raheli, "Density Evolution-Based Analysis and Design of LDPC Codes with A Priori Information," in *Information Theory and Applications Workshop*, 2010.